Executive Stock Options with Effort Disutility and Choice of Volatility*

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Abstract

We consider the problem of an executive that receives call options as compensation in a dynamic setting. She can influence the stock price return with her effort. In addition, she determines the level of volatility of the stock through the choice of projects. The executive is risk-averse and experiences disutility from the effort. In this framework, we introduce the problem of the company that wants to maximize the final expected value of the price of the stock minus the cost of the compensation package. The company has to design a compensation package such that the executive reaches the minimum level of utility or opportunity cost (individual rationality constraint). We characterize the optimal strike price the company should choose, and compute it numerically for the logarithmic case. When the executive can affect the mean return of the stock, we find that in the vast majority of cases options should be granted out-of-the-money. A key parameter in the model is the parameter that measures the quality of the projects. When this parameter is very high, it is optimal to grant stock rather than options. We distinguish between "large" companies, that emphasize the expected value of the stock, and "small" companies, that are more interested in keeping the cost of the compensation low. We find that stock is more likely to be the optimal contract for small companies.

1 Introduction

A call option gives its owner the right to buy one share of the underlying stock at a predetermined price. Options are similar to a leveraged portfolio invested in the underlying security. For that reason, options have become a main ingredient in compensation packages with the objective of aligning the interests of the executives with those of the company stockholders. Hall and Leibman (1998) and Murphy (1999) report statistics about the number of companies that use stock options as the main component of the compensation of their executives.

As a result of the increasing importance of stock options as compensation there has been a surge of interest on this topic in the economics literature. Among the first references in this area are Lambert, Lanen and Larcker (1989) that argue that executive stock options induce a reduction in dividend payments, Jensen and Murphy (1990) that show that the part of the compensation of executives linked to company performance depends mostly on their holdings of stock and options rather than on bonuses (more on this topic can be found in Murphy (1993), Hall and Leibman (1998) and Carpenter (1998)). More recently, Hall and Murphy (2000b) consider the problem of valuation of executive stock options, Detemple and Sundaresan (1999) develop a pricing model for options that can be applied to executive options, Hall and Murphy (2000a) study the problem of choosing the strike price of the options, Brenner, Sundaram and Yermack (2000) consider the problem of resetting the strike price and develop a valuation model for resettable options. A strand of the literature addresses the broader problem of the optimality of stock options versus other contracts with the manager, e.g. Aseff and Santos (2001). Related work is also Martellini and Urŏsević (2001) that tries to provide a uniform setting for the valuation of different compensation schemes. Jin (2000) studies empirically the tradeoff between incentives and risk, since the executive cannot trade the options.

We take as given that the executive will receive call options as part of her compensation. However, given that assumption, we try to develop a richer framework in order to address some of the problems mentioned above. In particular, the existent literature considers the incentives of options in a static framework: that framework does not seem appropriate to address some of the questions considered in the literature. One of the potential problems of stock options compensation (see, for instance, Johnson and Tian (2000a, 2000b)) is the incentive for the executive to increase volatility, since options values increase with volatility. Johnson and Tian (2000a) compare different option plans and study the incentive to increase volatility by computing the vega, the derivative of the option with respect to volatility, of the specific type of option: a higher vega would indicate a higher incentive for the executive to increase the volatility of the stock. Similarly, performance incentives are measured by the delta of the call option, the derivative of the option price with respect to the price of the underlying: higher delta would imply a higher incentive for the executive to try to increase the price of the stock (see Hall and Murphy (2000a), for example). Câmara

(2001) prices a broad range of relative performance based compensation packages, but he does not consider either the possibility of a choice of effort and/or volatility from the part of the executive. However, volatility is a dynamic concept and it seems appropriate to consider problems like these in a dynamic setting. Another element that is missing in the current literature is the computation of the effort of the executive and the tradeoff between the effect of the effort on the price of the stock and the disutility resulting from that effort.

In this paper we construct a dynamic framework that includes those ingredients and tries to address some of the problems considered in the literature. More specifically, we study the optimal strike price of executive stock options in a dynamic setting where the effort of the executive affects the dynamics of the stock. In addition, the executive can choose among a menu of projects with a tradeoff between expected return and volatility: projects with higher volatility offer a higher expected return. The executive receives call options on the stock as compensation. The executive is risk-averse with respect to the options payoff. Moreover, her effort induces disutility. The executive will choose effort and volatility so as to maximize expected utility minus effort disutility. The company cares both about final value of the stock and the cost of the compensation package, and chooses a strike price of the option to induce the executive to achieve an optimal mix of effort and volatility. The company has to grant a number of options at a strike price such that the utility of the executive is at least as high as a given reservation value (individual rationality constraint). This paper complements Carpenter (2000), in which, in a dynamic setting, the optimal choice of volatility is studied, but the agent cannot affect the stock mean return. Moreover, strike prices are given exogenously in that work.

For an agent that maximizes logarithmic utility from the options, minus the quadratic disutility from the effort, we compute the optimal level of effort and volatility in closed form. Based on that, we can compute the optimal strike price to be set by a company that cares about the expected final price of the stock and cost of the compensation package. In general, there is a range of possible optimal strike prices, depending on the parameters of the model. A surprising result in our model, however, is the fact that the optimal strike price is mostly out of the money. For parameter values that would induce a choice of a lower strike price, we find that it is usually optimal to grant stock rather than options. We also find that the strike price should be higher the higher the type (quality) of the executive. Also, the "size" of the company seems to have an effect on the optimal strike price. A larger company will award a more expensive compensation package with more options but with a higher strike price. A smaller company is more likely to grant stock rather than options. A key parameter of our model is a measure of the quality of the projects the executive can choose from. That parameter has a small effect on the optimal strike price level, but, when it takes sufficiently high values, it has the effect to make optimal a compensation package formed by stock rather than options. Small companies are more sensitive to this effect than large companies.

In our model the executive can affect the mean return of the stock independently of the volatility. This may be the case in which investors have incomplete information. With perfect information the mean return of the stock is determined in equilibrium by the level of systematic risk. In the latter case the executive cannot affect the mean return independently of the volatility, and we find that in this case it is optimal to grant stock rather than options.

We also compute the value of the option, both for the risk-neutral company and for the risk-averse investor. When the parameter that measures the quality of the projects is high, the ratio of the option value for the company and the option value for the executive increases. This is consistent with the rest of our conclusions that indicate that when that parameter is high, it is optimal to grant stock rather than options.

Finally, we also compute the optimal effort of the executive when cash is a part of the compensation package, but we are not able to get explicit expressions for the optimal strike price in this case. Moreover, in the Appendix, we indicate how the problem can be solved, up to numerical computations, for utility functions other than the logarithmic utility.

The paper is structured as follows. In section 2 we describe the dynamics of the stock and the effects of the actions of the executive on those dynamics as well as the objectives of the two players in our model: the executive and the company. In section 3 we derive the optimal effort and volatility to be chosen by the executive, as well as the optimal strike price to be set by the company. In section 4 we perform some numerical computations and comparative statics. In Section 5 we compute the option values from the perspectives of both the company and the executive. Section 6 considers the case of additional compensation in cash. We close the paper with some conclusions.

2 The Setting

We consider the problem of a company that grants stock options to an executive as a part of her compensation package. The options are given as a performance incentive. The executive that receives the options is risk-averse. Besides, the decisions of the executive affect the dynamics of the stock. We consider a dynamic setting. As it will become clear later, this is the only interesting case. In a static setting, the results would be trivial and not very interesting.

2.1 Stock Dynamics

Our benchmark stock has a price that follows a geometric Brownian motion process,

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

with starting value S_0. The process W is a standard Brownian motion process and μ and σ are exogenous constants. However, when the company is managed by the executive whose problem we address in the next section, the dynamics of the stock price S is given by

$$dS_t = \mu S_t dt + \delta u_t dt + \alpha v_t S_t dt + v_t S_t dW_t, \tag{1}$$

where u and v are adapted stochastic processes chosen by the executive, $\delta \in [0, \infty)$ and $\alpha \in (0, \infty)$. We shall assume that $E[\int_0^T |u_t|^2 dt] < \infty$ and $E[\int_0^T |v_t S_t|^2 dt] < \infty$. The control u is the level of effort the executive puts in the management of the company. The higher the u, the higher the expected value of the stock. We will assume later that the effort produces disutility for the executive. On the other hand, the choice of v is equivalent to the choice of the volatility of the stock, although it also has an impact on the expected value. We interpret the choice of v as a choice of projects. We assume that the executive can choose different projects or strategies that are characterized by a level of risk and expected return. Since $\alpha > 0$, the higher the risk of a project, the higher its expected return. The parameter α is a characteristic of the type of the company. For example, it may be argued that companies in new industries will in general be characterized by a large α. One possible interpretation of α (and, potentially, a way to estimate it empirically) would be the slope of the equivalent of the "Capital Market Line" resulting from all the projects available to the company (more about the distinction between firm-specific and market risk can be found below). On the other hand, δ is a measure of the impact of the effort of the executive on the value of the company. It can be interpreted as an indicator of the type (quality) of the executive. Carpenter (2000) studies the optimal choice of v for the case in which $\delta = 0$. Bolton and Harris (2001) also study a model where the agent has control of the dynamics of the asset, but for a different purpose.

