1. A bank offers a 5-year instrument that promises to repay the initial invested amount (with zero interest) if S&P 500 index experiences a gain during the 5-year period, and, in addition to that amount, if S&P 500 experiences a loss, it promises to pay 80% of the relative return r on the invested amount, where the relative return r is computed as the absolute value of the percentage loss in S&P 500.

Is this contract equivalent to a position in S&P 500, in a bond, in a combination of a bond and a call option, or in a combination of a bond and a put option?

2. At time zero you enter a short position in a forward contract on

8 shares of the stock XYZ at the forward price of 52.00. Moreover, you

sell (write) 2 exotic options each of which gives the holder

one share of the stock (only) if the price of one share is above 55.00 and which

pays the holder 50.00 (only) if the price is below 55.00. The today's selling price

of one option of this kind is 53.00. You also buy one zero-coupon

bonds with continuously compounded annual yield of 5%, with face

value of 100.00.

The maturity of all of your positions

is \(T=3\) months.

Assume that, after those trades are put in place, the initial capital you

have (need) is invested (borrowed) at zero interest rate.

Enter your total profit or loss if

at maturity the price of one stock share is 58.00

Enter your total profit or loss if

at maturity the price of one stock share is 47.00

3. Suppose company A can borrow USD at the rate of 6%, and euros at the rate of 11%. Company B can borrow USD at the rate of 4%, and euros at the rate of 10%. Suppose that company A needs

to borrow USD, and company B needs to borrow euros. In the following, you may

ignore the exchange rate risk and the credit risk.

Enter the expected percentage aggregate gain for all parties, if the two companies enter a swap contracts (possibly via a third party, a bank):

Suppose a bank offers the following swap contracts:

Receiving 5% on USD from A, and paying 11% on euros to A

Receiving 9% on euros from B, and paying 4% on USD to B.

Enter the percentage profit-loss with these contracts for all, negative numbers for loss

4. Suppose you promised your child a hundred shares of stock A for her eighteenth birthday, one year ahead. Which one of the following you may want to do to hedge this promise:

Buy shares of stock A today,

Sell call options on stock A today,

Sell put options on stock A today

All of the above

5. We want to show that the payoff of the bear spread can be

created from long/short positions in put options and cash. You can ignore the initial cost of the

positions, and you can assume that the interest rate is zero, for simplicity.

For short positions, use negative numbers. Suppose the bear spread points are K1 and K2, that is, the payoff is zero when the price \(S\) of the underlying is below K1, it is K1-K2 when S is above K2, and it is K1 – S for S between K1 and K2.

Enter the number of required put options with strike price equal to K1

Enter the number of required put options with strike price equal to K2

2. What is the present value of \$100.00 paid nine months from now if

a. the annual nominal rate is 6%, and compounding is done quarterly?

b. the effective annual rate is 6%, and compounding is done quarterly?

c. the annual continuous rate is 6\%?

3. During your last year as a college student your parents give you two options:

a.

They will pay you \$6,000 at the beginning of the year, \$5,500 after three months, and another \$5,500 six months after the beginning of the year. They expect you to find a good job when you complete your degree and request that you pay them \$9,000 two years from now, as a partial payback.

b. Alternatively, they will pay you \$6,500, \$6,000 and \$6,000 and you have to pay back \$11,000, at the same times as in part a. Assume that the nominal annual rate for the next two years is 4%, and

compounding is done quarterly.

What is the difference of the present value of the deal in part a minus the present value of the deal in part b?

4. You take a loan on $\$500,000$ for 30 years, at the annual nominal interest rate of 6%, compounded monthly. The loan installments also have to be paid monthly. The bank's APR is 6.6%. What is the amount of fees the bank is charging you?

5. A default-free coupon bond maturing in one year, that pays a coupon of $ \$5.00$ after six months, and makes a final payment of $ \$105.00$ (the last coupon and the principal), trades at $\$104.00$ today.

Moreover, a six-month default-free zero-coupon bond is traded at \$98, and pays \$100.00 at maturity. What should the price P of the one-year default-free zero-coupon bond be in order to prevent arbitrage opportunities, if the bond pays \$ 100 at maturity?

5. The one-year zero-coupon bond with nominal $\$100$ trades at $\$98$, and the four-year zero coupon bond with nominal $\$100$ trades at $\$90$. What is the (annualized) forward rate for the period between one year and four years from today?

1. Suppose the price of European put plus the price of the underlying stock (no dividends) is less than the present value of the strike price. To take advantage of this and make arbitrage profit, you would
(a) Borrow from bank, sell put and buy stock
(b) Borrow from bank, sell put and sell stock
(c) Borrow from bank, buy put and stock
(d) Sell put and stock and deposit proceeds to bank.
(e) None of the above

.

2. Suppose the price of an American put option is strictly less than the difference of the strike price and the price of then underlying. Which one of the following would result in arbitrage profit:
   (a) Buy the put and exercise it immediately
   (b) Buy the put and wait
   (c) Sell the put
   (d) Sell short the underlying
   (e) None of the above
   (f)
3. The price of the three-month US Treasury bills is $\$98.90$. A given stock trades at $\$99$, and the European calls on the stock with strike price 100 and maturity three months are trading at $\$1.5$. What is the no-arbitrage price of the put option with same strike and maturity?

