

Optimal Allocation to Hedge Funds : An Empirical Analysis

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Abstract

What percentage of their portfolio should investors allocate to hedge funds? The only available answers to the above question are set in a static mean-variance framework, with no explicit accounting for uncertainty on the active manager's ability to generate abnormal return, and usually generate unreasonably high allocations to hedge funds. In this paper, we apply the model introduced in Cvitanić, Lazrak, Martellini and Zapatero (2002b) for optimal investment strategies in the presence of uncertain abnormal returns to a database of hedge funds. We find that the presence of model risk significantly decreases an investor's optimal allocation to hedge funds. Another finding of this paper is that low beta hedge funds may serve as natural substitutes for a significant portion of an investor risk-free asset holdings.

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1 Introduction

One of the by-products of the bull market of the 90's has been the consolidation of hedge funds as an important segment of financial markets. The value of the hedge fund industry worldwide is estimated at more than 500 billion dollars distributed among over 5,000 hedge funds.¹ The majority of institutional investors seems to be moving towards holding hedge funds in their portfolios.² There seem to be two main reasons behind the success of hedge funds (see Amenc, Curtis and Martellini (2001) and Schneeweis and Spurgin (1999) for a detailed study). On the one hand, hedge funds seem to provide diversification with respect to other existing investment possibilities (beta benefit). On the other hand, it is argued that hedge funds provide an abnormal return adjusted by risk (alpha benefit).

In a nutshell, the diversification argument states that most hedge funds have a low beta with respect to traditional stock and bond indexes. This is reported for example in Schneeweis and Spurgin (1999, 2000) and Agarwal and Naik (2001), at least for some hedge fund strategies (the so-called "non directional strategies").³ The reason is that hedge funds can take advantage of shortselling and include derivatives and other non-traditional asset classes in a way that is not allowed to mutual funds. The second argument says that the alpha of hedge funds is positive. There is a growing literature on the measurement of hedge fund risk-adjusted performance. We mention here Ackermann, McEnally, and Ravenscraft (1999), Agarwal and Naik (2000a, 2000b, 2001), Amenc, Curtis and Martellini (2001), Amin and Kat (2001), Bares, Gibson and Gyger (2001), Brown and Goetzmann (1997, 2001), Brown, Goetzmann, and Ibbotson (1999), Brown, Goetzmann and Park (1997), Edwards and Liew (1999), Fung and Hsieh (1997a, 1997b, 2000), Lhabitant (2001)) Liang (2000, 2001), Schneeweis and Spurgin (1999, 2000) and Schneeweis, Spurgin and McCarthy (1996). Although these studies differ in the data and models they use and their results are not completely homogeneous, most of them conclude that there is some evidence of abnormal performance, at least in some segments of the hedge fund industry.

A standard way to present the benefits of hedge fund investing is to show the improvement they allow for in a mean-variance analysis. For example, Schneeweis and Spurgin (1999) construct a mean-variance frontier with the S&P500, the Lehman Brothers Bond Index and a hedge fund index, the EACM 100.⁴ The conclusion of this study is that the inclusion of hedge funds greatly improves the mean-variance frontier of investment possibilities. In fact,

¹Frank Russell-Goldman Sachs survey (1999).

²According to the Gollin, Harris, Ludgate (2001) survey, 64% of the European institutions included in the study invest in hedge funds. This is up from 56% in 2000.

³Agarwal and Naik (2001) report evidence of higher correlation between hedge fund returns and equity market returns in bear markets as opposed to bull markets. This suggests that conditional betas tend to be higher than unconditional betas.

⁴The EACM 100 is, arguably, the most widely used index for the performance of hedge funds. For more information on this index and an overview of existing hedge fund indices, see Amenc and Martellini (2002b).

the in-sample Sharpe ratio of hedge funds seems to be so superior to the Sharpe ratios of the two other funds considered in this paper that the optimal investment strategy of an investor that uses mean-variance analysis should be to invest almost all the risky part of the portfolio in hedge funds. We view this normative prescription of an almost exclusive investment in hedge funds as evidence of the failure of in-sample static mean-variance analysis to generate a reasonable asset allocation including both traditional and alternative investment vehicles.⁵

Although the existing literature seems to grant the interest of hedge funds as valuable investment alternatives, there seems to be several other shortcomings in the presentation of the advantages of including hedge funds in an investor's asset allocation. On one hand, as it is clear from the arguments presented above, the analysis is in general cast in a mean-variance setting. This is very restrictive because of all the well known assumptions on the preferences of the investors and/or returns of the securities that are necessary in order to make this setting appropriate. Furthermore, the analysis is static and rules out the possibility of non-myopic behavior. On the other hand, this analysis relies heavily on good estimates of expected returns. This problem seems to be far from solved (see Britten-Jones (1999)). A related issue is the difficulty of measuring the alphas of hedge funds. Amenc, Curtis and Martellini (2001) estimate alphas across several models and conclude that their quantification varies greatly with the different models.

What percentage of their portfolio should investors allocate to alternative investment vehicles given their low betas and uncertain potential for positive alphas? In this paper we apply the model developed and analyzed in Cvitanić, Lazrak, Martellini and Zapatero (2002a,b) to a database of hedge funds. Hedge fund offers some superior performance measured by its alpha. In the model we use, a non-myopic investor with incomplete information allocates wealth between a riskfree security, a passive portfolio and an actively managed portfolio. A related paper is Baks, Metrick and Wachter (2001). They test a model of actively managed funds with heterogeneous priors about the alphas. However, investors in that model maximize quadratic utilities: the hedging component of the investment strategy of the investor in our paper is a key factor of the investment decision. Also, we use a different database for our test.⁶ An important finding of this paper is that low beta hedge funds may serve as natural substitutes for a significant portion of an investor risk-free asset holdings.

The paper is organized as follows. In the next section we present the model and optimal investment strategy of an investor with incomplete information. In section 3, we present the data, and perform the calibration of the model and derive empirical results in section 4. We close the paper with some conclusions. Some information on hedge fund strategies can be

⁵Amenc and Martellini (2002a) perform out-of-sample testing of an improved asset return covariance matrix estimate and hedge fund allocation decisions.

⁶We focus on hedge funds while Baks, Metrick and Wachter (2001) focus on mutual funds.

found in a dedicated appendix.

2 Optimal Investment with Uncertain Alphas

In this section we briefly present the model and optimal investment strategy. The details, derivation and comparative statics can be found in Cvitanić, Lazrak, Martellini and Zapatero (2002b).