We emphasize that this is a partial equilibrium setting and we do not compare the dynamics of the stock of this company with the dynamics of other stocks. If all the investors had perfect information, the return of the stock would be decided in equilibrium, and the executive would not be able to influence the drift independently of the volatility, so that δ would be equal to zero. We consider the case $\delta = 0$ in section 4 of the paper. A model with positive δ may correspond to the case of partial information. different An equilibrium model where investors have partial information on the effect of the actions of different executives on the dynamics of the stocks of the companies that employ them is beyond the scope of this paper.

Without loss of generality, we will assume that $\mu = 0$. We then re-express equation (1) as

$$dS_t = \delta u_t dt + \alpha v_t S_t dt + v_t S_t dW_t. \tag{2}$$

In this paper we only consider one source of uncertainty (a single Brownian motion process). It would be more realistic to distinguish between idiosyncratic and systematic risk. In our setting, we could do that by including in the dynamics of S a second

Brownian motion process, independent of the first one. The diffusion term of the stock price dynamics would be of the type $vSdW_1 + \sigma SdW_2$. The process W_1 would describe the idiosyncratic risk, while W_2 would represent the systematic or market risk. The executive would choose v, as above, while σ would be exogenous. However, to make this a complete market setting, we should then allow the executive to trade in a security that depends on W_2. This is realistic, since executives can neither sell the options nor sell short the stock of their own company, but they are allowed to trade in other securities. In particular, as shown by Jin (2000), they can diversify the market risk component of the underlying stock by trading (a portfolio that replicates) the market index. As Jin (2000) shows, the market risk becomes irrelevant. Hence, the model we suggest in this paper can be considered a reduced but equivalent form of the model described in this paragraph, it is tractable, and it considers the relevant risk component.

2.2 The Executive

In the existing literature, the effort level is not modeled, while volatility is exogenous, with the exception of Carpenter (2000), Bolton and Harris (2001). In our model, the executive chooses u and v so as to maximize her utility. The executive is risk-averse. Besides, she experiences disutility as a result of the effort. The objective of the executive is

$$\max_{u,v} E\left[\log\left\{n(S_T - K)^+\right\} - \frac{1}{2}\int_0^T u_t^2 dt\right] \tag{3}$$

In (3), n is the number of call options the executive receives as a part of her compensation package. The second term of the objective function of the executive represents the disutility from effort. In the existing literature, the incentives to the executive are measured by the delta of the option weighted by its price: a higher delta for dollar spent in options means a stronger incentive for the executive (see, for instance, Hall and Murphy (2000a)). However, this is not compared to the disutility that arguably would arise from a higher effort level. That disutility might be, for example, the result of spending more time working for the company. We assume that the disutility is an increasing, convex function, with quadratic cost being a convenient choice. The executive will optimally choose a non-negative level of effort. The other control, v, involves the choice of projects the company will undertake and has no effect on the disutility of the executive, since it does not require any effort: the executive has a menu of projects and decides the level of risk to undertake. The projects are, in principle, comparable in quality since projects with higher risk also offer a higher expected return. By allowing the executive to choose the level of volatility we intend to address the problem of potential incentives to increase volatility that options might trigger, since in a Black and Scholes (1973) setting the price of call options is increasing with the volatility. We do not introduce a weighting parameter that would measure the relative importance of the utility from the options package versus the disutility from the effort. The reason is that this constant would be equivalent to the δ that represents the type of the executive: an executive with a high δ is equivalent to an executive that does not care much about the disutility induced by the effort.

We also point that in this specification the executive only receives compensation linked to incentives (stock or options). In the last section of the paper we consider the case in which the executive also receives a cash compensation.

Finally, we emphasize the need to address this problem in a dynamic setting: for the utility considered here the solution in a static setting would be trivial (zero volatility) and not very interesting.

2.3 The Company

The company will compensate the executive with stock shares, or call options on the stock, with maturity T. The company will choose both the number of options n and the strike price K. A strike price of 0 indicates that the company is giving stock to the executive. We assume that the company is risk neutral, and cares about the final value of the stock, as well as about the value of the compensation. On the other hand, the company has to guarantee that the utility of the executive is at least as large as a reservation utility R. This can be interpreted as the utility that the executive would achieve in the best alternative offer she has. This restriction amounts to an *individual rationality constraint*, standard in the Principle-Agent problem literature. Let us define

$$h(K,n) := \lambda E[S_T] - nE[(S_T - K)^+] \tag{4}$$

and

$$A(R) := \left\{(K,n) \in [0,\infty)^2 : \max_{u,v} E\left[\log\left\{n(S_T - K)^+\right\} - \frac{1}{2}\int_0^T u_t^2 dt\right] \geq R\right\}. \tag{5}$$

The objective of the company is, then,

$$\max_{(K,n)\in A(R)} h(K,n). \tag{6}$$

where λ is an exogenous constant that represents the "size" of the company: a large value of λ indicates a company that has a large capitalization value and for which a marginal increase in the value of the stock is very important with respect to the value of the compensation package. A small λ would indicate that the company devotes a large proportion of its cash flow to compensation. The objective horizon of the company matches the objective horizon of the executive and the maturity of the options. The value R, as we said above, represents the minimum utility the executive has to be able to achieve through the optimal choice of effort and volatility in order to work for the company.

3 Optimal Strategies

Previous papers that study the incentives that stock options represent for executives have concentrated on the delta and vega of the options (see, for instance, Johnson and Tian (2000a)). The idea is that when the options have a high delta, their price is very sensitive to an increase in the value of the stock and, therefore, the executive has a bigger incentive to try to increase the value of the stock. Similarly, a large vega indicates high sensitivity of the price of the option with respect to the volatility of the stock and, therefore, a larger incentive for the executive to increase volatility. In this paper we study simultaneously the optimal choice of effort and volatility in a dynamic setting. We point out that given the logarithmic specification of the compensation part of the utility of the executive, the number of options n will not affect her optimal effort and volatility choices. We first present the solution of the problem of the executive for a given strike price. Then we will discuss the optimal choice of the strike price by the company.

3.1 Optimal Effort and Volatility

We will present the optimal effort $\hat{u}$ and the optimal choice of volatility $\hat{v}$ of the executive. First we introduce the following auxiliary exponential martingale Z,

$$Z_t \;=\; \exp\left\{-\frac{1}{2}\alpha^2 t - \alpha W_t\right\}, \tag{7}$$

where α is the parameter in (2) that represents the tradeoff between volatility and expected return of the projects the executive can choose among. Also, consider the following function of time $\bar{T}$,

$$\bar{T}_t = \frac{e^{\alpha^2(T-t)} - 1}{\alpha^2}. \tag{8}$$

Using the previous notation and given the following quadratic equation in z,

$$\delta^2 \bar{T}_0 z^2 + (S_0 - K)z - 1 = 0, \tag{9}$$

where δ is the parameter that measures the type of the executive and K is the strike price of the options, we denote by $\check{z}$ the positive solution of (9):

$$\check{z} \;=\; \frac{1}{2\delta^2\bar{T}_0}\left((K - S_0) + \sqrt{(K - S_0)^2 + 4\delta^2\bar{T}_0}\right). \tag{10}$$

We now find the optimal controls of the executive:

Proposition 1 *Consider the problem of the executive described in Section 2.2. Consider also the exponential martingale of (7), $\check{z}$, the positive number given by (10), and $\bar{T}$, the time function of (8). The optimal effort $\hat{u}$ of the executive is*

$$\hat{u}_t = \delta \check{z} Z_t. \tag{11}$$

The optimal choice of volatility $\hat{v}$ is given by

$$\hat{v}_t S_t = \frac{\alpha}{\check{z} Z_t} + \alpha \check{z} \delta^2 Z_t \bar{T}_t. \tag{12}$$

The optimal effort and volatility determine that the price of the stock be given by the equation

$$S_t = \frac{1}{\check{z} Z_t} + K - \check{z} \delta^2 Z_t \bar{T}_t. \tag{13}$$

Proof: See the Appendix. $\diamond$

We observe that the optimal effort and volatility can also be written as functions of the price of the stock. That is,

$$\hat{u}_t = \frac{1}{2\delta^2 \bar{T}_t} \left((K - S_t) + \sqrt{(K - S_t)^2 + 4\delta^2 \bar{T}_t} \right).$$

and

$$\begin{aligned} \hat{v}_t S_t &= \frac{\alpha \delta}{\hat{u}_t} + \alpha \delta \hat{u}_t \bar{T}_t \\ &= \frac{2\alpha\delta^2 \bar{T}_t}{(K - S_t) + \sqrt{(K - S_t)^2 + 4\delta^2 \bar{T}_t}} + \frac{\alpha}{2} \left[(K - S_t) + \sqrt{(K - S_t)^2 + 4\delta^2 \bar{T}_t} \right]. \end{aligned}$$

With respect to the optimal effort, as expected, $\hat{u}$ is increasing in the strike price K: as K goes to infinity, the effort goes to infinity, as well. Besides, we note that $\check{z}$ is decreasing in T, the time to maturity of the option ($\bar{T}_0$ is increasing in T and $\check{z}$ is decreasing in $\bar{T}_0$). Therefore, the larger the maturity of the option, the lower the effort of the executive. The intuition is clear: a larger T has a similar effect on the executive as lowering of the strike price. The effect of δ (the "type" of executive) depends on whether the option is in-, out-, or at-the-money. When the option is at-the-money, the optimal effort is independent of δ, as we can see by substituting (10) in (11). We can also check that when the option is in-the-money the effort is increasing in δ, and when the option is out-of-the-money the optimal effort decreases with δ.