4. Suppose the discounted value of the forward price is strictly less than the difference between the price of the underlying stock and the present value of its (non-random) dividends to be paid during the life of the forward contract. Suppose you sell the stock short go long in the forward contract. Doing which of the following can you make arbitrage profit:
   (a) When the dividends are due, sell the forward contract to pay the dividends you owe
   (b) When the dividends are due, sell short more stock
   (c) Borrow cash to pay for dividends

5. Consider two European call options on the same underlying and with the same maturity, but with different strike prices, K_1 and K_2 respectively. Suppose that $K_2 > K_1$. Suppose you write a call with strike price K_2, buy a call with strike price K_1, and deposit K_2-K_1 in the bank. Your final payoff is negative (ignoring the initial option prices):
   (a) Sometimes
   (b) Always
   (c) Never

1. Consider a Cox-Ross-Rubinstein binomial tree model

With the expected value of relative stock return S(t)/S(t-1)-1 equal to $1.02

$, and variance equal to 0.0216. Suppose the up factor is u=1.2.

What is the value of the down factor d?

What is the probability of up move p?

{\bf 2.} Suppose you have written an option that pays the value of the squared difference between the stock price at maturity and $\$100.00$; that is, it pays $(S(1)-100)^2$.

The stock currently trades at S(0)=100. Your model is a single period binomial tree with up value for the stock equal to 101 and the down value equal to 98. The interest rate is 0.01. What is

the cost $C(0)$ of the replicating portfolio?.

How many shares does the replicating portfolio hold? (If selling short, use the minus sign)

{\bf 3.} Suppose the market consists of the bank account with $r=0$ and two stocks. We consider a single-period model with three possible states in the future. Stock S_1 starts at $S_1(0)=1$ and $S_1(1)$ takes possible values $(3,3,4)$ in the future, with positive probabillties. Stock S_2 starts at $S_2(0)=2$ and $S_2(1)$ takes possible values $(6,8,8)$ in the future, with positive probabilities, where the values are ordered according to the future states of the world. Is there an equivalent martingale measure for this model, that is, are there probabilities for the futures states under which both stocks are martingales?

Is there arbitrage in the market?

4. In a two-period CRR model with \(r=0.01\) per period, \(S(0)=100\), \(u=1.02\), and \(d=0.98\), consider a European derivative that expires after two periods, and pays the value of the squared stock

price, $S^2(T)$, if the stock price $S(T)$ is higher than 100.00 at maturity. Otherwise (when $S(T)$ is less or equal to 100), the option pays zero. What is the price of this option.

Practice problem 4.

Consider a model in which the price of the stock can go up by a factor of 1.12 or down by a factor of 0.92 per period. The interest rate is 5% per period. Compute the price of a security that pays $1,000$ if the stock goes down for five consecutive periods and pays 40 if the stock goes up for five consecutive periods. Otherwise, it pays zero.

5. Consider a single-period binomial model with two periods where the stock has an initial price of 100 and can go up by a factor of 1.15 or down by a factor of 0.95 in each period. The price of the European call option on this stock with strike price 115 and maturity in two periods is $5.424.$

What should be the price of the risk-free security that pays 1 after one period regardless of what happens? We assume, as usual, that the interest rate r per period is constant.

PRACTICE PROBLEM:

5. Consider a single-period binomial model with two periods where the stock has an initial price of 100 and can go up by a factor of 1.15 or down by a factor of 0.95 in each period. The price of the European put option on this stock with strike price 94 and maturity in two periods is $1.297578.$

What should be the price of the risk-free security that pays 1 after one period regardless of what happens? We assume, as usual, that the interest rate r per period is constant.

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1. Let Z be a Brownian motion process. Consider the expression

   $dZ^4 = AZ^Bdt+CZ^D dZ$

   what is A equal to?

   B?

   C?

   D?

PRACTICE PROBLEM 1. Let Z be a Brownian motion process. Consider the expression

$dZ^3 = AZ^Bdt+CZ^D dZ$

what is A equal to?

B?

C?

D?

1. Let S be the stock in the Black-Scholes-Merton model,

   $dS=S(\mu dt+\sigma dW)$

   Consider the expression

   $dS^5 = (A\mu+B\sigma^2) S^Cdt+D\sigma S^K dW$

   what is A equal to?

   B?

   C?

D?
K?

2. PRACTICE PROBLEM 2. Let S be the stock in the Black-Scholes-Merton model,

   $dS=S(\mu dt+\sigma dW)$

   Consider the expression

   $dS^4 = (A\mu+B\sigma^2) S^Cdt+D\sigma S^K dW$

   what is A equal to?

   B?

   C?

   D?
   K?

3. Consider the previous two problems with $\mu=0$ and $\sigma=1$. Suppose the correlation between Z and W is 0.9. Consider the dynamics of the ratio Z/S:

$d(Z/S)= (AZ^B +C)S^Ddt + \frac{1}{S} dZ + KZ S^L dW$

What is A, B, C, D, K, L

PRACTICE PROBLEM 3

Consider the previous two practice problems with $\mu=0$ and $\sigma=1$. Suppose the correlation between Z and W is 0.9. Consider the dynamics of the product ZS:

$d(ZS)= AZ^BS^Cdt + S dZ + DZ S^K dW$

What is A, B, C, D,

4. In the Merton-Black-Scholes model with \(S(0)=1\), \(\mu=0.1\) and \(\sigma=0.2\), find the expected value of the square root of stock price cubed \(\sqrt{S(1)}\) at time one.

PRACTICE PROBLEM:

In the Merton-Black-Scholes model with \(S(0)=1\), \(\mu=0.1\) and \(\sigma=0.2\), find the expected value of stock price squared \(S^2(1)\) at time one.