2.1 The Model

Uncertainty about risky asset prices in the economy is represented by a standard filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$, on which is defined a 2-dimensional Brownian motion $W = (W^1, W^2)$. We assume that the investor can choose among three assets, a risk-free asset B_t , and two risky assets. The first of them has a price that we denote by P_t and we interpret it as a passive fund, a fund that tracks a broad-based index such as the S&P500, that is a proxy for the market portfolio. The second security, whose price we denote by A_t , is a hedge fund. The dynamics of the price of these assets are given by

$$\begin{aligned} dB_t &= B_t r dt \\ dP_t &= P_t (\tilde{\mu}_P dt + \sigma_P dW_t^1) \\ dA_t &= A_t (\tilde{\mu}_A dt + \sigma_1 dW_t^1 + \sigma_2 dW_t^2) \end{aligned}$$

where the invertible volatility matrix $\sigma = \begin{pmatrix} \sigma_P & 0 \\ \sigma_1 & \sigma_2 \end{pmatrix}$ as well as the interest rate r are assumed to be constant. Alternatively, we could rewrite the dynamics of the prices as depending in a single Brownian motion each, but with correlation $\rho = \frac{\sigma_1}{\sqrt{\sigma_1^2 + \sigma_2^2}}$ and volatility of the active fund $\sigma_A := \sqrt{\sigma_1^2 + \sigma_2^2}$. The previous notation allows simpler representation and interpretation of the optimal investment strategy. We focus on lognormal processes, although the model can in principle be solved for more general processes with stochastically time-varying drift and volatilities.⁷

In this setting, there is a risk averse investor with initial wealth X_0 who has access to the three securities described above and that maximizes utility of final wealth, given by the expression

$$u(X_T) = \frac{(X_T)^a}{a} \tag{1}$$

For $a = 0$ this would be logarithmic -myopic- utility. We focus in the more interesting case $a < 0$.

⁷There is evidence of departure from normality in hedge fund returns (Agarwal and Naik (2000), Fung and Hsieh (1997a, 2000), Amin and Kat (2001) or Lo (2001)).

We assume that the investor does not observe neither the constant mean returns vector $\tilde{\mu} = (\tilde{\mu}_1, \tilde{\mu}_2)$ nor the source of noise W but observes the price processes (P, A) . Therefore the investor's information consists of the \mathbb{P} -augmentation of the filtration

$$\mathcal{F}_t := \sigma(P(s), A(s); \quad 0 \leq s \leq t)$$

generated by the price process $(P(t), A(t)); t \in [0, T]$. Define the “risk premium” vector process

$$\theta(t) := \sigma^{-1}(t)[\tilde{\mu}(t) - r(t)\mathbf{1}]$$

where $\mathbf{1} = (1, 1) \in \mathbb{R}^2$. In other words, $\theta = (\theta_P, \theta_A)$ with

$$\begin{aligned} \theta_P &= \frac{\tilde{\mu}_P - r}{\sigma_P} \\ \theta_A &= -\frac{\sigma_1}{\sigma_P \sigma_2}(\tilde{\mu}_P - r) + \frac{\tilde{\mu}_A - r}{\sigma_2} \end{aligned}$$

We assume that vector θ has a normal prior distribution

$$\theta \sim \mathcal{N} \left(\phi = \begin{pmatrix} \phi_P \\ \phi_A \end{pmatrix}, \Delta = \begin{pmatrix} \delta_P & 0 \\ 0 & \delta_A \end{pmatrix} \right)$$

with mean vector ϕ and variance-covariance matrix Δ and we assume that θ is independent of W . Here we assume that the priors are independent, as can be seen from the fact that the off-diagonal terms in Δ are zero. In Cvitanić, Lazrak, Martellini and Zapatero (2002b) we consider the general case of correlated priors.

As explained in the introduction, the growth of hedge funds is partially motivated by the possibility of risk-adjusted abnormal returns. With that motivation in mind, we further specify the dynamics of the active portfolio. In particular, we decompose the drift of the active portfolio into the sum of two elements, a normal return component and an abnormal return component, or

$$\tilde{\mu}_A = \underbrace{r + \beta(\tilde{\mu}_P - r)}_{\text{normal return}} + \underbrace{\tilde{\alpha}}_{\text{abnormal return}} \quad (2)$$

where $\beta = \frac{\sigma_1 \sigma_P}{\sigma_P^2} = \frac{\sigma_1}{\sigma_P}$ is the standard beta of the active fund with respect to the passive fund, regarded as a broad-based index, a proxy for the market portfolio (the S&P500 in the empirical tests of the model). Therefore, in order to measure risk-adjusted abnormal returns we use the CAPM framework.⁸

⁸We do not assume that CAPM holds, but merely use its terminology.

2.1.1 Optimal Allocation to Hedge Funds

We assume that the investor considers the assets with prices P and A as possible investment vehicles, with uncorrelated priors on their expected returns m_P and m_A , respectively. The following proposition provides a simple expression for the optimal holding in the passive and active portfolios.

Proposition 1 *Under assumptions and notation specified in section 1, in the uncorrelated case ($\gamma = 0$), the optimal holdings in the active and passive portfolios can be expressed in the following form*

$$\pi_A(0) = \frac{\alpha_0}{\sigma_2^2(1 - a - a\delta_A T)} \quad (3)$$

$$\pi_P(0) = \frac{m_P - r}{\sigma_P^2(1 - a - a\delta_P T)} - \beta\pi_A(0) \quad (4)$$

where $\alpha_0 = m_A - r - \beta(m_P - r)$ is the date 0 expected value of the abnormal return alpha of the active fund for the investor with incomplete information, $\sigma_2 = \sqrt{\sigma_A^2 - \beta^2\sigma_P^2}$ is the residual, or specific, component in the volatility of the active portfolio, and m_P (resp. m_A) is the prior on the passive (resp. active) portfolio expected return.

P roof. See Cvitanić, Lazrak, Martellini and Zapatero (2002b). ■

As expected, an increase in the expected alpha leads the investor to hold more of the active portfolio, everything else equal. On the other hand, an increase in the uncertainty around alpha leads the investor to hold less (or short less) of the active portfolio, everything else equal.⁹ An increase in the time-horizon also leads the investor to hold less (or short less) of the active portfolio. On the other hand, when there is no uncertainty around alpha, the solution is time-horizon independent. Finally, an increase in the specific risk of the active portfolio leads the investor to hold less (or short less) of it, everything else equal.

An important question that investors in hedge funds often ask is where should they take the money they are planning to allocate to the hedge fund from. The following proposition provides very simple insights into the question.

Proposition 2 *Define the optimal holdings in the passive fund and risk-free asset in the absence of the active portfolio as $\pi_P^0(0)$ and $\pi_B^0(0)$. The changes in holdings due to the introduction of the active portfolio are*

$$\begin{aligned} \Delta\pi_P(0) & : = \pi_P^0(0) - \pi_P(0) = \beta\pi_A(0) \\ \Delta\pi_B(0) & : = \pi_B^0(0) - \pi_B(0) = (1 - \beta)\pi_A(0) \end{aligned}$$

⁹Note that δ_A is the variance of the prior on the risk-premium, not on the expected return. The uncertainty of the prior on the active portfolio is $\sigma_{\mu_A}^2 = \sigma_A^2\delta_A$. This is also equal to the uncertainty on α in the absence of uncertainty on μ_P .

with $\pi_A(0)$ given in equation (3). Moreover, when $\pi_A(0)$ is positive (i.e., when α_0 is positive), we have that

$$\Delta\pi_B(0) \geq \Delta\pi_P(0) \iff \beta \leq \frac{1}{2}$$

P roof. See Cvitanić, Lazrak, Martellini and Zapatero (2002b). ■

We find that the introduction of the active fund leads investors to optimally withdraw an amount from the money market account larger than that taken out of the passive fund when the active fund has a beta lower than 1/2. Intuitively, this is because the active portfolio becomes less (more) comparable to the passive fund as its beta decreases (increases). In other words, this result suggests that low beta hedge funds may actually serve as natural substitutes for a significant portion of an investor risk-free asset holdings, while high beta hedge funds can be regarded as substitutes for a portion of equity holdings.