Since Z is a martingale, the expected value of the effort at any point in time is,

$$E[\hat{u}_t] = \delta \check{z}. \tag{14}$$

With respect to the effect of α, we note that $\bar{T}_0$ is increasing in α and, therefore, $\check{z}$ is decreasing in α. Expected effort is, then, decreasing in α (everything else constant): the better the menu of projects the executive can choose among, the lower the expected effort of the executive.

The analysis of the volatility is more complicated. Since $\bar{T}_T = 0$, the second term of (12) decreases in expected value as we approach maturity, and will tend to be negligible relative to the first term. Therefore, for short maturities, optimal volatility will tend to decrease with higher strike price. For maturities long enough, the relation will tend to be the opposite. We also see that the volatility is increasing in the type of executive δ ($\check{z}$ is decreasing in δ, and $\check{z}\delta^2$ is increasing in δ). The economic intuition is straightforward: a high-type executive can afford more volatility because her effort will be more effective to counteract drops in the value of the stock. It is straightforward to see that the expected value of the volatility at a future date t is

$$E[\hat{v}_t S_t] = \frac{\alpha}{\check{z}} e^{\alpha^2 t} + \alpha \check{z} \delta^2 \bar{T}_t. \tag{15}$$

Since $\bar{T}_t$ is increasing in α, the expected volatility is increasing in α. In other words, the higher the expected return-risk tradeoff, the higher the risk the executive will be willing to undertake.

Some of these comparative statics are illustrated in the results of Table 1 (that we analyze in detail in the next section). However, in that table the strike price is always the optimal one, and some of the previous conclusions hold for changes in a given parameter with constant strike price.

It is also interesting to study the correlation between optimal effort and optimal volatility. By Ito's lemma, and equation (12), the dynamics of the optimal volatility are

$$d(\hat{v}_t S_t) = (\cdot)dt + \alpha^2 \left(\frac{1}{\check{z} Z_t} - \delta^2 \check{z} Z_t \bar{T}_t \right) dW_t. \tag{16}$$

Equation (11) yields,

$$d\hat{u}_t = (\cdot)dt - \alpha \hat{u}_t dW_t. \tag{17}$$

It is clear that their correlation can be either positive or negative. Considering only the instantaneous correlation and ignoring the drift terms, we see that for a short maturity of the option they tend to be negatively correlated, and increases in optimal effort will be typically associated with decreases of the optimal level of volatility.

3.2 Optimal Strike Price and Number of Options

We assume that the company has full information about the parameters that characterize the dynamics of the stock, as well as the preferences of the executive. The objective of the company is given by (6). In order to characterize the optimal choice of K and n for the company, we introduce the function

$$V(z, n) = \frac{1}{z}((\lambda - n)e^{\alpha^2 T} - \lambda) + \lambda z \delta^2 \bar{T}_0 + \lambda S_0 \quad . \tag{18}$$

It will be shown in the Appendix that this is the objective function of the company, using $z = \check{z}$ and n as arguments. We see that the optimal n is the smallest n that we can take, namely the smallest n such that the executive's rationality constraint is satisfied. As shown in the Appendix, the executive's rationality constraint (5) is

$$\frac{1}{2}\delta^2\bar{T}_0 z^2 + \log z - \log n - \frac{\alpha^2}{2}T + R = 0 \quad . \tag{19}$$

From this it follows that the company will take the value of n equal to

$$n(z) = z e^{\{R-\alpha^2 T/2+\delta^2\bar{T}_0 z^2/2\}} \quad . \tag{20}$$

Substituting back in V, we get the objective function of the company as the function of one argument only:

$$V(z) = \frac{1}{z}\lambda(e^{\alpha^2 T} - 1) - e^{\{R+\alpha^2 T/2+\delta^2\bar{T}_0 z^2/2\}} + \lambda z\delta^2\bar{T}_0 + \lambda S_0 \quad . \tag{21}$$

We now introduce the following values of z:

We define z_0 by

$$z_0 = \frac{-S_0 + \sqrt{S_0^2 + 4\delta^2\bar{T}_0}}{2\delta^2\bar{T}_0}. \tag{22}$$

If $\delta = 0$ we define $z_0 = 1/S_0$. It is seen from (13) with $t = 0$ that this value corresponds to $K = 0$.

We denote by $z = z_1, \ldots, z_k$ the solutions to the nonlinear equation (if any)

$$0 = V'(z) = \frac{\lambda}{z^2}(1 - e^{\alpha^2 T}) - z\delta^2\bar{T}_0 e^{R+\alpha^2 T/2+\delta^2\bar{T}_0 z^2/2} + \lambda\delta^2\bar{T}_0 \quad . \tag{23}$$

In other words, at these values the derivative of the objective function V of (21) is equal to zero.

We observe that, when $\delta = 0$, $V'(z) \leq 0$. Furthermore, when $\delta > 0$, $V'(\infty) = -\infty$. Hence, for any $\delta \geq 0$, $V'(z)$ cannot be positive for all $z \geq 0$.

Recall from (13) that a given value of z corresponds to the value of K given by

$$K = S_0 - \frac{1}{z} + z\delta^2\bar{T}_0 \quad . \tag{24}$$

Denote by $K_0 = 0, K_1, \ldots K_k$, the values of K corresponding to $z_0, z_1, \ldots, z_k$.

We now state the result about the optimal strike price and the number of options for the company.

Proposition 2 *Consider the company whose objective is given by (6). The optimal strike price and number of options are given by $\hat{K} = K_j$ and $\hat{n} = n(z_j)$ (as determined from* (20)*), where $V(z_j)$ is the largest of the values $V(z_0), V(z_1), \ldots, V(z_k)$. We shall denote by $\bar{z}$ the value of z corresponding to $\hat{K}$.*

Proof: See the Appendix. $\diamond$

We will perform some numerical exercises in the next section. Here we consider the case $\delta = 0$ for which we can easily obtain the solution.

Corollary 1 *Suppose that the executive cannot affect the drift independently of the volatility, i.e., $\delta = 0$. Then it is optimal to issue stocks, i.e.,*

$$\hat{K} = 0. \tag{25}$$

Proof: See the Appendix. $\diamond$

4 Numerical Computations of the Optimal Strike Price

In the previous section we derived the optimal exercise price for a company that cares both about the expected final value of the stock and the cost of the compensation package. As it is shown in Proposition 2, the optimal exercise price will be the better (according to the objective of the company of (6)) of 0 (the company gives stock) or the K of equation (24). In order to find the latter, we have to solve for z in equation (23). In this section we find the optimal $z = \bar{z}$ numerically, in order to derive some properties of the optimal strike price. The results are included in Tables 1 and 2.

We study the strike price as a function of the parameters of the model: δ, that measures the type of the executive (higher δ means a higher type); α, that measures the quality of the projects the company can undertake (higher α means a better contribution in expected return for each additional unit of risk); R or reservation utility of the executive: higher R means that the company will have to compensate the executive more in order to entice her to work for it; finally, we study the optimal strike price with respect to λ that we interpret as a proxy for the size of the company: a larger company will devote a smaller share of its resources to executive compensation and will be more sensitive to the expected value of the price of the stock. In order to simplify our analysis, we present two tables. In Table 1 we consider the case of a "large" company (with high λ) while in Table 2 we focus on the case of a "small" company (with a low λ). The effect of the time to maturity is obvious, thus we fix it at $T = 5$. The initial value of the stock is 100.

The first key observation is that, in general, optimal strike prices are either zero (stock) or above 100 (out-of-the-money). In a few cases (we report one in Table 2) the optimal option is barely in-the-money, but as we change slightly the parameters in the direction that produces a decrease of the optimal strike price, the optimal contract becomes to grant stock. This result seems to be at odds with the fact that options are usually granted at-the-money.

With respect to the type of the executive, we observe that the higher the type of the executive (higher δ), the higher the optimal strike price. A higher type can have more impact with lower effort and the company can set a higher strike price that will not violate the individual rationality constraint.

The quality of the projects α has an interesting effect on the optimal compensation package. Higher α (higher quality) makes it easier for the executive to improve the price of the stock and the company can set a higher strike price: the executive will substitute volatility for effort. However, the executive is risk averse and will be reluctant to take on higher risk. As a result, for a sufficiently high level of α it will be optimal for the company to give stock rather than options. This lowers the leverage of the executive and allows her to take on more risk.

The effect of R, the reservation value for the executive, is obvious and unambiguous: higher reservation value for the executive requires a lower strike price. We find that it is optimal for the company to lower the strike price rather than give more options, in order to add the necessary compensation to entice the executive to work for the company.