5. Consider the following interest rate model: the continuous interest rate of a bank account in the time period \([0,1]\) is a time dependent process given by \(r(t)=0.05-0.06e^{-t}.\)

a. In the expression

\(\int_0^tsdr(s) =tr(t)+A\int_0^tr(s)ds \)

what is the value of \(A\)?

b. How much is one dollar invested at time \(t=0\) worth at time

\(t=1\)$? That is, enter the value of \(e^{\int_0^1r(s)ds\).

c. In the expression

\(dr(t)=(A-r(t))dt\) what is the value of \(A\)?

PRACTICE PROBLEM:

6. Consider the following interest rate model: the continuous interest rate of a bank account in the time period \([0,1]\) is a time dependent process given by \(r(t)=1-0.92e^{-t}.\)

b. In the expression

\(\int_0^tr(s)ds= tr(t) + A\int_0^tsdr(s) \)

what is the value of \(A\)

b. How much is one dollar invested at time \(t=0\) worth at time

\(t=1\)$? That is, enter the value of \(e^{\int_0^1r(s)ds\).

c. In the expression

\(dr(t)=(A-r(t))dt\) what is the value of \(A\)?

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1. Suppose that the stock price today is \(S(t)=3.00\), the annual volatility is \(\sigma=0.15\), and the time to maturity is \(4\) months. Consider a contingent claim whose Black-Scholes price is given by the function

\(V(t,s)=s^3e^{0.2(T-t)}\)

where the time is in annual terms.

What is the claim price today?
What is the interest rate equal to?
If the stock at maturity is \(S(T)= 4.00\), what is the payoff of the claim at maturity?

PRACTICE PROBLEM:
Suppose that the stock price today is \(S(t)=2.00\), interest rate is \(r=0\), and the time to maturity is \(3\) months. Consider a contingent claim whose Black-Scholes price is given by the function

$V(t,s)=s^2 e^{2(T-t)}$

where the time is in annual terms.

What is the claim price today?
What is the interest rate equal to?
If the stock at maturity is $S(T)= 2.00$, what is the payoff of the claim at maturity?

PRACTICE PROBLEM:

.Consider a Merton-Black-Scholes model with $r=0.05$, $sigma=0.2$, $T=0.5$ years, $S(0)=100$, and a call option with the strike price $K=100$. Using the normal distribution table, or a software program that computes normal distribution values, find the price of the call option, when there are no dividends. However, you are not allowed to use an option price calculator. In other words, you are required to use the Black-Scholes formula.

2. Let $C(t,s)$ be a price function (the value of the replicating portfolio) for the option with payoff $g(S(T))$ in the Black-Scholes-Merton model in which money can be borrowed from the bank at interest rate R. If money is deposited in the bank, it returns interest at rate $r<R$.

The replication portfolio requires borrowing money at rate R for those values of (t,s) for which

$sC_s(t,s)>C(t,s)$

$sC_s(t,s)<C(t,s)$

For all values of $t,s)$, no matter what function $g(s)$ is.

Which one of the following is the Black-Scholes pricing partial differential equation that is valid no matter with function $g(s)$ is

$C_t+\frac{1}{2}\cdot 2s^2C_{ss}+r(sC_s-C) =0 .$

$C_t+\frac{1}{2}\cdot 2s^2C_{ss}+R(sC_s-C) =0 .$

$C_t+\frac{1}{2}\cdot \sigma^2s^2C_{ss}+r\min\{0, sC_s-C\} +R\max\{0,sC_s-C\}=0 .$

$C_t+\frac{1}{2}\cdot \sigma^2s^2C_{ss}+R\min\{0, sC_s-C\} +r\max\{0,sC_s-C\}=0 .$

PRACTICE PROBLEM
Consider the Black-Scholes-Merton model in which you don't know the value of volatility σ, you only know that it is a constant that belong to the interval $[\sigma_{\min},\sigma_{\max}]$. Suppose you sell the option with payoff $g(S(T))$. You want to be sure that you charge the price $C(t,s)$ that is just large enough to enable you to have enough funds to pay $g(S(T))$ at maturity.

Which one of the following is the Black-Scholes pricing partial differential equation that is valid no matter with function $g(s)$ is?

$rC= rsC_s+C_t+\frac{1}{2}\cdot \sigma_\max^2s^2C_{ss}.$

$rC= rsC_s+C_t+\frac{1}{2}\cdot \sigma_\min^2s^2C_{ss} .$

$rC= rsC_s+C_t+\frac{1}{2}\cdot \sigma_\max^2s^2\max\{0,C_{ss}\}+\frac{1}{2}\cdot \sigma_\min^2s^2\min\{0,C_{ss}\}.$

$rC= rsC_s+C_t+\frac{1}{2}\cdot \sigma_\min^2s^2\max\{0,C_{ss}\}+\frac{1}{2}\cdot \sigma_\max^2s^2\min\{0,C_{ss}\} .$

6. Consider a Merton-Black-Scholes model with $r=0.07$, $\sigma=0.3$, $T=0.5$ years, $S(0)=100$, and a call option with the strike price $K=100$. Using the normal distribution table (or an appropriate software program)

4. Find the Black-Scholes formula for the derivative paying at maturity T the amount $\$A$ if $S(T)\le K_1$ or if $S(T)\ge K_2$, and zero otherwise, in the Black-Scholes continuous-time model.