Neither the prior on the expected return of the passive fund asset, nor volatility of that fund, have any impact on that decision. It should be noted that the condition $\beta \leq \frac{1}{2}$ holds for most non-directional hedge fund strategies. It is for example satisfied for all 10 hedge fund indices in our sample (see table 1 below). This, on the other hand, would be relatively unusual for traditional long-only active strategies.

In the next section we calibrate the model to hedge fund returns data.

3 Methodology and Description of the Data

We use a proprietary data base of individual hedge fund managers, the Managed Account Reports or MAR database, to calibrate and test the model. The MAR database contains monthly returns on more than 1,500 offshore and onshore hedge funds and managers usually select their own categories. There are 9 categories (“medians”), some of which are divided into sub-categories (“submedians”): Event- Driven Median (Distressed securities sub-median, Risk arbitrage sub-median), Global Emerging Median, Global International Median, Global Established Median (Global Established growth sub-median, Global Established small-cap sub-median, Global Established value sub-median), Global Macro Median, Zürich Market Neutral Median (Market Neutral arbitrage sub-median, Market Neutral long/short sub-median, Market Neutral mortgage-backed sub-median), Sector Median, Short-Sellers Median, Fund of Funds Median (Fund of Funds diversified sub-median, Fund of Funds niche sub-median). MAR has recently been acquired by Zürich Capital Market.

In this study, we focus on 581 hedge funds + 10 indices and sub-indices in the MAR database that have performance data as early as 1996. It should be noted that using a specific sample from an unobservable universe of hedge funds introduces biases in performance measurement. There are three main sources of difference between the performance of hedge funds in the

data base and the performance of hedge funds in the population (see Fung and Hsieh (2001a)): survivorship bias, selection bias, instant history bias.¹⁰ Overall, it is probably a safe assumption to consider that these biases account for a total approaching at least 4.5% annual (see Park, Brown and Goetzmann (1999) and Fung and Hsieh (2001a)).

More specifically, the explicit solution derived in section 2 allows us to quantify the relationship between the optimal allocation to hedge funds and managerial skill with uncertainty around this managerial skill. There are actually at least three reasons why the abnormal return, or $\tilde{\alpha}$, generated by managers can not be known with certainty by investors.¹¹

- Model risk: for a given fund and a given sample, estimates around alpha vary with the model under consideration
- Sample risk: for a given fund, and a given model, estimates of alpha vary with the sample under consideration
- Selection risk: for a given model and a given sample, estimates of alpha vary with the fund under consideration

The first source of uncertainty around managerial skill is due to the fact that investors do not have a dogmatic belief in one particular model but instead are uncertain about the true model they should use to measure risk-adjusted performance. In that sense, uncertainty around managerial skill may be calibrated from the variation of performance measurement across models. The second source of uncertainty around managerial skill is estimation risk that affects both the passive and the active funds. The third source of uncertainty arises from the fund picking problem.¹² Even if investors are ready to believe that there are fund managers who are able to generate positive alphas, they do not necessarily know which ones, and past risk-adjusted performance, while providing some guidance, is not enough to ensure that fund picking risk can be hedged away.¹³

In the absence of meaningful estimates of the magnitude of selection risk, we focus on model and sample risks in this paper. Sample risk is measured in terms of usual parameter

¹⁰Survivorship bias occurs when unsuccessful managers leave the industry, and their successful counterparts remain, leading to the counting of only the successful managers in the database. Selection bias occurs if the hedge funds in the database are not representative of those in the universe. Besides, when a hedge fund enters into a vendor data base, the fund history is generally backfilled. This gives rise to an instant history bias.

¹¹Investors in hedge funds typically try and reduce uncertainty around estimates of alphas by pursuing a lengthy and costly qualitative due diligence process.

¹²It is a very different situation for an investor to pick with certainty a fund behaving like the average fund in the data base or randomly pick some fund in the data base.

¹³See Brown, Goetzmann and Ibbotson (1999) or Agarwal and Naik (2000) for results on the persistence of hedge fund performance.

uncertainty, using t-stat values as a measure of dispersion around point estimates. Obtaining an estimate of model uncertainty, on the other hand, is less straightforward. In this paper, we use 5 different asset pricing models to compute a fund abnormal return and use the dispersion in alphas across models as a measure of model uncertainty. In other words, we use as a prior for the unknown alpha of a given fund an equally-weighted average of posterior estimates for that alpha from different competing models that have been used in the literature on hedge fund performance. These models are listed below.

1. CAPM. This is a standard version of Sharpe (1964) CAPM. We use the S&P 500 as a proxy for the market portfolio.
2. CAPM with stale prices. We adjust standard CAPM market beta by running regressions of returns on both contemporaneous and lagged market returns given that, in the presence of stale or managed prices, simple market model types of linear regressions may produce estimates of beta that are biased downward (Scholes and Williams (1977), Dimson (1979), Asness, Kraill and Liew (2001)).
3. CAPM with non-linearities. Because hedge fund portfolios typically involve nonlinear and/or dynamic positions in standard asset classes, we also apply Leland (1999) performance measurement for situations in which the portfolio returns are highly nonlinear in the market return.
4. Explicit single-index factor model. We test a single-factor model, where the return on an equally-weighted portfolio of hedge funds in the same style category is used as a factor (we perform objective cluster-based classification, as opposed to rely on managers' self-proclaimed styles).
5. Explicit multi-index factor model. Building on an approach initiated by Sharpe (1964, 1988, 1992), or Fama and French (1992), we use market indices as proxies for true unknown factors. Since hedge fund returns exhibit non-linear option-like exposures to standard asset classes, traditional style analysis offer limited help in evaluating the performance of hedge funds (Fung and Hsieh (1997, 2000)). A possible remedy has been suggested to try and capture such non-linear dependence is to include new regressors with non-linear exposure to standard asset classes to proxy dynamic trading strategies in a linear regression.¹⁴ Natural candidates for new regressors are buy-and-hold positions in derivatives (Schneeweis and Spurgin (2000), Agarwal and Naik (2001) or Fung and Hsieh (2001)), or hedge fund indices (Lhabitant (2001)). Here, we follow the latter

¹⁴Alternatively, one may allow for a non-linear analysis of standard asset classes. A portfolio interpretation, may, however, no longer be available.

approach and use the CSFB/Tremont indices which are currently the industry’s only asset-weighted hedge fund indices.¹⁵

We also introduce a “method 0” alpha for each fund, which is simply the excess mean return. This is the common practice for hedge fund managers who claim the risk-free rate should be used as a benchmark, and receive incentive fees based on performance of their fund over the risk-free rate. Note that, while commonly used in practice, the mean excess return is a meaningful definition of alpha only under the two restrictive assumptions that CAPM is the true model and the beta of the fund is zero.

