Recap of Basic Electronics

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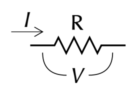
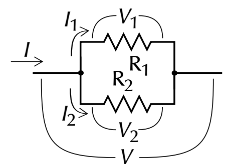
Introduction

The first part of “electrophysiology” is electronics. For many of you it may have been a while since you had a resistor in your hands, so this tutorial provides a refresher.

Two of the most central concepts in electronics are *voltage* (V) and *current* (I). Current flows *through* circuit elements, voltage stands *across* circuit elements:



That means that when two circuit elements are connected in a *parallel* fashion:



the total current is divided (not necessarily fifty-fifty) between the two branches:

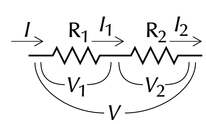
$$I = I_1 + I_2, \qquad \text{(parallel)}$$

while the voltage across each of the two elements is the same:

$$V_1 = V_2 = V. \qquad \text{(parallel)}$$

A helpful analogy is a watershed. If you turn on a firehose on the top of Mount Wilson, a certain amount of water will flow toward the edge of the plateau. There, the flow will split among several channels, and the total rate of flow in all of those channels will be the same as the total flow across the plateau. Current in electronic circuits behaves in a precisely analogous way. In the same analogy, elevation plays the role of voltage, and the above equation for voltages in parallel branches is simply the observation that whether you hike up to Mt Wilson from Caltech by way of the Toll Road, the Mt Wilson Trail, or some other route, you always end up with the same elevation gain.

Circuit elements can also be connected in *series*:



Just like the rate of water flow in two waterfalls of the same stream is going to be the same[1], the current flowing through these two circuit elements will be the same:

$$I_1 = I_2 = I. \qquad \text{(series)}$$

And just as the total elevation gain along the whole Toll Road is the sum of the part below and the part above Henninger Flats, voltage adds across two circuit elements connected in series:

$$V = V_1 + V_2. \qquad \text{(series)}$$

[1] If we ignore evaporation and water seeping into the rock; electrons do not evaporate and do not disappear into the air.

Resistors

The above equations hold irrespective of the details of the particular circuit elements involved, but to relate currents with voltages, those details are important. In the diagrams above, you may have recognized -\/\/\/- as the symbol for a *resistor*. A resistor is usually viewed as a device that will impede current, resulting in a voltage in proportion to its resistance value:

$$V = IR.$$

That is the classic formulation of Ohm's law. A complementary view is to characterize a resistor as a conductor (albeit not as good a conductor as a wire). In that view, we can define *conductance* as the inverse of resistance:

$$g = 1/R,$$

and, flipping Ohm's law upside down, say that the resistor allows a current to flow in proportion to its conductance:

$$I = gV.$$

This second view is often useful in electrophysiology, where a cell membrane contains a large collection of ion channels, which are essentially resistors in parallel. To see why that is, let us consider how the voltage drop across series and parallel combinations of resistors work out.

As we saw above, in a series connection, the total voltage drop is given by $V = V_1 + V_2$, while the current through each of the resistors is the same. For each of the resistors we have $V_i = IR_i$, which means that $V = IR_1 + IR_2$. This can be written as $V = I(R_1 + R_2)$, which has the same form as Ohm's law. Thus we can say that the "equivalent" resistance of the entire circuit is

$$R = R_1 + R_2. \qquad \text{(series)}$$

By contrast, for the parallel connection, the voltage drop is the same across both elements, while the current adds: $I = I_1 + I_2$. Thus, it is convenient to consider that $I_i = g_i V$, which means that $I = g_1 V + g_2 V = (g_1 + g_2)V$. This allows us to write the equivalent conductance of the entire circuit as:

$$g = g_1 + g_2. \qquad \text{(parallel)}$$

Exercise: Write out the equation for the parallel combination of two resistors in terms of their resistance values.

Capacitors

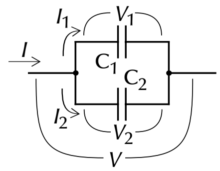
Let's consider one other circuit element that is important for electrophysiology: the capacitor (−||−). A capacitor is a device that consists of two parallel conducting plates with an insulator between them. Current can flow into and out of it, but not through it. When current (I) flows into the capacitor, charge (Q) builds up on one of the plates:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = I.$$

Because like charges repel and opposite charges attract, the result is a build-up of opposite charge on the other plate. Depending on the size of the plates and the spacing between them, this charge build-up corresponds to a certain amount of voltage drop across the capacitor: $V = \frac{1}{C}Q$. The coefficient C is the *capacitance* of the device, and we can usefully summarize the behavior of the capacitor as:

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = I.$$

Let's consider how capacitances add in circuit combinations.