The price of a European stock is given, in Euros, by

$dS(t)/S(t)=\mu dt+\sigma dW_1(t)$

The exchange rate Dollar/Euro is given by

$dQ(t)/Q(t)=\beta dt + \delta dW_2(t)$

where W_1 has correlation ρ with W_2.

(a) Select the Brownian motions W_1^* and W_2^* such that the discounted dollar value of the euro $e^{-rt}Q(t)e^{r_ft}$ is a martingale, and the discounted dollar value of the European stock $e^{-rt}Q(t)S(t)$ is also a martingale under the corresponding probability P^*, where r and r_f are the US and the Euro risk-free interest rates.

$W_1^*(t)= \frac{\mu-r_f}{\sigma} t + W_1(t)$

$W_2^*(t)= \frac{\beta+r_f-r_d}{\delta} \times t + W_2(t)$

$W_1^*(t)= \frac{\mu-r_f-\rho-\sigma\delta}{\sigma} \times t + W_1(t)$

$W_2^*(t)= \frac{\beta+r_f-r_d}{\delta} \times t + W_2(t)$

$W_1^*(t)= \frac{\mu-r_f}{\sigma} \times t + W_1(t)$

$W_2^*(t)= \frac{\beta-r_d}{\delta} \times t + W_2(t)$

$W_1^*(t)= W_1(t)$

$W_2^*(t)= W_2(t)$

PROBLEM 4.

We consider the payoff $C(T)$ in dollars in the amount

$C(T)=\log\left(\sqrt{Z(T)}\right),$

where $Z(t)$ is the price in dollars of the European stock at time t. Suppose you know from the previous problem the values of A,B,C such that

$Z(T)=Q(T)S(T)=Q(t)S(t)e^{A[T-t]+B[W_1^*(T)-W_1^*(t)]+C[W_2^*(T)-W_2^*(t)]}$

The price of the claim $C(T)$ is equal to

$e^{-r(T-t)}Q(t)S(t)N(d_1)$

$e^{-r(T-t)}\{\frac12\log Q(t)S(t) +\frac12\{A(T-t) + \frac12(B^2+C^2)(T-t)\}$

$e^{-r(T-t)}\{\frac12\log Q(t)S(t) \}$

$e^{-r(T-t)}\{\frac12\log Q(t)S(t) +\frac12\{A(T-t) \}$

PRACTICE PROBLEM:

Consider a foreign stock,

$dS(t)/S(t)=\mu_S dt +\sigma_S dW_S(t),$

and the exchange rate

$dQ(t)/Q(t)=\mu_Qdt +\sigma_QdW_Q(t),$

where W_S, W_Q are Brownian Motions with correlation ρ. Thus, QS is the value of the foreign stock in domestic currency. Denote by r_f the (constant) foreign risk-free rate.

Find the Brownian motions W_Q^*,W_S^* corresponding to the probability P^* under which the discounted domestic value of the foreign stock $e^{-rt}Q(t)S(t)$ and the discounted domestic value of one unit of foreign currency $e^{-rt}Q(t)e^{r_ft}$ are both martingales.

$W_S^*(t)= \frac{\mu-r_f-\rho-\sigma\delta}{\sigma} t + W_1(t)$; $W_Q^*(t)= \frac{\beta+r_f-r}{\delta} t + W_Q(t)$

$W_S^*(t)= W_S(t)$; $W_Q^*(t)= W_Q(t)$

$W_S^*(t)= \frac{\mu-r_f}{\sigma} t + W_S(t)$; $W_Q^*(t)= \frac{\beta-r}{\delta} t + W_Q(t)$

$W_S^*(t)= \frac{\mu-r_f}{\sigma} t + W_S(t)$; $W_Q^*(t)= \frac{\beta+r_f-r}{\delta} t + W_Q(t)$

PRACTICE PROBLEM 4.

We consider the payoff $C(T)$ consisting of

$C(T)= F \times S(T)$

units of domestic currency (that is, constant F is the exchange rate specified in the contract, not related to the actual exchange rate $Q(T)$. Suppose you know from the previous problem the values of A,B,C such that

$S(T)=S(t)e^{A(T-t)+B(W_Q^*(T)-W_Q^*(t))+C(W_S^*(T)-W_S^*(t))}$

Also recall that, for any numbers x,y, and Brownian motions W_1 and W_2 with correlation ρ.

$E_t\left(e^{x(W_1(T)-W_1(t))+ y(W_1(T)-W_2(t))}\right)=e^{\frac12\left(x^2+y^2+2xy\rho(T-t)\right)}$

Select the domestic price of the claim at time t:

$F Q(t)S(t)\ e^{(A-r)(T-t)+\frac12\left(B^2+C^2+2BC\rho\right)(T-t)}$

$FS(t)\ e^{(A-r)(T-t)+\frac12\left(B^2+C^2+2BC\rho\right)(T-t)}$

$F\ e^{ (A-r)(T-t)+\frac12\left(B^2+C^2+2BC\right)\rho(T-t)}$

$F S(t)\ e^{(A-r)(T-t)+\frac12\left(B^2+C^2\right)(T-t)}$

(d)
Bonus problem. This problem is provided just for fun. No solution is provided.

Denote by $V(t)$ the replicating portfolio of the payoff in the above problem, that does replication by investing the domestic currency in the foreign stock, the foreign bank account and the domestic bank account. Can you say, in terms of $V(t)$, how much the replicating portfolio invests in each of these three assets?