For the purpose of illustrating the model of asset allocation between a passive and an active fund, we apply these 5 models to hedge fund indices and individual hedge funds on the period 1996-2000. Because we can not present results on as many as 581 funds, we focus on 10 indices. These indices represent the return on an equally-weighted portfolio of hedge funds pursuing respectively event driven strategies (with sub-categories distressed and risk), market neutral strategies (with sub-categories arbitrage and long/short), short-sales strategies, and fund of funds strategies (with sub-categories niche and diversified). We refer the reader to the Appendix for more information on hedge fund strategies).

In table 1, we present the summary statistics for these indices (beta with respect to the S&P500, mean return, total volatility, systematic volatility and specific volatility), as well as for an average fund, which is a hypothetical fund exhibiting the average characteristics of all funds in the data base.¹⁶

We check, for example, that betas for market neutral indices are very close to zero, while the beta on the short-sale index is negative, as it should be. This suggests that significant diversification benefit might be generated from the inclusion of that asset class in an equity portfolio.

In table 2, we present the alpha on each index obtained using the various afore-mentioned models, as well as the mean and standard deviation of these alphas. The latter quantity can be regarded as a real-world empirical estimate of uncertainty about alpha driven by model risk. As can be seen from these numbers, model risk induces a significant amount of uncertainty

¹⁵The CSFB/Tremont indices cover nine strategies (convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, fixed-income arbitrage, global macro, long-short equity, managed futures), and is based on 340 funds representing \$100 billion in invested capital, selected from a database, the TASS database, which tracks over 2,600 funds. Amenc and Martellini (2002b) have introduced a set of “pure style indices” and tested their superior power in the context of style analysis. We do not, however, use these pure style indices because data is not available before 1998.

¹⁶The reader should be cautioned that this average fund can not be regarded as an equally-weighted index of all funds in the data base. For example the 6.63% volatility of the so-called average fund is the average of all funds volatility, and not the volatility of a fund posting performance the equally-weighted average of the performance on each fund.

Strategy	Beta	Mean Return	Volat.	Systematic Risk	Specific Risk
Ev. Dist.	0.23	10.94	6.56	3.71	5.41
Ev. Risk	0.14	13.14	3.98	2.21	3.31
Ev. Driven	0.16	12.28	4.71	2.63	3.91
FoF Div.	0.24	12.31	6.26	3.83	4.95
FoF Niche	0.15	11.87	4.36	2.41	3.63
FoF	0.22	11.22	5.60	3.53	4.35
Mkt Neutr. Arb	0.06	16.62	10.58	0.90	10.54
Mkt Neutr. L/S	0.04	12.01	2.08	0.61	1.99
Mkt Neutr.	0.02	11.02	1.42	0.39	1.36
Short Sale	-0.91	6.37	20.71	14.62	14.68
Average	0.03	11.78	6.63	0.56	6.60

Table 1: Summary Statistics. This table displays in percentage terms the mean return and total, specific and systematic volatility on each index obtained from monthly data on the period 1996-2000 and converted into annual numbers. It also displays the beta of these indices with respect to the S&P500.

around the estimate for alpha. That uncertainty can be as high as 10% on this sample and for these indices, and higher than 30% for some funds in the data base (see figure 2)).

To get a better insight about how these average alpha values for the indices compare to those obtained on the sample of funds, we compute the distribution of average alpha across all hedge funds in the data base (see figure 1). The mean of that distribution is 2.77%, the standard deviation is 11.13%. (Note that we do not include here method 0; this is why the numbers are lower than in the following tables.) This seems to indicate that the average hedge fund is likely to generate positive risk-adjusted return, when the risk-adjustment is performed with an average of asset pricing models. The conclusion that hedge funds yield on average positive alpha needs, however, to be balanced by the presence of survivorship, selection and instant history biases, which account for a total approaching at least 4.5% annual, as recalled earlier. Therefore, the average alpha net of these biases is a negative $-1.73\% = 2.77\% - 4.5\% < 0$. On the other hand, 261 (out of 581 hedge funds) have an average alpha across methods larger than 5%, which seems to indicate the presence of positive abnormal return for at least some funds in the sample, even after accounting for the presence of the biases.

In the same vein, we compute the distribution of standard deviation of alpha across the sample of hedge funds (figure 2). The mean of that distribution is 5.77%, the standard deviation is 5.33%. It should be noted that one fund has a dispersion of alpha across methods larger than 30%.

We first perform an ex-post experiment, where we focus on model risk and assume away estimation risk. In other words, we address the following question: given ex-post alpha es-

Distribution of Average Alpha Across Hedge Funds

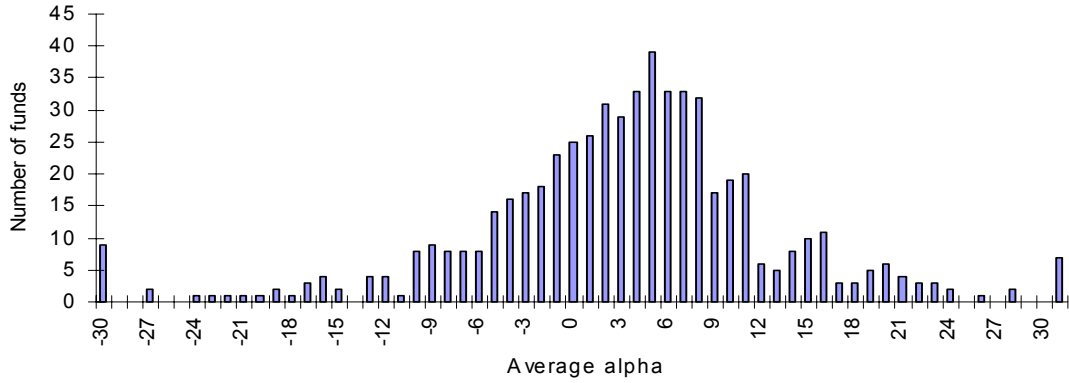


Figure 1: Cross-Section of Average Alphas. The mean of that distribution is 2.77%, the standard deviation is 11.13%.

Distribution of Standard Deviation of Alpha Across Hedge Funds

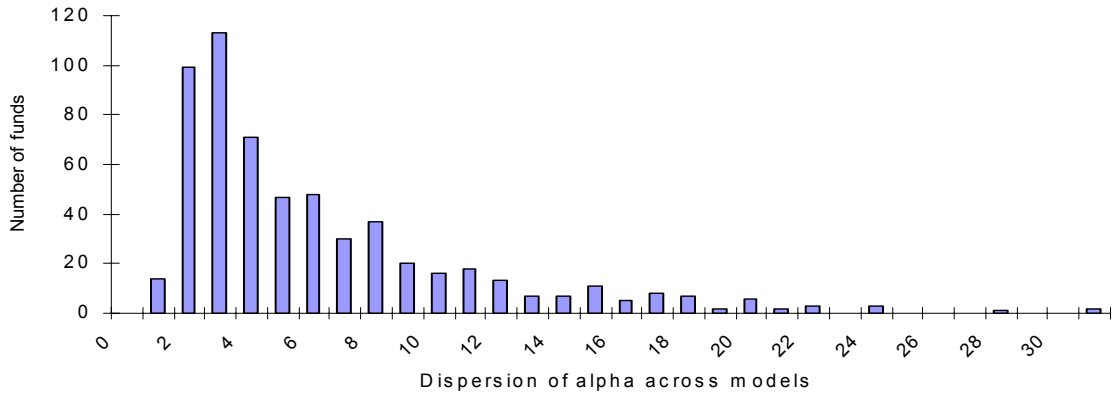


Figure 2: Cross-Section of Standard Deviation of Alphas. The mean of that distribution is 5.77%, the standard deviation is 5.33%.