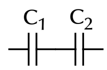


Just as for the parallel combination of two resistors, we have $I = I_1 + I_2$ while $V_1 = V_2 = V$. In this case, that means $I = C_1 \frac{\mathrm{d}V}{\mathrm{d}t} + C_2 \frac{\mathrm{d}V}{\mathrm{d}t} = (C_1 + C_2) \frac{\mathrm{d}V}{\mathrm{d}t}$. Thus we can usefully write:

$$C = C_1 + C_2 \qquad \text{(parallel)}$$

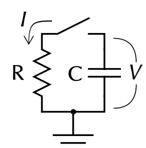
for the equivalent capacitance of two capacitors in parallel.

Exercise: figure out how capacitances add in a series combination:



Applications

Now, let's consider some practical circuits that combine resistors and capacitors and see how they behave in time. Let's start very simply:

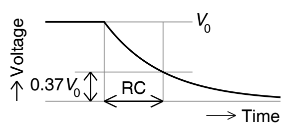


Assume that we have previously charged up the capacitor to a voltage V_0, and that at time $t = 0$ we close the switch. Prior to that time, no current flows, so the voltage across the capacitor is preserved. After the switch closes, current starts to flow out of the capacitor and through the resistor. The relationship between current and voltage for the resistor is $V_R = I_R R$, while for the capacitor we have $C \frac{\mathrm{d}V_C}{\mathrm{d}t} = I_C$. From the way the diagram is drawn, it is clear that $V_R = V_C$ (remember how the total elevation gain between campus and Mt Wilson is independent of your path), while $I_R + I_C = 0$ (since there is no current flow into or out of the total circuit). In summary:

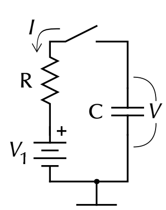
$$C \frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{R} V.$$

Together with the initial condition $V(t = 0) = V_0$, this may readily be integrated to

$$V(t) = V_0 \, \mathrm{e}^{-t/(RC)}.$$



Exercise: A battery (−||+) is a device that maintains a constant voltage between its terminals and that produces whatever current necesary to do so. What would the above equation become if we add a battery (voltage: V_1) to the circuit, as in the diagram below:



Hint: think of what the voltage across the capacitor will be once current stops flowing.

Practical considerations: identifying resistors and capacitors

The value of a resistor is encoded in the colored bands around the resistor. Typically, there are four bands. The resistor code is read with the gold band to the right. The other bands are read as XY x $10^Z\Omega$, where the values of X, Y, and Z are: black: 0, brown: 1, red: 2, orange: 3, yellow: 4, green: 5, blue: 6, violet: 7, grey: 8, white: 9. Thus, a resistor labeled yellow-violet-red-gold has a a value of 47 x $10^2\Omega$ = 4.7 kΩ. (The gold means that the resistor is defined to 5% accuracy.)

In practice, only resistor values of 10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, and 82 times some power of ten are common. That knowledge can help if the color is not too clear. For instance, if you are unsure if a resistor is colored brown-black-green or violet-black-green, it is probably brown-black-green.

The value of a capacitor is written on it in tiny print. There are two styles: one has written XYZ on it, which means XY x 10^Z pF. For instance "223" means 22 nF. The other style looks like XYJ, which means XY pF. For instance 56J means 56 pF. Note that there are commonly lots of other numbers and letters printed on capacitors that indicate (largely irrelevant) material properties. It can be somewhat challenging to find the relevant set of scribblings.

Answers to exercises

- The parallel combination of two resistors results in:

$$R = \frac{1}{1/R_1 + 1/R_2}. \qquad \text{(parallel)}$$

  (Note that the two forms can be interconverted through the relationship $g = 1/R$.)

- The series combination of two capacitors results in:

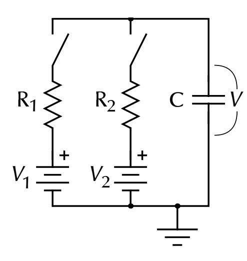
$$C = \frac{1}{1/C_1 + 1/C_2}. \qquad \text{(series)}$$

  That is, capacitances add like conductances rather than like resistances.

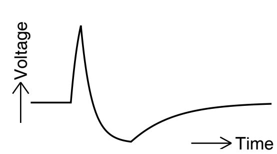
- Adding a battery to the RC circuit results in:

$$V(t) = V_1 + (V_0 - V_1)\,\mathrm{e}^{-t/(RC)}.$$

  Taking it one step further, with appropriate timing of opening and closing the two switches, a circuit like this:



  can lead to voltage across the capacitor like this:



  It is not an accident that this looks a lot like a classic action potential shape (and if you allow the switches to open and close gradually, the approximation becomes even better).