5.
Consider two stocks,

$$dS_1(t)=S_1(t)[\mu_1 dt +\sigma_1 dW_1(t)],$$

$$dS_2(t)=S_2(t)[\mu_2dt +\gamma_1 d W_1(t)+\gamma_2dW_2(t)],$$

where W_2, W_1 are independent Brownian Motions.

(a) Find a Brownian Motion W_1^* corresponding to a probability under which the process $\frac{1}{S^2_1(t)}$ is a martingale. Write the dynamics $dS_1(t)$ of S_1 under that probability.

(a) Find the Black-Scholes formula for the option paying at maturity T the amount $\$A$, if the ratio of squared assets $\frac{S^2_2(T)}{S_1^2(T)}$ is less than K, and zero otherwise.

PRACTICE problem. Consider a Black-Scholes model with two stocks,

$dS_i(t)=S_i(t)\left(\mu_idt+\sigma_idW_i(t)\right)$

$i=1,2$, where W_1 and W_2 are two independent Brownian motions.

The Black-Scholes formula for the option paying D dollars if $\min[S_1(T),S_{2}(T)]>K$, and zero otherwise, is, using self-explanatory notation :

$De^{-rT}N(d_2(\sigma_1))N(d_2(\sigma_2))$

$De^{-rT}N(d_1(\sigma_1))N(d_1(\sigma_2))$

$De^{-rT}{\bf 1}_{\{\min\{S_1(0)S_2(0)>K\}}$

$De^{-rT}\min\{0, S_1(0)S_2(0)-K\}$

6. Consider the claim which pays $\frac{S(T_1)}{ S(T_0)}$ at time T_1, $T_1>T_0>0$. You can assume that the discount process is e^{-rt}, and a Black-Scholes model, if it helps.

Enter the price of the claim at time zero:

$e^{-rT_0}S(0)$

$e^{-rT_1}S(0)$

e^{-rT_0}

e^{-rT_1}

----

2. Consider a two period binomial tree with the stock price starting at $S(0)=100$, with the up factor $u=1.1$ the down-factor $d=0.9$, and simple discounting $\frac{1}{1+r}=\frac{1}{1.05}$ in each period.

Enter the price of the American put option with the strike price $K=101$, and maturing at the end of the two periods:

Problem 2. In a two-period binomial tree model with \(r=1%\) per period, \(S(0)=100\), \(u=1.02\) and \(d=0.98\), consider an American option that expires after two periods, and, when exercised at time \(t\), it pays the value of the squared stock price, \(S^2(t)\), if the stock price \(S(t)\) is higher than \(101.00\). Otherwise, when \(S(t)\) is less or equal to \(101.00\), the option pays zero when exercised at time \(t\).

Enter the price of this option.

{\bf 1.} Consider the following two-period setting: the price of a

stock is \$52.

Interest rate per period is 2\%. After one period the price of the stock can go up

to \$56 or drop to \$46. If it goes

up the first period, the second period it can go up to \$58

or down to \$48. If it goes

down the first period,

the second period it can go up to \$49 or down to \$40.

Compute the price of an American put option with strike price $K = 52$ that matures

at the end of the second period.

SET 7 PROBLEM 5.

{\bf 11.}

In this problem, it may be useful to know that the random variable \(\int_0^t\sigma(u)dW(u)\) with \(\sigma\)

deterministic, has normal distribution, with mean zero and variance $int_0^t\sigma^2(u)du.$

The formula for the call option priceat time $t=0$ in the Black-Scholes-Merton model in which the volatility is a deterministic function of time $\sigma(t)$ is obtained by replacing everywhere in the Black-Scholes formula

σ^2 by $\int_0^T\sigma^2(u)du$

σ^2 by $\frac{1}{T}\int_0^T\sigma^2(u)du$

σ by $\int_0^T\sigma(u)du$

σ by $\frac{1}{T}\int_0^T\sigma(u)du$

11. PRACTICE PROBLEM 5

The formula for the call option price at time $t=0$ in the Black-Scholes-Merton model in which the risk-free rate is a deterministic function of time $r(t)$ is obtained by replacing everywhere in the Black-Scholes formula

r by $r(t)$

r by $r(T)$

r by $\frac{1}{T}\int_0^T r(u)du$

r by $\int_0^T r(u)du$

PRACTICE PROBLEM.

Suppose company \(A\) today enters a contract with company \(B\) for delivering \(10000\) units of commodity \(c\) three months from today. Company \(A\) wants to hedge the risk of this position. Unfortunately, there is no futures market for commodity \(c\). There is, however, a futures market for commodity \(d\) that includes a futures contract with maturity of three months. Suppose that it is found from historical data that the standard deviation of the commodity \(c\) price is \(\sigma_{c}=0.1\), the standard deviation of the commodity \(d\) 3-month futures price is (\sigma_{d}=0.05\), and the correlation between the two is \(0.6\). Find the hedging strategy that results in the minimal variance, and compute this minimal variance.

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Suppose that the stock price today is $S(t)=100$, the interest rate is $r=0\%$, and the time to maturity is six months. Consider a claim whose Black-Scholes price is given by the function $C(t,s)=s^2e^{0.02(T-t)},$ where time is in annual terms.

If your portfolio is long 10 of these claims and holds N shares of the stock, what should the value of N be so that the portfolio is delta-neutral? Use the negative sign for shorting.

Suppose that another claim with the same maturity is available with the Black-Scholes price given by the function $c(t,s)=s^3e^{0.06(T-t)}.$ If you still hold 10 units of the first claim, how many claims of the second type and how many shares of the stock would you trade to create a portfolio that is both delta neutral and gamma-neutral (gamma equal to zero)? Use the minus sign for selling.