Strategy	Meth 0	Meth 1	Meth 2	Meth 3	Meth 4	Meth 5	Mean Alpha	St. Dev. Alpha
Ev. Dist.	10.42	2.83	-0.68	2.20	1.53	-0.14	2.69	4.02
Ev. Risk	12.62	6.26	4.67	5.84	7.82	6.67	7.31	2.80
Ev. Driven	11.76	5.05	2.77	4.55	5.74	4.07	5.66	3.15
FoF Div.	11.80	4.10	0.93	3.73	-0.82	-2.01	2.96	4.96
FoF Niche	11.35	4.83	2.32	4.42	5.70	3.26	5.31	3.19
FoF	10.70	3.26	0.13	2.86	-0.20	-3.06	2.28	4.72
Mkt Neutr. Arb	16.10	10.82	9.66	10.47	7.16	12.04	11.04	2.96
Mkt Neutr. L/S	11.50	6.46	6.45	6.45	9.50	9.41	8.30	2.15
Mkt Neutr.	10.51	5.64	4.60	5.53	9.12	8.61	7.33	2.39
Short Sale	5.85	13.34	13.90	14.19	1.59	31.57	13.41	10.27
Average	11.26	6.26	4.48	6.02	4.72	7.04	6.63	2.47

Table 2: Hedge Fund Abnormal Performances. This table displays in percentage terms the annual alpha on each index obtained using different asset pricing models. It also displays the average alpha across methods, as well as the dispersion (standard deviation). Method 0 is the excess mean return. Method 1 is the CAPM alpha. Method 2 is an extension of CAPM to account for the presence of stale prices. Method 3 is Leland (1999) modification on CAPM alpha. Methods 4 and 5 are, respectively, a single- and multi-index factor models.

timates for 1996-2000, what would have been the optimal allocation to hedge funds in the period, accounting for the fact that different models disagree on alpha estimates? We then introduce estimation risk.

4 Empirical Results

We now proceed to the computation of optimal portfolio strategies for an investor facing an investment set including the risk-free asset (T-Bill), the S&P500, and one hedge fund index.

4.1 In-Sample Test

We first assume away sample risk and selection risk to focus on the impact of model risk. We introduce those effects in another sub-section.

4.1.1 The Base Case

While it is commonly assumed that a “standard” level of risk-aversion is captured by a value $1 - a = 3$ or 4 and a conservative *upper bound* for the value of the risk-aversion parameter should be 10, it is well-known that values as high as 20 are needed to fit the equity premium (this is the *equity premium puzzle* from Mehra and Prescott (1985)). In this paper, we use

$a = -15$ as a base case value; this is consistent with a $\left(1 - \frac{m_p - r}{\sigma_P^2(1-a)}, \frac{m_p - r}{\sigma_P^2(1-a)}\right) = (68.2\%, 31.8\%)$ Merton (1973) allocation to the risk-free versus risky asset. We also perform some comparative static analysis on the risk aversion a parameter. We use the mean T-Bill rate on the period (5.06%) as an estimate of the risk-free rate r . The average return on the S&P500 on the period is 18.23%, and will be regarded as the value for m_p , which we assume identical to the true population value for the moment ($\delta_P = 0$). The annual volatility on the S&P500 volatility is estimated at $\sigma_P = 16.08\%$ from monthly data over the period 1996-2000. Finally, we arbitrarily fix the time-horizon at a $T = 5$ value.¹⁷

Based on the above estimates and assumptions on parameters, we compute in table 3 the base case optimal allocation to T-Bills, S&P500 and hedge funds such as predicted by the model (equations (3) and (4)). We also detail the origin of the funds invested in hedge funds (see Proposition 2).

Strategy	Holding in Passive	Holding in Active	Relative Holding A versus P	Holding in Risk-Free	Delta Passive	Delta Risk-Free Risk-Free
Ev. Dist.	28.14%	16.03%	36.29%	55.83%	3.70%	12.33%
Ev. Risk	18.62%	96.02%	83.76%	-14.64%	13.22%	82.80%
Ev. Driven	22.49%	57.13%	71.76%	20.38%	9.35%	47.78%
FoF Div.	28.70%	13.20%	31.51%	58.10%	3.14%	10.06%
FoF Niche	23.66%	54.54%	69.75%	21.80%	8.19%	46.35%
FoF	29.30%	11.56%	28.30%	59.13%	2.54%	9.02%
Mkt Neutr. Arb	29.31%	45.33%	60.73%	25.36%	2.53%	42.79%
Mkt Neutr. L/S	24.23%	202.21%	89.30%	-126.44%	7.61%	194.60%
Mkt Neutr.	27.91%	160.45%	85.18%	-88.35%	3.94%	156.51%
Short Sale	42.57%	11.80%	21.71%	45.62%	-10.73%	22.54%
Av. Fund	30.84%	28.64%	48.15%	40.51%	1.00%	27.64%

Table 3: Optimal Portfolio Strategies - The Base Case. This table features the percentage holding in the passive portfolio (column 2), active portfolio (column 3) and the risk-free asset (column 5), as well as the relative holdings in the active versus passive portfolio (column 4). It also features the changes in holdings due to the introduction of the hedge fund (see Proposition 2) (called delta in columns 6 and 7). We use the following set of base case parameter values: $r = 5.06\%$; $m_P = 18.23\%$; $\delta_P = 0$ (we assume away sample risk); $T = 5$; $a = -15$. The average fund is the fund with characteristics computed as the average of the indexes.

From the results in table 3, we find that the average hedge fund receives a 28.64% allocation. The strategies that exhibited spectacular performances, e.g., the market neutral long/short index posting an average return higher than 12% for a volatility around 2%, with a 8.30% average alpha and a low 2.15% dispersion around that value, receives a share of the portfolio that goes beyond the 100%. A rational investor would essentially want to borrow at the risk-free rate to invest in such a fund.

¹⁷ Given the presence of lockup periods imposed by most hedge fund managers, it is hardly feasible in practice to invest in hedge funds over a very short horizon.

More generally, our analysis provides formal backing to the widely spread notion that investors should use hedge funds as substitutes for some portion of their risk-free asset holdings (see Proposition 2). The origin of the 28.64% now allocated to the average active portfolio are 27.64% from the risk-free asset versus 1.00% from the passive portfolio (remember that the beta of the average fund is 0.03).