All these effects are apparent, both in Table 1 and Table 2. In order to analyze the effect of changes in λ (size of the company) we compare Table 1 and Table 2. From this comparison, we see that the larger the company, the higher the strike price and also the higher the number of options (so as to satisfy the individual rationality constraint). The larger company cares (relatively) more about the value of the stock than about the value of the compensation and sets a higher strike price, that will require more effort from the part of the executive. These results are driven by the sensitivity of the executive to the strike price. Since options are equivalent to a leveraged position in the stock, the utility of the executive is greatly affected by changes in the strike price: as the strike price decreases, so does the delta of the option (the implicit leverage of the executive) and this has a positive effect (even if we keep the value of the package constant) on the utility of the executive. However, the company is risk neutral and only cares about the value of the compensation package. A big company will pay a very expensive package, but will expect a high effort.

As we mentioned above, an important conclusion of our results is that, in general, the optimal strike price is out-of-the-money. Our results shed some light on the debate about whether options should be at-the-money or out-of-the-money (Hall and Murphy (2000a)). We also recall from our discussion about the dynamics of the stock of equation (1) that the case $\delta = 0$ could correspond to the situation of perfect information about the effect of the actions of the executive on the dynamics of the stock. In that case, from Corollary 1, it will be optimal for the company to grant stock, rather than options.

5 Options Values

Valuing the option granted to the executive is important for several reasons. First, very often the company and the executive decide to change the characteristics of the compensation package. Typically, the executive will surrender her option package in exchange for another package of options with different characteristics (especially, different strike price) and same value. In order to do that, the company needs to value the options (see Brenner, Sundaram and Yermack (2000)). Besides, another way to consider the incentives to the executive would be an analysis of the comparison of the value of the option to the company and to the executive. More explicitly, the executive of the problem considered in this paper faces incomplete markets as a result of the fact that she cannot sell the options and cannot trade in the underlying security. On the other hand, the company does not face the same type of restrictions, although it is not clear that the company faces complete markets. In this paper, for tractability reasons, we assume that the company is risk neutral (which is standard in the literature) and in its objective function we subtract the expected option payoff under the objective probability measure, rather than the true complete markets value of the options. There is no obvious way of computing the difference between the value of the option for the executive and for the company. The approach taken in the literature with respect to the executive (that we follow here) is to compute the certainty equivalent: a constant amount of money that would leave the executive at the same utility level as the option. As for the company, we compute the option value under the original objective probability measure, consistent with the company's objective function. These two numbers will provide us with useful information, since we are then able to compare these values and asses the appropriateness of using options for compensation schemes.

We point out that the Black and Scholes (1973) formula is not appropriate for our setting, for several reasons. First, we have a model with stochastic volatility, that the Black and Scholes (1973) formula cannot address. Second, and probably more important from a practical point of view, it is well known that the drift of the stock process does not appear in the Black and Scholes (1973) formula, since it is an irrelevant parameter, given the underlying equilibrium. However, that is not the case in our setting. As it should be clear by now, the strike price induces an optimal effort level which is key in determining the payoff of the option. Besides, as we argued in the description of our model, for our setting to be compatible with an equilibrium, we would need asymmetry of information between the company/executive and the rest of the investors in the market. This would deviate our setting from Black and Scholes (1973).

We first define the *certainty equivalent* (CE) of the executive as the constant payoff x ("cash") that, if paid instead of an option with a given strike price, would leave the executive with the same utility level, everything else (including the effort level and choice of volatility) equal. The utility of the individual, for the same effort level and utility choice, will be the same with the options or the amount of cash x. The rationale is the following: if the company gives the executive a certain amount of options with a given strike price, the executive will choose an optimal effort and volatility. Here, we look for the amount of money that, if paid to an executive that exercises that effort and chooses that volatility (as a result of a contract, not of the endogenous incentives) would reach the same utility. The exercise is the following, consider a strike price K and a number of options n that will induce an optimal effort process $\hat{u}$ and an optimal volatility process $\hat{v}$, with the final price of the stock S_T.

The CE of the investor x is determined by

$$\log x = E\left[\log(S_T - K)^+\right]. \quad (26)$$

We now introduce the main result in the following proposition. We note that, as throughout the rest of the paper, we have assumed that the interest rate is zero.

Proposition 3 *Consider the problem described in section 2. The value of the option for the company is*

$$E\left[(S_T - K)^+\right] = \frac{e^{\alpha^2 T}}{\bar{z}}, \quad (27)$$

where $\bar{z}$ is the optimal value of Proposition 2. The value of the option for the executive (CE), that we denote by x, as described by (26), is,

$$x = \frac{e^{\alpha^2 \frac{T}{2}}}{\bar{z}} \quad (28)$$

with the same $\bar{z}$ as above.

Proof: See the Appendix. $\diamond$

From (27) and (28), the ratio of the value of the option for the company and the value for the executive is given by $e^{\alpha^2 \frac{T}{2}}$. This ratio reinforces our results of Tables 1 and 2. We saw that the optimal strategy of the company is very sensitive to α the parameter that we interpret as the measure of the quality of the projects. When this parameter is high enough, we saw in Tables 1 and 2 that it is optimal for the company to give stock, rather than options. ¿From (27) and (28), we see that when α is high, the value of the option is considerably higher for the company than for the executive. This points in the direction of the suboptimality of giving options when α is very high.

In Table 3 we present some results. Options values are in the last two columns. C_0 represents the value of the option for the company and CE the value of the option for the executive. Since we are interested in the sensitivity of the option value to the underlying parameters, we have not chosen the optimal strike price. In fact, we fix the strike price at $K = 120$ and move the initial price of the stock (we consider three possibilities, $100, 110$ and 119) in order to analyze the effect of changes to the parameter values depending on how far out of the money the option is. We only consider cases in which the option is out of the money since this seems to be the best choice for the realistic values of the parameters. In order to compute the value $\bar{z}$ of (27) we use equation (13) (where, of course, $Z_0 = 1$).

The first observation is the fact that the value of the option is not greatly affected by α, the quality of the projects of the company. This is somewhat surprising since, as we argued in the previous section, the optimality of giving options versus stock as compensation depends heavily on the value of α. On the other hand, the value of the option is very sensitive to the type of executive, δ. This sensitivity is bigger the farther out of the money. Finally, the sensitivity of the value of the option to changes in the value of the stock seems to be larger the higher the type of the executive (δ).

6 The Case of Additional Cash Compensation

In this section we consider the case in which the executive receives, additionally to the stock options, some cash compensation. The problem of the executive is,

$$\max_{u,v} E\left[\log\left\{w + n(S_T - K)^+\right\} - \frac{1}{2}\int_0^T u_t^2 dt\right] \tag{29}$$

where $w > 0$ represents some cash compensation independent of the price of the stock of the company. The rest of the parameters are as in section 2. Carpenter (2000) considers the problem of optimal volatility for more general utility functions, when $\delta = 0$.

Alternatively, this case can be interpreted as the situation in which the executive has some wealth whose value is independent of the price of the stock. The main difference between this case and the case considered before is the fact that the number of options received in the compensation package becomes now relevant. The problem becomes considerably more difficult, but we can still compute numerically the optimal effort and volatility. We include in this section the main result and some numerical examples.

We start by considering the auxiliary function g defined by

$$g(t,y) \ = \ \left(K - \frac{w}{n}\right) N(d_2(y)) + yN(d_1(y)), \tag{30}$$

where

$$d_1(t,y) \ := \ \frac{\log\{\frac{yn}{cw}\} + \frac{1}{2}\alpha^2(T-t)}{\alpha\sqrt{T-t}}, \tag{31}$$

$$d_2(t,y) \ := \ \frac{\log\{\frac{yn}{cw}\} - \frac{1}{2}\alpha^2(T-t)}{\alpha\sqrt{T-t}}, \tag{32}$$

and

$$N(x) \ := \ \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{z^2}{2}\right\} dz. \tag{33}$$

We observe that

$$\begin{aligned}\frac{\partial}{\partial y} g(t,y) \ &= \ \left(K - \frac{w}{n}\right) \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(d_2(t,y))^2\right\} \frac{1}{y} \frac{1}{\alpha\sqrt{T-t}} \\ &+ \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(d_1(t,y))^2\right\} \frac{1}{\alpha\sqrt{T-t}} + N(d_1(t,y)).\end{aligned}$$

Let us consider the function $f : [1,\infty) \mapsto \mathbf{R}$ defined by

$$f(x) \ = \ \log\{x\} - 1 - \left(\frac{nK}{w} - 1\right)\frac{1}{x}.$$

We observe that $f(1) = -\frac{nK}{w} < 0$, $f(\infty) = \infty$, and f is strictly increasing (indeed, for every $y > 1$, $f'(y) = \frac{1}{y} + (\frac{nK}{w} - 1)\frac{1}{y^2} > 1/y - 1/y^2 > 0$). Thus, there exists a constant $c = c(n, K, w) \in (1, \infty)$ such that

$$f(c) = 0 \quad \text{and} \quad y > c \iff f(y) > 0. \tag{34}$$

Let $\tilde{z}$ be the solution of the nonlinear equation

$$S_0 = g\left(0, \frac{1}{z}\right) - z\delta^2 \bar{T}_0. \tag{35}$$

Proposition 4 *Consider an executive who wants to solve (29). Consider the stochastic process Z and the function $\bar{T}$ defined in section 3. Then the optimal effort is given by*

$$\hat{u}_t = \delta \tilde{z} Z_t \tag{36}$$

and the optimal volatility is given by

$$\hat{v}_t S_t = \alpha \tilde{H}_t \frac{\partial}{\partial y} g(t, \tilde{H}_t) + \alpha \tilde{z} \delta^2 Z_t \bar{T}_t. \tag{37}$$

The optimal effort and volatility determine that the price of the stock be given by

$$S_t = g(t, \tilde{H}_t) - \tilde{z}\delta^2 Z_t \bar{T}_t. \tag{38}$$

Here,

$$\tilde{H}_t = \frac{1}{\tilde{z} Z_t}. \tag{39}$$

Proof: See the Appendix. ◇

From the discussion above, it is clear that both the optimal effort $\hat{u}$ and the optimal volatility $\hat{v}$ are completely determined by $\tilde{z}$. This has to be found numerically from equation (35). We present several examples in Table 4. We see that the optimal effort increases with the number of options granted; it does not change much when the strike price changes; and it may be decreasing or increasing with respect to the time to maturity, depending on the number of options granted.