Suppose that the stock price today is $S(t)=1$, the interest rate is $r=0\%$, the volatility is $\sigma=\sqrt{0.02}$ and the time to maturity is three months. Consider a claim whose Black-Scholes price is given by the function $C(t,s)=s^2e^{\sigma^2(T-t)},$ where time is in annual terms.

If your portfolio is long 10 of these claims and holds N shares of the stock, what should the value of N be so that the portfolio is delta-neutral? Use the negative sign for shorting.

Suppose that another claim with the same maturity is available with the Black-Scholes price given by the function $c(t,s)=s^3e^{3\sigma^2(T-t)}.$ If you still hold 10 units of the first claim, how many claims of the second type and how many shares of the stock would you trade to create a portfolio that is both delta neutral and vega-neutral (vega equal to zero)? Use the minus sign for selling.

PRACTICE PROBLEM 3.

The Black-Scholes price of a three-month European call with strike price 100 on a stock that trades at 100 is 5.50, and its delta is 0.56. Annual continuous risk-free rate is 5%. You sell the option for 6.00 and hedge your position by standard Black—Scholes hedging. One month later (the hedge has not been adjusted), the price of the stock is 98, the market price of the call is 3.40. You liquidate the portfolio (buy the call and undo the hedge). Compute the net profit or loss resulting from the trade.

PROBLEM 3.

You take a short position in one European call option contract, with strike price 100 and maturity 3 months, on a stock that is trading at 100. The annual volatility of the stock is constant and equal to 25%. The annual risk-free interest rate is constant and equal to 5%. Suppose that you sold the option at a premium of 8% over the Black-Scholes price, that is, for 1.08 times the Black-Scholes price. You hedge your portfolio with the underlying stock and the risk-free asset. The hedge is rebalanced monthly. After two months the portfolio is liquidated.

Compute the final profit or loss, if the price of the stock is 98 at the end of the first month and 101 at the end of the second month, and assume that the option is traded at exactly the Black-Scholes price at the end of the first month and at the end of the second month.

4.

You take a short position in a European contingent claim

contract that pays the squared stock value $S^2(T)$ at

maturity $T= 3$

months. The stock is trading at $\$50$. Its annual volatility is

constant and equal to 20%. The dividends are zero.

The annual continuous risk-free interest rate is constant and equal to 5%.

Suppose that you sold the claim at

the Black-Scholes price, which is of the form, for some a,b,c,

$$C(t,s)=s^ae^{(br+c\sigma^2)(T-t)}$$

You hedge your position

%(i.e., you replicate the payoff)

with the underlying stock and the risk-free asset.

The hedge is rebalanced weekly, using the Black-Scholes model.

After two weeks the portfolio is liquidated.

Assume that one week is exactly 1/52 years.

(a) Find the values of a,b and c.

(b) Compute the final profit or loss if the price of the stock is \$51.00 at the end of the first week, and \$50.50 at the end of the second week. Assume that everyone agrees on the Black-Scholes model.

(c) Suppose that instead of trading in the bank and the stock, you hedge by trading in the bank and the corresponding put option with the same maturity and strike equal to 50. How many put options would you hold in your hedging portfolio at time zero?

PRACTICE PROBLEM 4.

A portfolio manager manages portfolio whose current value is two million. She assumes the portfolio value satisfies the Black-Scholes model, with $\sigma=0.2$. The interest rate is $r=0.03$. For simplicity, we assume zero dividend rate. The manager, wants to protect the portfolio by replicating a put option on its value, with strike price equal to 1.9 billion and maturity one year.

What amount does the manager have to sell from the portfolio today, or add to the portfolio, in order to start replicating the put option?

5. A fund manager has a portfolio that is currently worth 100 million. The manager decides to invest in an index, but she would like to make sure that the value of the portfolio does not end up more than 5% below the initial value after one year. At the same time, she wants to make profits if the index moves sufficiently high. To do this, the manager will invest $\alpha \times 100$ million in the index ($\alpha<1$), and $(1-\alpha)\times 100$ million in replicating a put option with payoff $K-\alpha I(T)$, where $I(T)$ is the index value in six months and K is the strike price equal to 95% of 100 million The risk-free annual rate is 5%. The dividend yield on the index is 3%. Dividends are continuously reinvested. The volatility of the index is 30% per year. Assume that the Black-Scholes model holds, and that one can do perfect replication by continuous rebalancing.

What is the value of α for which the budget constraint

$$100\alpha + P(\alpha) = 100$$

holds, where $P(\alpha)$ is the Black-Scholes put option value in millions, written on the asset with initial value $\alpha \times 100$?

(Hint: This has to be solved numerically by trial and error, or a search routine, for example, using Solver in Excel.)

What amount of the index does the manager have to sell at initial time, to start replicating the put option?

Suppose you know that $120\alpha \ge 95$. What is her profit/loss (in millions) if the index goes up by a factor of 1.2 after one year?

What is the profit/loss if the index goes down by a factor of 0.90 after one year?