4.1.2 Impact of Biases

We now penalize the estimated expected return on all hedge funds by 4.5%, a reasonable estimate of effect of survivorship, selection and instant history biases, and obtain the following results (under the same base case parameter values).¹⁸

Strategy	Holding in Passive	Holding in Active	Relative Holding A versus P	Holding in Risk-Free	Delta Passive	Delta Risk-Free Risk-Free
Ev. Dist.	34.32%	-10.75%	-45.60%	76.43%	-2.48%	-8.27%
Ev. Risk	26.76%	36.94%	57.99%	36.31%	5.09%	31.85%
Ev. Driven	29.93%	11.67%	28.05%	58.40%	1.91%	9.76%
FoF Div.	33.49%	-6.90%	-25.96%	73.42%	-1.64%	-5.26%
FoF Niche	30.59%	8.34%	21.43%	61.07%	1.25%	7.09%
FoF	34.31%	-11.22%	-48.61%	76.91%	-2.46%	-8.76%
Mkt Neutr. Arb	30.34%	26.86%	46.96%	42.80%	1.50%	25.36%
Mkt Neutr. L/S	28.36%	92.52%	76.54%	-20.88%	3.48%	89.04%
Mkt Neutr.	30.32%	62.01%	67.16%	7.67%	1.52%	60.49%
Short Sale	38.97%	7.84%	16.75%	53.19%	-7.13%	14.97%
Av. Fund	31.52%	9.20%	22.59%	59.28%	0.32%	8.88%

Table 4: Optimal Portfolio Strategies - The Impact of Biases. This table features the percentage holding in the passive portfolio (column 2), active portfolio (column 3) and the risk-free asset (column 5), as well as the relative holdings in the active versus passive portfolio (column 4). It also features the changes in holdings due to the introduction of the hedge fund (see Proposition 2) (called delta in columns 6 and 7). We use the following set of base case parameter values: $r = 5.06\%$; $m_P = 18.23\%$; $\delta_P = (7.19\%)^2 = .52\%$; $T = 5$; $a = -15$. We penalize the estimated expected return on a hedge fund by 4.5%, a reasonable estimate of effect of survivorship, selection and instant history biases.

From the results in table 4, we find that the willingness of investors to hold hedge funds has severely declined as a result of expected returns being deflated by 4.5%. The holdings in the average hedge fund have decreased from 28.64% down to 9.20%, which is consistent with what most investors allocate to hedge funds.¹⁹ Moreover, the optimal holdings in any hedge fund exhibiting an average alpha lower than 4.5% are now negative, as can be expected from equation (3).

¹⁸In principle, the presence of biases not only affect expected return on hedge funds, but also beta and volatility estimate (for more details on this, see Amenc, Martellini and Vaissie (2002)).

¹⁹From informal conversations with various people in industry, it actually appears that most asset allocators would heuristically argue for a 10 to 20% allocation to hedge funds as a reasonable number.

4.1.3 Impact of Time-Horizon

We have argued that, for non trivial uncertainty around the alpha of the active portfolio, an increase in time horizon leads to an increase in the holdings in the passive portfolio, and therefore a decrease in the relative holdings in the active portfolio. The intuition is that time allows for uncertainty around alpha to unfold and hurt the risk-averse investor. On the other hand, when there is no uncertainty around alpha (see equation (3)), the solution is time-horizon independent.

To check the magnitude of the effect, we perform a comparative static analysis: we set the time-horizon to 10 years (instead of 5 years), and leave other parameter values unchanged.

Strategy	Holding in Passive	Holding in Active	Relative Holding A versus P	Holding in Risk-Free	Delta Passive	Delta Risk-Free Risk-Free
Ev. Dist.	29.69%	9.31%	23.87%	60.99%	2.15%	7.16%
Ev. Risk	24.37%	54.25%	69.00%	21.38%	7.47%	46.78%
Ev. Driven	26.51%	32.59%	55.15%	40.90%	5.34%	27.26%
FoF Div.	30.12%	7.24%	19.37%	62.64%	1.72%	5.51%
FoF Niche	27.25%	30.59%	52.88%	42.16%	4.59%	26.00%
FoF	30.47%	6.26%	17.05%	63.27%	1.37%	4.89%
Mkt Neutr. Arb	29.85%	35.68%	54.45%	34.47%	2.00%	33.69%
Mkt Neutr. L/S	27.72%	109.55%	79.81%	-37.27%	4.12%	105.43%
Mkt Neutr.	29.81%	82.92%	73.56%	-12.72%	2.03%	80.88%
Short Sale	38.17%	6.96%	15.42%	54.87%	-6.33%	13.28%
Av. Fund	31.25%	16.86%	35.04%	51.88%	0.59%	16.27%

Table 5: Optimal Portfolio Strategies - Impact of Time-Horizon. This table features the percentage holding in the passive portfolio (column 2), active portfolio (column 3) and the risk-free asset (column 5), as well as the relative holdings in the active versus passive portfolio (column 4). It also features the changes in holdings due to the introduction of the hedge fund (see Proposition 2) (called delta in columns 6 and 7). We use the following set of base case parameter values: $r = 5.06\%$; $m_P = 18.23\%$; $\delta_P = 0$ (we assume away sample risk); $a = -15$. The time-horizon is increased to $T = 10$ years.

From the results in table 5, we find that the impact is very significant. This suggests that the optimal allocation in hedge funds should be much smaller for private investors, who often have long-term horizons, than for a shorter-term institutional investor.

4.1.4 Impact of Risk-Aversion

We have argued that an increase in the absolute value of the risk-aversion parameter a leads to an increase in the relative fraction dedicated to the passive portfolio, as well as in the risk-free asset. To check the magnitude of the effect, we perform a comparative static analysis: we set the risk-aversion parameter a to -7 (instead of -15), and leave other parameter values unchanged. This value for the risk-aversion parameter corresponds to a less conservative $\left(1 - \frac{m_p - r}{\sigma_p^2(1-a)}, \frac{m_p - r}{\sigma_p^2(1-a)}\right) = (36.3\%, 63.7\%)$ Merton allocation to the risk-free versus risky asset,

in the absence of an active fund.

