

Heat Capacity of a Current Carrying Superconductor

David L. Goodstein^a, Alexa W. Harter^a, and Talso C. P. Chui^b

^aCondensed Matter Physics, California Institute of Technology, Pasadena, CA 91125

^bJet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109

Abstract

We present an analysis of the heat capacity of a superconductor carrying a constant applied electric current. We find that the heat capacity diverges with an exponent of 0.5 at a depressed transition temperature. This result is similar to a recent calculation of the heat capacity of superfluid helium under an applied heat current.

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The Ginzburg-Landau (GL) equations¹ give a remarkably accurate description of superconductors in the vicinity of the superconducting transition. It is well known that the GL equations predict that superconductivity will break down at a critical current that occurs when the velocity of the Cooper pairs, v_s , is²

$$v_{s,c} = \frac{1}{\sqrt{3}} \frac{\Delta}{p_F} \quad (1)$$

where Δ is the energy gap and p_F the Fermi momentum. It does not seem to have been noticed, however, that in a current-biased sample, the same physics leads to a divergent heat capacity. The argument is as follows.

The free energy density of a uniform sample is given by GL as

$$f = f_0 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \gamma|\nabla\psi|^2 \quad (2)$$

where f_0 pertains to the normal state. If the sample carries a uniform current, the order parameter is given by

$$\psi = \psi_0 e^{i\vec{k}\cdot\vec{r}} \quad (3)$$

where ψ_0 and \vec{k} are constant. Then, minimizing f with respect to ψ^* yields

$$f = f_0 - \frac{(\alpha + \gamma k^2)^2}{2\beta} \quad (4)$$

where, according to the usual GL arguments, β is a constant,

$$\alpha = -a_0(T_c - T) \quad (5)$$

where a_0 is a positive constant,

$$\gamma k^2 = \frac{1}{2} m^* v_s^2 \quad (6)$$

where m^* is the mass of a Cooper pair, and the number density of Cooper pairs is given by

$$n_s = -\left(\frac{\alpha + \gamma k^2}{\beta}\right). \quad (7)$$

The super current is then given by

$$J_s = n_s e^* v_s \quad (8)$$

where e^* is the charge on a Cooper pair. It follows that,

$$\left(\frac{\partial J_s}{\partial v_s}\right)_T \sim -(\alpha + 3\gamma k^2). \quad (9)$$

At small k (or v_s , which is equivalent to k according to Eq (6)) this derivative is positive and nearly constant, but because n_s is reduced by increasing k according to Eq (7), the derivative is driven to zero at a critical value of k , and the super current becomes unstable along a curve in the $T - k$ plane given by

$$\alpha + 3\gamma k^2 = 0. \quad (10)$$

Equation (10) reduces to Eq (1) if we recognize that $(-\alpha/\gamma)^{1/2} = \Delta/\hbar v_F$ where v_F is the Fermi velocity.

It follows from Eq (4) that the entropy density in the $T - k$ plane is given by

$$s = s_0 + \frac{a_0}{\beta}(\alpha + \gamma k^2) \quad (11)$$

At any constant value of k , this gives rise to the usual finite step in the heat capacity,

$$C_s = C_n + \frac{a_0^2 T}{\beta} \quad (12)$$

where C_s and C_n are the superconducting and normal heat capacities. However, if instead of holding k constant we hold J_s constant (as in a current-biased sample) then

$$C_{J_s} = C_n + \frac{a_0^2 T}{\beta} + \frac{2a_0 \gamma k}{\beta} \left(\frac{\partial k}{\partial T}\right)_{J_s}. \quad (13)$$

But, using Eqs. (7) and (8),

$$\left(\frac{\partial k}{\partial T}\right)_{J_s} = -\frac{a_0 k}{\alpha + 3\gamma k^2}. \quad (14)$$

It follows that C_{J_s} diverges along the critical current curve given by Eq (10). If we write the

solution of Eq (10) as a curve in the $T-k$ plane, $T_c(k)$ then it is easily shown that³ the singular contribution is proportional to $(T_c(k) - T)^{-1/2}$.

Behavior analogous to this has been predicted to occur in superfluid helium in a constant heat flux.³ However, in the case of superfluidity, this kind of mean field theory is not dependable. The phenomenon is particularly interesting in the case of superconductivity, because in this case the GL equations are thought to be valid without correction for thermal fluctuations, and because the heat capacity at constant current is relevant to device applications such as transition-edge bolometers.⁴ We know of no other instance in which a mean field theory predicts a power-law divergence in the heat capacity. It should also be noted that the same analysis leads one to expect that the mean-square velocity fluctuations, $\langle v_s^2 \rangle$, should also diverge along $T_c(k)$.

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