7 Conclusions

We present a model where an executive is granted stock options as compensation. Her decisions can affect the dynamics of the stock of the company in two ways: through her effort she can increase the expected return of the stock, but this effort produces disutility; also, she can choose the level of risk of the stock price. Higher level of risk will also result into higher expected return. The executive is risk averse. We get closed form solutions for the optimal effort and volatility level of the individual. The company chooses the strike price of the call options. The company cares both about the expected value of the stock price and about the cost of the compensation package. However, the company has to guarantee that the compensation package is such that the utility of the individual exceeds some reservation level. We find that there is a large range of optimal strike prices, depending on the values of the parameters of the model, but for most cases, it will be optimal to issue options out-of-the-money. We find that the optimal strike price is increasing with the type of the executive. Also, with the size of the company: it is optimal for larger companies to set higher strike prices.

A key parameter in our model is a measure of the quality of the projects of the company. We find that, although it does not have an important effect on the optimal strike price, when the quality of the projects is very high, it is optimal for the company to grant stock rather than options. Smaller companies are more sensitive to this effect and, therefore, more likely to grant stock than options.

In order to derive closed form solutions for the optimal policies of the executive we have to assume that the utility is logarithmic and the only source of wealth of the executive is the package of options granted as compensation. We use a dynamic setting because in a static setting, the solution is trivial (the executive chooses zero volatility) and not very interesting.

We can also extend some of our results to the case in which the executive receives cash, besides options and we find explicitely the optimal effort strategy. However, a procedure to derive the optimal strike price in a simple numerical way does not seem to be feasible. Another possible extension of the present work is to include tax effects in the model.

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A Appendix: Proofs

A.1 Proof of Proposition 1.

We consider the more general case in which the executive maximizes

$$\max_{u,v} E\left[F(S_T) - \int_0^T G(u_s)ds\right],$$

where

$$F(s) = \frac{1}{\gamma}[n(s-K)^+]^\gamma, \quad G(u) = \frac{u^2}{2}$$

and $\gamma < 1$ is the risk-aversion parameter. The log-utility case $F(x) = \log(x)$ corresponds to $\gamma = 0$. We approach this problem by familiar duality/martingale techniques, as introduced by Cox and Huang (1989), Karatzas, Lehozky and Shreve (1987). Consider the dual function

$$\tilde{F}(z) = \max_{s\geq 0}[F(s) - sz].$$

The maximum is attained at the points of the form

$$\hat{s} = \hat{s}(z,a) = \left(\left(\frac{z}{n^\gamma}\right)^{\frac{1}{\gamma-1}} + K\right)\mathbf{1}_{\{(\frac{z}{n^\gamma})^{\frac{1}{\gamma-1}} > \frac{K\gamma}{1-\gamma}\}} + a\mathbf{1}_{\{(\frac{z}{n^\gamma})^{\frac{1}{\gamma-1}} = \frac{K\gamma}{1-\gamma}\}},$$

where a is either 0 or $(\frac{z}{n^\gamma})^{\frac{1}{\gamma-1}} + K$. Consider also the dual function

$$\tilde{G}(z) = \max_u[-G(u) + \delta uz],$$

where the maximum is attained at

$$\hat{u} = \hat{u}(z) = \delta z.$$

Define the stochastic process

$$M_t = Z_t S_t - \delta\int_0^t Z_s u_s ds. \tag{40}$$

where Z is the exponential martingale defined in (7). Applying Ito's rule, we get

$$dM_t = (v_t - \alpha)S_t Z_t dW_t \quad \text{and} \quad M_0 = S_0. \tag{41}$$

Obviously, M is a local martingale, but we would like to prove that M is also a martingale. For that purpose, it is good enough to verify the condition

$$E\left[\sup_{0\leq t\leq T}|M_t|\right] < \infty.$$

According to the Burkholder-Davis-Gundy inequality (see, for instance, Theorem 3.3.28 of Karatzas and Shreve (1991)), it is enough to check that

$$E\left[\left(\int_0^T (v_t-\alpha)^2 S_t^2 Z_t^2 dt\right)^{1/2}\right] < \infty.$$

We observe that, according to Theorem 6.1.6 of Yong and Zhou (1999), $E\left[\sup_{0\le t\le T} Z_t^2\right] < \infty$. Since $E[\int_0^T |v_t S_t|^2 dt] < \infty$, that theorem applied to equation (1) gives $E\left[\sup_{0\le t\le T} S_t^2\right] < \infty$. Applying Hölder's inequality (see, for instance, Theorem 4.2 of Chow and Teicher (1988)) and again the condition $E[\int_0^T |v_t S_t|^2 dt] < \infty$, we note that

$$\begin{aligned}
E\left[\left(\int_0^T (v_t S_t Z_t)^2\, dt\right)^{1/2}\right] &\le E\left[\left(\sup_{0\le t\le T} Z_t^2 \int_0^T (v_t S_t)^2\, dt\right)^{1/2}\right] \\
&= E\left[\left(\sup_{0\le t\le T} Z_t^2\right)^{1/2}\left(\int_0^T (v_t S_t)^2\, dt\right)^{1/2}\right] \\
&\le \left(E\left[\sup_{0\le t\le T} Z_t^2\right]\right)^{1/2}\left(E\left[\int_0^T (v_t S_t)^2\, dt\right]\right)^{1/2} \\
&< \infty.
\end{aligned}$$

This implies that

$$E\left[\left(\int_0^T ((v_t-\alpha) S_t Z_t)^2\, dt\right)^{1/2}\right] < \infty,$$

and therefore that M is a martingale. Thus,

$$E[M_T] = S_0.$$

By definitions, we get

$$E\left[F(S_T) - \int_0^T G(u_s) ds\right] \le E\left[\tilde{F}(zZ_T) + \int_0^T \tilde{G}(zZ_s) ds\right] + zE[M_T],$$

where we can replace $E[M_T]$ by S_0. Therefore, the above inequality gives an upper bound for our maximization problem. The upper bound will be attained if the maximums are attained, and if $E[M_T] = S(0)$. In other words, the optimal terminal stock price and the optimal effort $\hat{u}$ are given by

$$S_T = \hat{s}(\check{z} Z_T, A) \quad \text{and} \quad \hat{u}_t = \delta \check{z} Z_t, \tag{42}$$

where A and $\check{z}$ are chosen so that A is any $\mathcal{F}_T$ measurable random variable taking only two possible values, 0 and $(\frac{\check{z} Z_T}{n^\gamma})^{\frac{1}{\gamma-1}} + K$, and so that $E[M_T] = S(0)$.

For $\gamma = 0$, we can choose $A \equiv 0$, and we see that

$$S_T = \frac{1}{\check{z} Z_T} + K.$$

Using this and the martingale property of M, we get

$$Z_t S_t = E\left[\frac{1}{\check{z}} + K Z_T - \check{z}\delta^2 \int_t^T Z_s^2 ds \;\Big|\; \mathcal{F}_t \right] = \frac{1}{\check{z}} + K Z_t - \check{z}\delta^2 Z_t^2 \bar{T}_t, \tag{43}$$

in the notation of (8). In other words,

$$S_t = \frac{1}{\check{z} Z_t} + K - \check{z}\delta^2 Z_t \bar{T}_t. \tag{44}$$

Using Ito's rule we see that the diffusion term of S is given by

$$\hat{v}_t S_t = \frac{\alpha}{\check{z} Z_t} + \alpha \check{z}\delta^2 Z_t \bar{T}_t,$$

as claimed in (12). We observe that the $\hat{u}$ and $\hat{v}$ defined above are adapted stochastic processes with $E[\int_0^T |\hat{u}_t|^2 dt] < \infty$ and $E[\int_0^T |\hat{v}_t S_t|^2 dt] < \infty$. Finally, the requirement $E[M_T] = S_0$, obtained by setting $t = 0$ in (44), gives

$$S_0 = \frac{1}{\check{z}} + K - \check{z}\delta^2 \bar{T}_0. \tag{45}$$

This is equivalent to (9), and we are done. ◇

A.2 Proof of Proposition 2

Our first objective is to compute the objective function of the company

$$h(K, n) := \lambda E[S_T] - n E[(S_T - K)^+], \tag{46}$$

and

$$e = e(n, K) := \max_{u,v} E\left[\log\left\{ n(S_T - K)^+ \right\} - \frac{1}{2}\int_0^T u_t^2 dt \right] \quad . \tag{47}$$