Strategy	Holding in Passive	Holding in Active	Relative Holding A versus P	Holding in Risk-Free	Delta Passive	Delta Risk-Free Risk-Free
Ev. Dist.	55.92%	33.68%	37.59%	10.41%	7.77%	25.91%
Ev. Risk	35.81%	202.43%	84.97%	-138.24%	27.88%	174.55%
Ev. Driven	43.99%	120.30%	73.23%	-64.29%	19.70%	100.60%
FoF Div.	57.03%	27.95%	32.89%	15.02%	6.66%	21.29%
FoF Niche	46.41%	115.09%	71.26%	-61.50%	17.27%	97.82%
FoF	58.30%	24.51%	29.60%	17.19%	5.38%	19.13%
Mkt Neutr. Arb	58.52%	92.32%	61.20%	-50.84%	5.16%	87.16%
Mkt Neutr. L/S	47.55%	428.59%	90.01%	-376.14%	16.13%	412.46%
Mkt Neutr.	55.29%	342.23%	86.09%	-297.52%	8.40%	333.83%
Short Sale	86.19%	24.76%	22.31%	-10.95%	-22.51%	47.27%
Av. Fund	61.59%	60.08%	49.38%	-21.67%	2.10%	57.99%

Table 6: Optimal Portfolio Strategies - Impact of Risk-Aversion. This table features the percentage holding in the passive portfolio (column 2), active portfolio (column 3) and the risk-free asset (column 5), as well as the relative holdings in the active versus passive portfolio (column 4). It also features the changes in holdings due to the introduction of the hedge fund (see Proposition 2) (called delta in columns 6 and 7). We use the following set of base case parameter values: $r = 5.06\%$; $m_P = 18.2322\%$; $\delta_P = 0$ (we assume away sample risk); $T = 5$ year. The risk-aversion coefficient is set to $a = -7$.

From the results in table 6, we find that the average fund generates a 60.08% to hedge funds, which compares to a 28.64% for the base case. On the other hand, we find a relatively trivial impact on the relative holdings in the passive fund versus the active fund; the impact is essentially an increase in the fraction invested in the risk-free asset.

4.2 Introducing Estimation Risk

One key element that was left aside in the previous results was estimation risk. We know, however, that there is a large uncertainty around the sample estimates both for $\tilde{\mu}_P$ and $\tilde{\alpha}$. So far, we have only focused on uncertainty around alpha driven by model risk, which was measured by the standard deviation of alphas in table 2. In this section, we discuss how the introduction of estimation risk impacts optimal holdings. The numbers we obtain here are not to be taken literally since we have to make a few simplifying assumptions and approximations in the process. The numbers we get should rather be regarded as indicative and illustrative of the magnitude of the impact.

We first recognize that an α for a given model is only an imperfect sample estimate of the true population value; different samples generate different alpha values for a given fund and a given model. Similarly, the sample mean value of the return on the S&P is an imperfect estimate of the true expected return on the S&P.

Uncertainty around the S&P 500 mean return is simply measured from monthly data as the

dispersion around the expected return point estimate. Assessing the magnitude of estimation risk for the active portfolio is less straightforward because it involves both estimation and model risks. In an attempt to measure the combined influence of model and estimation risks, we use a *decomposition of variance* formula ($V(Y) = E[V(Y|X)] + V[E(Y|X)]$), so that, for a given fund, the measure $\hat{\sigma}_{\tilde{\alpha}}^2$ of the variance on the estimator $\hat{\alpha}$ of $\tilde{\alpha}$ can be written as

$$\hat{\sigma}_{\tilde{\alpha}}^2 = \underbrace{\frac{1}{6} \sum_{i=1}^6 \hat{\sigma}_i^2}_{\text{measure of estimation risk}} + \underbrace{\frac{1}{6} \sum_{i=1}^6 \left(\hat{\alpha}_i - \frac{1}{6} \sum_{j=1}^6 \hat{\alpha}_j \right)^2}_{\text{measure of model risk}}$$

where we define $\hat{\alpha}_i$ as the estimate of alpha obtained from model i , and where $\hat{\sigma}_i^2$ is the estimated variance of the estimate of $\tilde{\alpha}$ for model i , $i = 1, \dots, 6$.

Let us define the t-value for $\hat{\alpha}_i$ as the usual $t_i = \frac{\hat{\alpha}_i}{\hat{\sigma}_i}$. When a t-value is available, an estimate of the variance of the estimate of alpha for model i is readily available and given by $\hat{\sigma}_i^2 = \frac{\hat{\alpha}_i^2}{t_i^2}$. Since t-values are not readily available for all models, for simplicity, we assume that they are all of the same order of magnitude as the one obtained for the CAPM model.²⁰

Column 3 in table 7 features $\hat{\sigma}_i^2 = \frac{\hat{\alpha}_i^2}{t_i^2}$, the variance of the CAPM estimate for alpha which serves as estimate of sample risk for alphas. Column 4 features $\frac{1}{6} \sum_{i=1}^6 \left(\hat{\alpha}_i - \frac{1}{6} \sum_{j=1}^6 \hat{\alpha}_j \right)^2$, i.e., the variance of alphas estimates across different models, which serves as an estimate of model risk for alphas. Finally, we compute in column 5 the total uncertainty around alpha, given by the squared-root of the sum of the terms in columns 3 and 4. A last step involves the computation of the uncertainty on the drift $[r + \beta (\tilde{\mu}_p - r) + \tilde{\alpha}]$ of the active portfolio. It is given by $\sqrt{\beta^2 Var(\tilde{\mu}_p) + Var(\tilde{\alpha})}$ under the assumption of no correlation between the priors on $\tilde{\mu}_p$ and the priors on $\tilde{\alpha}$. We argue below that this is a natural assumption given that $\tilde{\alpha}$ is a risk-adjusted residual return, measuring the abnormal return left after accounting for the normal return component $r + \beta (\tilde{\mu}_p - r)$.

The term $\sqrt{\beta^2 Var(\tilde{\mu}_p) + Var(\tilde{\alpha})}$ can be estimated by $\sqrt{\beta_P^2 \hat{\delta} \sigma_P^2 + \hat{\sigma}_{\tilde{\alpha}}^2}$, where β is the usual sample estimate of the fund exposure to market risk.²¹ This term appears in the last column of the previous table.

Based on these estimates and the base case parameter values, we compute the optimal allocation to T-Bills, S&P500 and hedge funds such as predicted by the model (see table 8).

From the results in table 8, we find that introducing sample uncertainty leads investors to decrease their holdings in the passive portfolio. On the other hand, introducing sample

²⁰We do not have specific reasons to believe that this should provide us with an upward- versus downward-biased estimate for that quantity.

²¹Again note that β_P and σ_P are assumed to be known and equal to their sample value, which is the reason why we do not use the notation $\hat{\beta}$ and $\hat{\sigma}_P$ in the previous expression.

Strategy	T-Test Alpha CAPM	Yearly Var Alpha CAPM	Variance Alpha	St. Dev. Alpha	Total Alpha Uncertainty	Uncertainty Drift Active
Ev. Dist.	1.12	0.16	16.16	4.02	4.04	4.37
Ev. Risk	4.07	0.42	7.82	2.80	2.87	3.04
Ev. Driven	2.78	0.30	9.94	3.15	3.20	3.41
FoF Div.	1.78	0.19	24.61	4.96	4.98	5.27
FoF Niche	2.87	0.35	10.17	3.19	3.24	3.42
FoF	1.61	0.24	22.30	4.72	4.75	5.00
Mkt Neutr. Arb	2.21	0.04	8.78	2.96	2.97	3.00
Mkt Neutr. L/S	7.08	1.20	4.63	2.15	2.41	2.43
Mkt Neutr.	9.04	2.57	5.70	2.39	2.88	2.88
Short Sale	1.97	0.02	105.49	10.27	10.27	12.18
Average	3.45	0.55	21.55	2.47	4.70	4.71

Table 7: The Magnitude of Alpha Uncertainty. This table displays for each hedge fund index in annual percentage terms an estimate of sample risk (column 3), the variance of alphas estimates across different methods (column 4) and the total uncertainty around alpha (column 5), given by the squared-root of the sum of the terms in columns 3 and 4). Uncertainty on the drift of the active portfolio is displayed in the last column.

risk does not increase much uncertainty around alpha and therefore does not affect much the holdings in the active fund. This is because, as can be seen from table 7, we actually find that the contribution of sample risk to uncertainty around alpha is much smaller than that of model risk, which can be intuitively explained by the fact that sample risk had been significantly reduced by the structure imposed by the model.