PRACTICE PROBLEM 5:

Consider foreign stock $S(t)$, exchange rate $Q(t)$, domestic risk-free rate r, and foreign risk-free rate r_f. Domestic value of the foreign stock is denoted $X(t)=Q(t)S(t)$ and it satisfies

$dX(t)/X(t)=\mu_X dt +\sigma_X dW_X(t).$

Domestic value of one unit of foreign currency is denoted $Y(t)=Q(t)e^{r_ft}$ and it satisfies

$dY(t)/Y(t)=\mu_Y dt +\sigma_Y dW_Y(t),$

where W_S, W_Q are Brownian Motions with correlation ρ. Thus, QS is the value of the foreign stock in domestic currency. Denote by r_f the (constant) foreign risk-free interest rate, and by r the domestic risk-free rate.

What is the PDE for the price $C(t,x,y)$ (in domestic currency) of a European claim with payoff of the form $C(X(T),Y(T))$?

( ) $C_t+\frac{1}{2}\sigma_X^2 x^2C_{xx}+r(xC_{x}-C)=0.$

( ) $C_t+\frac{1}{2}\sigma_X^2 x^2C_{xx}+ \frac{1}{2}\sigma_Y^2y^2C_{yy} +r(xC_{x}+yC_{y}-C)=0.$

( ) $C_t+\frac{1}{2}\sigma_X^2 x^2C_{xx}+\rho \sigma_X\sigma_Y$

$xyC_{x y} +r(xC_{x}-C)=0.$

( ) $C_t+\frac{1}{2}\sigma_X^2 x^2C_{xx}+ \frac{1}{2}\sigma_Y^2y^2C_{yy}+\rho \sigma_X\sigma_Y xyC_{x y} +r(xC_{x}+yC_{y}-C)=0.$

What is the number of shares of the foreign stock the replicating portfolio should hold?

( ) $C_x(t,X(t),Y(t))$

( ) $C_y(t,X(t),Y(t))$

( ) $Q(t)\times C_x(t,X(t),Y(t))$

( ) $C_x(t,X(t),Y(t))\times \frac{1}{Q(t)}$

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PROBLEM 5.

Suppose that, under the risk-neutral probability P^*, the stock price process S satisfies

$dS(t) = S(t)[r(t) dt +\sigma dW_1^*(t)]$

and the risk-free rate $r(t)$ is a stochastic process that satisfies

$dr(t) =(\alpha-\beta r(t)) dt +\rho_1 dW_1^*(t)+\rho_2 dW^*_2(t)$

where α, β, σ, ρ_1 and ρ_2 are positive constants, and W_1^* and W_2^* are independent Brownian motions.

What is the PDE for the price $C(t,s,r)$ (in domestic currency) of a European claim with the payoff of the form $C(S(T))$?

( ) $C_t+\frac{1}{2}\sigma^2 s^2C_{ss}+r(sC_{s}-C)=0$

( ) $C_t+\frac{1}{2}\sigma^2s^2C_{ss}+\frac{1}{2}(\rho_1^2+\rho_2^2) C_{rr}+\rho_1 \sigma sC_{ sr} +r(sC_{s}-C)+ (\alpha-\beta r)C_r =0$

( ) $C_t+\frac{1}{2}\sigma^2s^2C_{ss}+\frac{1}{2}(\rho_1^2+\rho_2^2) C_{rr}+\rho_1 \sigma sC_{ sr} +r(sC_{s}-C)=0$

( ) $C_t+\frac{1}{2}\sigma^2s^2C_{ss}+\rho_1 \sigma sC_{ sr} +r(sC_{s}-C)+ (\alpha-\beta r)C_r =0$

What is the number of shares of the stock a delta-neutral portfolio should hold?

( ) $C_s(t,S(t),r(t))$

( ) $C_r(t,S(t),r(t))$

( ) $C_s(t,S(t),r(t))+C_r(t,S(t),r(t))$

( ) $\sigma C_s(t,S(t),r(t))+\rho_1C_r(t,S(t),r(t))$

What is the amount in the risk-free asset that the delta-neutral portfolio which invests only in the stock and the risk-free asset should hold?

( ) $S(t)C_s(t,S(t),r(t))$

( ) $C_r(t,S(t),r(t))$

( ) $S(t)C_s(t,S(t),r(t))+C_r(t,S(t),r(t))$

( ) $C(t,S(t),r(t))-S(t)C_s(t,S(t),r(t))$

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5. Denote $P_{i-1}=P(T_{i-1},T_i)$, and let f be a given function. Which of the following is the time t-value of the payoff of $C_i=f(P_{i-1})$ dollars paid at time T_i?

( ) $f(P(t,T_i)$

( ) The same as the time t value of the payoff of $P_{i-1}C_i$ dollars paid at time T_{i-1}.

( ) The same as the time t value of the payoff of C_i dollars paid at time T_{i-1}.

( ) $f(P_{i-1})$

PROBLEM 5.

Consider a floating-rate coupon bond which pays coupons c_i at times T_i, $i=0,\ldots, n-1$ and it pays 1.00 at T_n, where the coupons are given by

$$c_i=(T_i-T_{i-1})L(T_{i-1},T_i)$$

and $T_i-T_{i-1}=\Delta T$ is constant. Denoting $P_{i-1}=P(T_{i-1},T_i)$, the LIBOR rate is given by

$$1+\Delta T L=\frac{1}{P_{i-1}}$$

Which of the following is the time $t<T_0$ value of this bond?

( ) $P(t,T_0)$

( ) The same as the time t value of the payoff of $P(t,T_n)$ dollars paid at time T_{n-1}.

( ) $P(t,T_n)$

( ) $P(t,T_n)-P(t,T_0)$

PRACTICE PROBLEM 4.