It is easily seen that

$$E[Z^2(t)] = e^{\alpha^2 t}, \quad E[Z(t)] = 1, \quad E[Z^{-1}(t)] = e^{\alpha^2 t}. \tag{48}$$

Thus, according to equation (13),

$$E[S_t] = \frac{1}{\check{z}} e^{\alpha^2 t} + K - \check{z}\delta^2 \bar{T}_t \quad .$$

We also see that $E[(S_T - K)^+] = e^{\alpha^2 T}/\check{z}$, and, using (24), we verify that the value $h(K,n)$ of (46) is equal to the value $V(z,n)$ of (18). Again using (24), we see that, indeed, the value z_0 of (22) corresponds to $K = 0$. Finally, using $\hat{u} = \delta\check{z}Z$ and (48), we can compute

$$e = \log(n/z) + \frac{\alpha^2}{2}T - \frac{1}{2}\delta^2\check{z}^2\bar{T}_0 \quad ,$$

in terms of z and n, and we can check that $e(n(z)) = R$, with $n(z)$ given in (20).

Now, the company wishes to maximize the function h as a function of K, so that the strike price is non-negative and the executive's rationality constraint is satisfied. Obviously, the function h can only attain a maximum on this interval at $\hat{K} = 0$, or $\hat{K} = K_i$, $i = 1, \ldots, k$.

$\diamond$

A.3 Proof of Corollary 1

In the case $\delta = 0$, the objective function of the company is

$$V(z) = \frac{1}{z}\lambda(e^{\alpha^2 T} - 1) - e^{R+\alpha^2 T/2} + \lambda S_0 \quad . \tag{49}$$

This is decreasing in z since $e^{\alpha^2 T} > 1$. Thus, it is optimal to take the smallest possible z, $z = z_0$, corresponding to $\hat{K} = 0$.

$\diamond$

A.4 Proof of Proposition 3

We introduce the notation

$$\bar{H}_t = \frac{1}{\bar{z}Z_t}$$

where Z is given by equation (7) and $\bar{z}$ is defined in Proposition 2. According to (13),

$$S_T = K + \bar{H}_T.$$

Thus,

$$E\left[(S_T - K)^+\right] = E[\bar{H}_T] = \frac{1}{\bar{z}}E\left[\frac{1}{Z_T}\right] = \frac{1}{\bar{z}}e^{-\alpha^2 T}.$$

In addition,

$$\begin{aligned} E\left[\log\left\{(S_T - K)^+\right\}\right] &= E\left[-\log\{\bar{z}\} - \log Z_T\right] \\ &= -\log\bar{z} + \frac{1}{2}\alpha^2 T. \end{aligned} \qquad \diamond$$

A.5 Proof of Proposition 4

We want to find $\hat{u}$ and $\hat{v}$ that solve the problem

$$\max_{u,v} \; E\left[F(S_T) - \int_0^T G(u_s)ds\right],$$

where

$$F(s) = \log\{w + n(s-K)^+\} \quad \text{and} \quad G(u) = \frac{u^2}{2}.$$

Introduce the stochastic process Z satisfying

$$dZ_t = -\alpha Z_t dW_t \quad \text{and} \quad Z(0) = 1.$$

Equivalently,

$$Z(t) = \exp\left\{-\frac{\alpha^2}{2}t - \alpha W(t)\right\}.$$

Applying the formula of integration by parts, we see that

$$Z_t S_t \;=\; S_0 + \int_0^t Z_s \delta u_s ds + \int_0^t S_s Z_s (v_s - \alpha) dW_s.$$

Let us consider the stochastic process M defined by

$$M_t := Z_t S_t - \delta \int_0^t Z_s u_s ds = S_0 + \int_0^t S_s Z_s (v_s - \alpha) dW_s.$$

Since $E[\int_0^T u_s^2 ds] < \infty$ and $E[\int_0^T v_s^2 S_t^2 ds] < \infty$, we can repeat the proof of Proposition 1 to verify that M is a martingale. Next, consider the dual function

$$\tilde{F}(z) = \max_{s \geq 0}[F(s) - sz].$$

The maximum is attained at the points of the form

$$\begin{aligned}
\hat{s} = \hat{s}(z,a) \;&=\; \left(K + \max\left(\frac{1}{z} - \frac{w}{n}, 0\right)\right) I_{\{\log\{w + \max(\frac{n}{z} - w, 0)\} - zK - \max(1 - \frac{wz}{n}, 0) > \log\{w\}\}} \\
&\quad + a I_{\{\log\{w + \max(\frac{n}{z} - w, 0)\} - zK - \max(1 - \frac{wz}{n}, 0) = \log\{w\}\}} \\
&=\; \left(K + \max\left(\frac{1}{z} - \frac{w}{n}, 0\right)\right) I_{\{\log\{w + n\max(\frac{1}{z} - \frac{w}{n}, 0)\} - zK - z\max(\frac{1}{z} - \frac{w}{n}, 0) > \log\{w\}\}} \\
&\quad + a I_{\{\log\{w + n\max(\frac{1}{z} - \frac{w}{n}, 0)\} - zK - z\max(\frac{1}{z} - \frac{w}{n}, 0) = \log\{w\}\}}
\end{aligned}$$

where a is either 0 or $K + \max\left(\frac{1}{z} - \frac{w}{n}, 0\right)$. Consider also the dual function

$$\tilde{G}(z) = \max_u[-G(u) + \delta u z],$$

where the maximum is attained at

$$\hat{u} = \hat{u}(z) = \delta z.$$

By definitions, we get

$$E\left[F(S_T) - \int_0^T G(u_s)ds\right] \leq E\left[\tilde{F}(zZ_T) + \int_0^T \tilde{G}(zZ_s)ds\right] + zE[M_T].$$

Since M is a martingale, $E[M(T)] = S_0$. Therefore the above inequality gives an upper bound for our maximization problem. The upper bound will be attained if the maximums are attained, and if $E[M_T] = S(0)$. In other words, the optimal solution is given by

$$\hat{S}_T = \hat{s}(\tilde{z}Z_T, A) \quad \text{and} \quad \hat{u}_t = \delta\tilde{z}Z_t,$$

where A and $\tilde{z}$ are chosen so that A is any $\mathcal{F}_T$ measurable random variable taking only two possible values, 0 and $K + \max\left(\frac{1}{z} - \frac{w}{n}, 0\right)$, and so that $E[M_T] = S(0)$. The optimal $\hat{v}$ is obtained from the martingale representation of

$$M_t = E[M_T|\mathcal{F}_t] = E\left[Z_T\hat{S}_T - \delta\int_0^T Z_s\hat{u}_s ds|\mathcal{F}_t\right] = S_0 + \int_0^t (\hat{v}_s - \alpha)S_s Z_s dW_s.$$

For the case $\alpha > 0$ that we are considering, we will set $A \equiv 0$. Introduce the stochastic process $\tilde{W}$ defined by

$$\tilde{W}_t = W_t + \alpha t.$$

We observe that $\tilde{W}$ is a Brownian motion under the measure $\tilde{P}$ defined by $d\tilde{P}/dP = Z_T$. Introduce also the notation

$$\tilde{H}(t) = \frac{1}{\tilde{z}Z_t} \quad \text{and} \quad \tilde{M}_t = S_t - \tilde{z}\delta^2\int_0^t Z_s ds.$$

We observe that

$$\tilde{H}(t) = \frac{1}{\tilde{z}}\exp\left\{\frac{\alpha^2}{2}t + \alpha W(t)\right\} = \frac{1}{\tilde{z}}\exp\left\{-\frac{\alpha^2}{2}t + \alpha\tilde{W}(t)\right\}.$$

Thus,

$$d\tilde{H}_t = \tilde{H}_t\alpha d\tilde{W}.$$

Hence, for every $0 \leq t \leq s$:

$$\tilde{H}_s = \tilde{H}_t\exp\left\{-\frac{1}{2}\alpha^2(s-t) + \alpha(\tilde{W}_s - \tilde{W}_t)\right\}.$$

We also note that

$$S_t - \tilde{z}\delta^2\int_0^t Z_s ds = \int_0^t \delta u_s ds + \int_0^t v_s S_s d\tilde{W}_s - \tilde{z}\delta^2\int_0^t Z_s ds.$$

Thus, the stochastic process $\tilde{M}$ is a $\tilde{P}-$martingale. Hence,

$$\tilde{M}_t = \tilde{E}[\tilde{M}_T|\ \mathcal{F}_t],$$

or equivalently

$$S_t = \tilde{E}\left[\hat{s}(\tilde{z}Z_T, 0) - \tilde{z}\delta^2 \int_t^T Z_s ds \mid \mathcal{F}_t\right].$$