In this section, the optimal holdings have been computed under the assumption that the prior on hedge fund alpha is uncorrelated with the prior the market expected return, because there is no obvious way to estimate the impact of such correlation empirically. This is actually a natural assumption in a standard CAPM setting where the residual return is by definition independent of the market return.

More generally, however, $\tilde{\mu}_p$ and $\tilde{\alpha}$ could potentially be correlated if one defines $\tilde{\alpha}$ though CAPM while recognizing at the same time that the true asset pricing model is a multi-factor model.²² At the theoretical level, one may relax the assumption of uncorrelated priors by using the more general set up in Cvitanic, Lazrak, Martellini and Zapatero (2001a,b).²³ The

²²For example, it has been shown that convertible arbitrage hedge fund strategies are attractive specially when the yield curve is moderately upward sloping and market volatility is at a moderate level (see Kazemi and Schneeweis (2001)). Based on such stylized facts, investors may build priors for alphas on convertible arbitrage hedge fund that might exhibit non trivial correlation with priors on market returns.

²³In a theoretical model with correlated priors, they conclude that when the mean prior $\bar{\alpha}$ is positive, and the prior on the alpha of the hedge fund is negatively correlated with the prior on the market portfolio expected return, it is optimal to invest in hedge funds more than in the uncorrelated case. Furthermore, optimal holdings in the hedge fund increase with an increase on the perceived Sharpe ratio of the market. When the priors on the alpha is positively correlated with the prior on the market expected return, optimal holdings in the hedge fund decrease when the perceived market Sharpe ratio increases.

Strategy	Holding in Passive	Holding in Active	Relative Holding A versus P	Holding in Risk-Free	Delta Passive	Delta Risk-Free Risk-Free
Ev. Dist.	23.54%	14.19%	37.60%	62.27%	3.27%	10.91%
Ev. Risk	15.20%	84.36%	84.74%	0.44%	11.62%	72.74%
Ev. Driven	18.52%	50.65%	73.22%	30.83%	8.29%	42.36%
FoF Div.	23.97%	11.95%	33.28%	64.08%	2.85%	9.11%
FoF Niche	19.49%	48.83%	71.48%	31.68%	7.33%	41.50%
FoF	24.51%	10.47%	29.94%	65.01%	2.30%	8.17%
Mkt Neutr. Arb	24.30%	45.05%	64.96%	30.66%	2.52%	42.53%
Mkt Neutr. L/S	20.64%	163.97%	88.82%	-84.61%	6.17%	157.80%
Mkt Neutr.	24.06%	112.45%	82.38%	-36.50%	2.76%	109.69%
Short Sale	35.18%	9.20%	20.74%	55.61%	-8.37%	17.57%
Av. Fund	25.83%	28.08%	52.09%	46.08%	0.98%	27.10%

Table 8: Optimal Portfolio Strategies with Estimation Risk. This table features the percentage holding in the passive portfolio (column 2), active portfolio (column 3) and the risk-free asset (column 5), as well as the relative holdings in the active versus passive portfolio (column 4). It also features the origin of the funds invested in the active portfolio (columns 6 and 7). We use the following set of base case parameter values: $r = 5.06\%$; $m_P = 18.23\%$; $\delta_P = (7.19\%)^2 = .52\%$; $T = 5$; $a = -15$.

challenge remains, however, to generate meaningful empirical estimates of such correlation.

5 Conclusion

Because the returns of alternative investment strategies exhibit in general low correlation with that of standard asset classes, it is expected that hedge funds will take on a significant share in active allocation strategies. While in its infancy the world of alternative investment strategies consisted of a disparate set of managers following very specific strategies, significant attempts at structuring the industry have occurred over the last decade which now allow active asset allocation models to apply to hedge funds as well as to traditional investment vehicles. In particular, investable portfolios replicating broad-based hedge funds indexes are today available with a sufficient level of liquidity.²⁴

In this paper we apply the model developed in Cvitanić, Lazrak, Martellini and Zapatero (2002a,b) to a database of hedge funds. Our results have important implications for investors who consider including alternative investment vehicles in their portfolios. In particular, they suggest that low beta hedge funds may serve as natural substitutes for a significant portion

²⁴In particular, Zurich Capital Markets launched a series of hedge fund indexes in 2001 that consist of equally weighted portfolios of funds that satisfy a number of qualitative criteria for institutional investment as well as a statistical classification procedure for style classification, under the supervision of an independent advisory board. Investable portfolios, i.e., replicating portfolios with an approximate 2.5% tracking error, are available for each of these 5 indexes with monthly liquidity assured by Zurich Capital Markets (see www.zcmgroup.com for more details).

of an investor risk-free asset holdings. Since the model we use can be generalized in several directions, this paper attempts to provide money managers with a tool to allocate assets among hedge funds.

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A Appendix: Information on Hedge Fund Strategies

We present some information on the hedge fund strategies discussed in the paper. We refer the reader to www.marhedge.com for more information about these strategies as well as about others not covered in this paper.

- **Event-Driven.** Investment theme is dominated by events that are seen as special situations or opportunities to capitalize from price fluctuations. Distressed Securities managers focus on securities of companies in reorganization and/or bankruptcy, ranging from senior secured debt (low-risk) to common stock (high risk). Risk Arbitrage managers simultaneously buy stock in a company being acquired and sell stock in its acquirers. If the takeover falls through, traders can be left with large losses.
- **Market Neutral.** The manager attempts to lock-out or neutralize market risk. In theory, market risk is greatly reduced but it is difficult to make a profit on a large diversified portfolio, so stock picking is critical. Long/short: net exposure to market risk is believed to be reduced by having equal allocations on the long and short sides of the market.

Convertible arbitrage: manager goes long convertible securities and short underlying equities, profiting from mispricing in the relationship of the two. Stock arbitrage: manager buys a basket of stocks and sells short stock index futures contract, or reverse. Fixed income arbitrage: manager buys bonds - often T-bonds, but also sovereign and corporate bonds - and goes short instruments that replicate the owned bond; manager aims to profit from mispricing of relationship between the long and short sides.

- Short-Seller. Manager takes a position that stock prices will go down. A hedge fund borrows stock and sells it, hoping to buy it back at a lower price. Manager shorts only overvalued securities.
- Fund of Funds. Capital is allocated among funds, providing investors with access to managers with higher minimums than individual might afford. Diversified funds of funds allocate capital to a variety of fund types. Niche funds of funds allocates capital to a specific type of fund.