EE/Ae 157b
Homework #2
Due Date: February 5, 2020

Problem 1. (10 points)

A thin cylinder of length l and radius a oriented at an angle α relative to the horizontal axis is characterized by the following scattering matrix

$$[S] = \frac{k_0^2 l^3}{3\left[\ln(4l/a) - 1\right]} \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix}$$

Derive the expressions for the received power for the co-polarized and cross-polarized cases for linear polarizations. For the co-polarized case, the antenna vectors are

$$\hat{\mathbf{p}}^t = \hat{\mathbf{p}}^r = \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}$$

And for the cross-polarized case, they are

$$\hat{\mathbf{p}}^t = \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}; \quad \hat{\mathbf{p}}^r = \begin{bmatrix} \sin\psi \\ -\cos\psi \end{bmatrix}$$

Now find the polarization orientation angle that would maximize and minimize the received power for each case. Is it possible to measure the orientation of the cylinder using a polarimetric radar?

Problem 2 (20 Points)

Calculate the covariance matrix for the cylinder with orientation as given in the previous problem in the backscatter case. Remember that in the *backscatter* case *only*, one can show that for linear media $S_{hv} = S_{vh}$. In this case, we define the scattering vector as

$$\vec{S} = \begin{pmatrix} S_{hh} \\ \sqrt{2}S_{hv} \\ S_{vv} \end{pmatrix}$$

Use this form when doing this problem. We define the covariance matrix of a scatterer as

$$C = \vec{S}\vec{S}^{\dagger}$$

Here the † sign means complex conjugate and transposed.

Now let us consider a collection of thin cylinders that are oriented statistically in a plane orthogonal to the direction in which the wave is propagation. We shall assume the cylinders are oriented according to the probability density function

$$p(\alpha)=\frac{1}{\pi}\cos^2\alpha; \quad 0\le\alpha\le 2\pi$$

Calculate the average covariance matrix for this collection of cylinders.

Next, repeat the exercise for the case where the cylinders are uniformly randomly oriented in the interval $[0,2\pi]$.

Problem 3. (10 points)

Use the expressions for the small perturbation model and plot the ratio σ_{hh}/σ_{vv} as a function of ε for incidence angles θ = 20°, 40° and 60°. Use the range 2 - 30 for the dielectric constant. Do you think this ratio can be used to measure the dielectric constant? For which dielectric constants will the measurement be most accurate?