We note that for every $0 \leq t \leq s$:

$$Z_s = \exp\left\{-\frac{\alpha^2}{2}s - \alpha W(s)\right\} = \exp\left\{\frac{\alpha^2}{2}s - \alpha\tilde{W}(s)\right\},$$

so for every $0 \leq t \leq s$:

$$\begin{aligned} \tilde{E}[Z_s|\ \mathcal{F}_t] &= \tilde{E}\left[Z_t \exp\left\{\frac{\alpha^2}{2}(s-t) - \alpha(\tilde{W}(s) - \tilde{W}(t))\right\} \mid \mathcal{F}_t\right] \\ &= Z_t \exp\left\{\alpha^2(s-t)\right\}. \end{aligned}$$

Denoting

$$\tilde{T}_t = \frac{e^{\alpha^2(T-t)} - 1}{\alpha^2},$$

we obtain

$$S_t = \tilde{E}[\hat{s}(\tilde{z}Z_T, 0)|\ \mathcal{F}_t] - \tilde{z}\delta^2 Z_t \tilde{T}_t.$$

Here,

$$\begin{aligned} \hat{s}(\tilde{z}Z_T, 0) &= \left(K + \max\left(\frac{1}{\tilde{z}Z_T} - \frac{w}{n}, 0\right)\right) I_{\{\log\{w+n\max(\frac{1}{\tilde{z}Z_T}-\frac{w}{n},0)\} - \tilde{z}Z_T K - \tilde{z}Z_T \max(\frac{1}{\tilde{z}Z_T}-\frac{w}{n},0) > \log\{w\}\}} \\ &= \left(K + \max\left(\tilde{H}(T) - \frac{w}{n}, 0\right)\right) I_{\{\log\{w+n\max(\tilde{H}(T)-\frac{w}{n},0)\} - \frac{1}{\tilde{H}(T)}K - \frac{1}{\tilde{H}(T)}\max(\tilde{H}(T)-\frac{w}{n},0) > \log\{w\}\}} \end{aligned}$$

Thus,

$$\begin{aligned} &\tilde{E}[\hat{s}(\tilde{z}Z_T, 0)|\ \mathcal{F}_t] \\ &= K\tilde{P}\left\{\log\{w + n\max(\tilde{H}(T) - \frac{w}{n}, 0)\} - \frac{1}{\tilde{H}(T)}K - \frac{1}{\tilde{H}(T)}\max(\tilde{H}(T) - \frac{w}{n}, 0) > \log\{w\}|\ \mathcal{F}_t\right\} \\ &\quad + \tilde{E}\left[\max\left(\tilde{H}(T) - \frac{w}{n}, 0\right) I_{\{\log\{w+n\max(\tilde{H}(T)-\frac{w}{n},0)\} - \frac{1}{\tilde{H}(T)}K - \frac{1}{\tilde{H}(T)}\max(\tilde{H}(T)-\frac{w}{n},0) > \log\{w\}\}} |\mathcal{F}_t\right]. \end{aligned}$$

To continue with the computations, let us consider the function $f : [1, \infty) \mapsto \mathbf{R}$ defined by

$$f(x) := \log\{x\} - 1 - \left(\frac{nK}{w} - 1\right)\frac{1}{x}.$$

Since $\frac{nK}{w} - 1 > -1$, there exists $c = c(n, K, w) \in (1, \infty)$ such that

$$f(c) = 0 \quad \text{and} \quad y > c \Longleftrightarrow f(y) > 0.$$

We note that

$$
\begin{aligned}
&\left\{\log\left\{w+n\max\left(\tilde{H}(T)-\frac{w}{n},0\right)\right\}-\frac{1}{\tilde{H}(T)}K-\frac{1}{\tilde{H}(T)}\max\left(\tilde{H}(T)-\frac{w}{n},0\right)>\log\{w\}\right\}\\
&= \left\{\left\{\log\left\{w+n\max\left(\tilde{H}(T)-\frac{w}{n},0\right)\right\}-\frac{1}{\tilde{H}(T)}K-\frac{1}{\tilde{H}(T)}\max\left(\tilde{H}(T)-\frac{w}{n},0\right)>\log\{w\}\right\}\right.\\
&\qquad\qquad \left.\cap\left\{\tilde{H}(T)-\frac{w}{n}>0\right\}\right\}\\
&\quad\bigcup\left\{\left\{\log\left\{w+n\max\left(\tilde{H}(T)-\frac{w}{n},0\right)\right\}-\frac{1}{\tilde{H}(T)}K-\frac{1}{\tilde{H}(T)}\max\left(\tilde{H}(T)-\frac{w}{n},0\right)>\log\{w\}\right\}\right.\\
&\qquad\qquad \left.\cap\left\{\tilde{H}(T)-\frac{w}{n}\le 0\right\}\right\}\\
&= \left\{\left\{\log\left\{w+n\left(\tilde{H}(T)-\frac{w}{n}\right)\right\}-\frac{1}{\tilde{H}(T)}K-\frac{1}{\tilde{H}(T)}\left(\tilde{H}(T)-\frac{w}{n}\right)>\log\{w\}\right\}\right.\\
&\qquad\qquad \left.\cap\left\{\tilde{H}(T)-\frac{w}{n}>0\right\}\right\}\\
&\quad\bigcup\left\{\left\{\log\{w+n(0)\}-\frac{1}{\tilde{H}(T)}K-\frac{1}{\tilde{H}(T)}(0)>\log\{w\}\right\}\right.\\
&\qquad\qquad \left.\cap\left\{\tilde{H}(T)-\frac{w}{n}\le 0\right\}\right\}\\
&= \left\{\left\{\log\{n\tilde{H}(T)\}-\frac{1}{\tilde{H}(T)}K-1+\frac{w}{n\tilde{H}(T)}>\log\{w\}\right\}\cap\left\{\tilde{H}(T)-\frac{w}{n}>0\right\}\right\}\\
&\quad\bigcup\left\{\left\{\log\{w\}-\frac{1}{\tilde{H}(T)}K>\log\{w\}\right\}\cap\left\{\tilde{H}(T)-\frac{w}{n}\le 0\right\}\right\}\\
&= \left\{\left\{\log\left\{\frac{n\tilde{H}(T)}{w}\right\}>\frac{nK}{w\left(\frac{n\tilde{H}(T)}{w}\right)}+1-\frac{w}{n\tilde{H}(T)}\right\}\cap\left\{\tilde{H}(T)-\frac{w}{n}>0\right\}\right\}\\
&= \left\{\left\{f\left(\frac{n\tilde{H}(T)}{w}\right)>0\right\}\cap\left\{\tilde{H}(T)-\frac{w}{n}>0\right\}\right\}\\
&= \left\{\left\{\frac{n\tilde{H}(T)}{w}>c\right\}\cap\left\{\tilde{H}(T)-\frac{w}{n}>0\right\}\right\}\\
&= \left\{\tilde{H}(T)>c\frac{w}{n}\right\}.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\tilde{E}[\hat{s}(\tilde{z}Z_T,0)|\,\mathcal{F}_t] &= K\tilde{P}\left\{\tilde{H}_T>c\frac{w}{n}|\,\mathcal{F}_t\right\}+\tilde{E}\left[\left(\tilde{H}_T-\frac{w}{n}\right)I_{\{\tilde{H}_T>c\frac{w}{n}\}}|\,\mathcal{F}_t\right]\\
&= K\int_{\frac{1}{\alpha}\log\{\frac{cw}{n}\frac{1}{\tilde{H}_t}\}+\frac{1}{2}\alpha(T-t)}^{\infty}\frac{1}{\sqrt{2\pi(T-t)}}\exp\left\{-\frac{1}{2}\frac{x^2}{(T-t)}\right\}dx
\end{aligned}
$$

$$+\int_{\frac{1}{\alpha}\log\{\frac{cw}{n}\frac{1}{\tilde{H}_t}\}+\frac{1}{2}\alpha(T-t)}^{\infty}\left(\tilde{H}_t\exp\left\{-\frac{1}{2}\alpha^2(T-t)+\alpha x\right\}-\frac{w}{n}\right)$$
$$\frac{1}{\sqrt{2\pi(T-t)}}\exp\left\{-\frac{1}{2}\frac{x^2}{(T-t)}\right\}dx.$$

To simplify the above equation, let us define the probability measure $\bar{P}$ by

$$\frac{d\bar{P}}{d\tilde{P}}=\exp\left\{\alpha\tilde{W}_T-\frac{1}{2}\alpha^2T\right\}=\frac{\tilde{H}(T)}{\tilde{H}(0)}.$$