Consider the Vasicek model

$dr(t)=(b-ar(t))dt+\sigma dW(t)$

Let $C(T)=C(r(T),Y(T))$ be a claim whose payoff is a function of the final value of the short rate $r(T)$ and of the "bank account" value at time T

$Y(T)=e^{\int_0^Tr(s)ds}$

Which of the following is the PDE for its price?

( ) $C_t+\frac{1}{2}\sigma^2 C_{r}+(b-ar)C_{r}-rC=0$

( ) $C_t+\frac{1}{2}\sigma^2 C_{r}+r(C_{r}-C)=0$

( ) $C_t+\frac{1}{2}\sigma^2 C_{r}+(b-ar)C_{r}-rC +ryC_y=0$

( ) $C_t+\frac{1}{2}\sigma^2 C_{r}+(b-ar)C_{r}-rC +ryC_y+\frac12 \sigma^2y^2C_{yy}=0$

Suppose you sell that claim and you trade in the bank account and zero-coupon bond with maturity T to hedge the short position. Which of the following is the number of units of the T-bond that you should hold in order to make equal to zero the Brownian motion, dW part of your overall portfolio?

PROBLEM 4.4

Consider the Vasicek model

$$dr(t)=(b-ar(t))dt+\sigma dW(t)$$

and the values $P_1(t)=P(t,T_1)$, $P_2(t)=P(t,T_2)$ of the zero-coupon bonds with maturities T_1,T_2, given by

$$P(t,T_i)=e^{A(t,T_i)-B(t,T_i)r(t)}.$$

(a) Find the dynamics $d(r(t)e^{kt})$ for a given value of k.

(b) What is the distribution of the random variable $r(t)$, and its mean and variance? (It helps to know that if f is deterministic then $\int_0^tf(s)dW(s)$ has normal distribution.)

Hint: Use part (a) with an appropriately chosen value of k, in order to make the drift independent of r.

(c) Suppose you hold a short position of one unit ofthe T_1-bond.

>>Which one of the following is the number of units of the T_2-bond that you should hold at time t in order to hedge the short position? (Hint: it may help to first compute $\frac{d}{dr}P_i$.)<<

( ) $\frac{P(t,T_1)}{ P(t,T_2)}$

( ) $\frac{P(t,T_2)}{ P(t,T_1)}$

( ) $\frac{B(t,T_1)P(t,T_1)}{B(t,T_2)P(t,T_2)}$

( ) $\frac{B(t,T_2)P(t,T_2)}{ B(t,T_1)P(t,T_1)}$

1. The one-month and two-month interest rates are 3.8% and 4.0%, respectively. Our model of the term structure says that one month from now the one-month interest rate will be either

3.3% or 4.3%. Compute the price of an interest rate derivative that pays $\$1$ in one month if the one-month interest rate is 4.3% and $\$0.5$ if the one-month interest rate is 3.3%. (**NOTE:** the interest rates quoted are the spot rates that have been annualized using compounding.)

PROBLEM 9.5

Consider the Hull-White model

$$dr=(b(t)-ar)dt+\sigma dW$$

Since this is an affine model, we know that the bond price is of the form

$P(t,T)=e^{A(t,T)-B(t,T)r(t)}$

>>What is the correct expression for $B(t,T)$?<<

( ) $B(t,T)=\frac{1}{a}(1-e^{-a(T-t)})$

( ) $B(t,T)=1-e^{-a(T-t)}$

( ) $B(t,T)=\frac{1}{a}e^{-a(T-t)}$

( ) $B(t,T)=\frac{1}{a}(1-e^{-a(T-t)}) -\int_t^Tb(s)ds$

In the corresponding Black-Scholes-Merton formula for the call option $\max(0,P(T_1,T_2)-K)$ on the T_2-bond, what do we have to use in place of $\sigma^2(T-t)$?

( ) $\int_0^T) b^2(u)du$

( ) $\int_0^T)(a^2(u)du$

( ) $\sigma^2 \int_0^T (A(u,T_1)-A(u,T_2))^2du$

( ) $\sigma^2 \int_0^T (B(u,T_1)-B(u,T_2))^2du$

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( ) $\int_0^T) b^2(u)du$

( ) $\int_0^T)(a^2(u)du$

( ) $\sigma^2 \int_0^T (A(u,T_1)-A(u,T_2))^2du$

( ) $\sigma^2 \int_0^T (B(u,T_1)-B(u,T_2))^2du$

PROBLEM 5.5

Consider the HJM model with

$\sigma(t,T)=\sigma e^{-a(T-t)}~~,$

for given positive constants σ and a. (In this model the forward rates volatility goes down with shorter time to maturity.) Then, we know that the drift of the forward rates, under the pricing probability, is

$\alpha(t,T)=\sigma(t,T)\int_t^T\sigma e^{-a(u-t)}du=\frac{\sigma^2}{a}\left(e^{-a(T-t)}-e^{-2a(T-t)}\right)$

What is the correct expression for the interest rate

$r(t)=f(0,t)$

$r(t)=\int_0^t\alpha(u,t)du+\int_0^t\sigma e^{-a(t-u)}dW(u)$

$r(t)=f(0,t)+\int_0^t\alpha(u,t)du+\int_0^t\sigma e^{-a(t-u)}dW(u)$

$r(t)=f(t,0)+\int_0^t\alpha(t,u)du+\int_0^t\sigma e^{-a(u-t)}dW(u)$

What short rate model does this HJM model correspond to?

Vasicek model

Hull-White model

Cox-Ingersoll-Ross model

Ho-Lee model