According to Girsanov's theorem, the stochastic process $\bar{W}$ defined by

$$\bar{W}_t=\tilde{W}_t-\alpha t=W_t$$

is a standard Brownian motion on $(\Omega,\mathcal{F},\bar{P})$. We note that according to Bayes' formula,

$$\begin{aligned}
\tilde{E}\left[\tilde{H}_T I_{\{\tilde{H}_T>c\frac{w}{n}\}}|\ \mathcal{F}_t\right] &= \exp\left\{\alpha\tilde{W}_t-\frac{1}{2}\alpha^2t\right\}\bar{E}\left[\tilde{H}_T\frac{1}{\exp\left\{\alpha\tilde{W}_T-\frac{1}{2}\alpha^2T\right\}}I_{\{\tilde{H}_T>c\frac{w}{n}\}}|\ \mathcal{F}_t\right]\\
&= \tilde{H}_t\bar{E}\left[I_{\{\tilde{H}_T>c\frac{w}{n}\}}|\ \mathcal{F}_t\right]\\
&= \tilde{H}_t\bar{P}\left\{\tilde{H}_T>c\frac{w}{n}|\ \mathcal{F}_t\right\}\\
&= \tilde{H}_t\bar{P}\left\{\tilde{H}_t\exp\left\{\alpha(\bar{W}_T-\bar{W}_t)+\frac{1}{2}\alpha^2(T-t)\right\}>c\frac{w}{n}|\ \mathcal{F}_t\right\}\\
&= \tilde{H}_t\bar{P}\left\{\alpha(\bar{W}_T-\bar{W}_t)+\frac{1}{2}\alpha^2(T-t)>\log\left\{\frac{1}{\tilde{H}_t}\frac{cw}{n}\right\}|\ \mathcal{F}_t\right\}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\tilde{E}[\hat{s}(\tilde{z}Z_T,0)|\ \mathcal{F}_t] &= \left(K-\frac{w}{n}\right)\tilde{P}\left\{\tilde{H}_T>c\frac{w}{n}|\ \mathcal{F}_t\right\}+\tilde{E}\left[\tilde{H}_T I_{\{\tilde{H}_T>c\frac{w}{n}\}}|\ \mathcal{F}_t\right]\\
&= \left(K-\frac{w}{n}\right)N(d_2(t,\tilde{H}_t))+\tilde{H}_tN(d_1(t,\tilde{H}_t)),
\end{aligned}$$

where

$$\begin{aligned}
d_1(t,\tilde{H}_t) &:= \frac{\log\{\frac{\tilde{H}_tn}{cw}\}+\frac{1}{2}\alpha^2(T-t)}{\alpha\sqrt{T-t}}\\
d_2(t,\tilde{H}_t) &:= \frac{\log\{\frac{\tilde{H}_tn}{cw}\}-\frac{1}{2}\alpha^2(T-t)}{\alpha\sqrt{T-t}},
\end{aligned}$$

and

$$N(x):=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{z^2}{2}\right\}dz.$$

This means that we can write

$$\tilde{E}[\hat{s}(\tilde{z}Z_T, 0)|\ \mathcal{F}_t] = g(t, \tilde{H}_t),$$

where the function g is defined by

$$\begin{aligned} g(t,y) \quad &:= \quad K\int_{\frac{1}{\alpha}\log\{\frac{cw}{n}\frac{1}{y}\}+\frac{1}{2}\alpha(T-t)}^{\infty} \frac{1}{\sqrt{2\pi(T-t)}}\exp\left\{-\frac{1}{2}\frac{x^2}{(T-t)}\right\}dx \\ &\quad + \int_{\frac{1}{\alpha}\log\{\frac{cw}{n}\frac{1}{y}\}+\frac{1}{2}\alpha(T-t)}^{\infty}\left(y\exp\{-\frac{1}{2}\alpha^2(T-t)+\alpha x\}-\frac{w}{n}\right) \\ &\qquad\qquad \frac{1}{\sqrt{2\pi(T-t)}}\exp\left\{-\frac{1}{2}\frac{x^2}{(T-t)}\right\}dx \\ &= \quad \left(K-\frac{w}{n}\right)N(d_2(t,y)) + yN(d_1(t,y)). \end{aligned}$$

Thus,

$$S_t = g(t, \tilde{H}_t) - \tilde{z}\delta^2 Z_t \bar{T}_t. \tag{50}$$

If we take $t = 0$ in the above equation, we get

$$S_0 = g(0, \tilde{H}_0) - \tilde{z}\delta^2 Z_0 \bar{T}_0 = g\left(0, \frac{1}{\tilde{z}}\right) - \tilde{z}\delta^2 \bar{T}_0. \tag{51}$$

Although this does not give an explicit solution for $\tilde{z}$, it is possible to obtain it numerically. Applying Ito's formula in (50), and comparing equations (2) and (50), we obtain

$$\hat{v}_t S_t = \alpha\tilde{H}_t \frac{\partial}{\partial y} g(t, \tilde{H}_t) + \alpha\tilde{z}\delta^2 Z_t \bar{T}_t. \tag{52}$$

Here,

$$\begin{aligned} \frac{\partial}{\partial y} g(t,y) \quad &= \quad \left(K-\frac{w}{n}\right)\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}(d_2(t,y))^2\right\}\frac{1}{y}\frac{1}{\alpha\sqrt{T-t}} \\ &\quad + \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}(d_1(t,y))^2\right\}\frac{1}{\alpha\sqrt{T-t}} + N(d_1(t,y)). \end{aligned}$$

We observe that the $\hat{u}$ and $\hat{v}$ defined above are adapted stochastic processes with $E[\int_0^T |\hat{u}_t|^2 dt] < \infty$ and $E[\int_0^T |\hat{v}_t S_t|^2 dt] < \infty$. Therefore, if $w > 0$ and $\alpha > 0$, the optimal drift $\hat{u}_t = \delta\tilde{z}Z_t$ and optimal volatility $\hat{v}_t$ are given by equations (51)-(52). $\diamond$

Table 1:
Optimal Strike Price
for a "Large" Company

$S_0 = 100;\ T = 5;\ \lambda = 1000$				
α	δ	R	$\hat{K}$	$\hat{n}$
0.03	1	1	106.758	995.1
0.03	1	2	106.053	995.02
0.03	1	5	103.377	994.395
0.03	2	1	114.425	995.145
0.03	2	2	113.096	995.081
0.03	2	5	108.205	994.641
0.03	5	1	138.891	995.189
0.03	5	2	135.793	995.14
0.03	5	5	124.730	994.843
0.1	1	1	106.825	946.793
0.1	1	2	106.825	945.912
0.1	1	5	0	1.447
0.1	2	1	114.571	947.255
0.1	2	2	113.224	946.554
0.1	2	5	108.259	941.724
0.1	5	1	139.294	947.739
0.1	5	2	136.155	947.204
0.1	5	5	124.934	943.952
0.2	1	1	0	0.0246
0.2	1	2	0	0.0668
0.2	1	5	0	1.343
0.2	2	1	0	0.0246
0.2	2	2	0	0.0668
0.2	2	5	0	1.341
0.2	5	1	140.673	805.674
0.2	5	2	137.393	803.652
0.2	5	5	125.619	791.23

Table 2:
Optimal Strike Price
for a "Small" Company

$S_0 = 100$; $T = 5$; $\lambda = 100$				
α	δ	R	$\hat{K}$	$\hat{n}$
0.03	1	1	105.022	99.486
0.03	1	2	104.107	99.464
0.03	1	5	99.779	99.056
0.03	2	1	111.178	99.496
0.03	2	2	109.509	99.48
0.03	2	5	102.348	99.284
0.03	5	1	131.380	99.505
0.03	5	2	127.614	99.495
0.03	5	5	112.713	99.399
0.1	1	1	0	0.0265
0.1	1	2	0	0.072
0.1	1	5	0	1.447
0.1	2	1	111.279	94.519
0.1	2	2	109.584	94.351
0.1	2	5	0	1.446
0.1	5	1	131.682	94.622
0.1	5	2	127.862	94.508
0.1	5	5	112.676	93.447
0.2	1	1	0	0.0246
0.2	1	2	0	0.0668
0.2	1	5	0	1.3425
0.2	2	1	0	0.0246
0.2	2	2	0	0.0668
0.2	2	5	0	1.3414
0.2	5	1	132.712	79.99
0.2	5	2	128.702	79.557
0.2	5	5	0	75.301

Table 3:
Option Price

$K = 120; T = 5$				
S_0	α	δ	C_0	CE
100	0.03	1	0.2486	0.2481
100	0.03	2	0.9608	0.9587
100	0.1	1	0.2661	0.2596
100	0.1	2	1.0278	1.0024
100	0.2	1	0.3335	0.3017
100	0.2	2	1.2846	1.1623
110	0.03	1	0.4804	0.4793
110	0.03	2	1.7193	1.7154
110	0.1	1	0.5139	0.5012
110	0.1	2	1.8355	1.7902
110	0.2	1	0.6423	0.5812
110	0.2	2	2.2790	2.0621
119	0.03	1	1.8019	1.7978
119	0.03	2	4.0231	4.0140
119	0.1	1	1.9121	1.8649
119	0.1	2	4.2641	4.1588
119	0.2	1	2.3270	2.1056
119	0.2	2	5.1688	4.6769

Table 4: Optimal Effort when $w > 0$			
$S_0 = 100$, $w = 50$ $\delta = 0.5$, $\alpha = 0.5$			
n	K	T	$\tilde{z}$
1	90	1	0.0062743
5	90	1	0.0114899
1	90	5	0.0071296
5	90	5	0.0106822
1	100	1	0.0061745
5	100	1	0.0113005
1	100	5	0.0071088
5	100	5	0.0106929
1	110	1	0.0060784
5	110	1	0.0111076
1	110	5	0.0070867
5	110	5	0.0106937