

Co-development Ventures: Optimal Time of Entry and Profit-Sharing

Jakša Cvitanić ^{*}, Sonja Radas [†] and Hrvoje Šikić [‡]

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Abstract

We find the optimal time for entering a joint venture by two firms, and the optimal linear contract for sharing the profits. We consider three contract designs, the risk-sharing, the timing-incentive and the asymmetric contract decisions design. An important result we establish is that if the firms are risk-neutral and if the cash payments are allowed, all three designs are equivalent. However, if at least one of the two firms is risk averse, the optimal contract parameters may vary significantly across the three designs and across varying levels of risk aversion, as illustrated in our numerical exercises. We also analyze a dataset of joint biomedical ventures, that exhibits general agreement with our theoretical predictions. In particular, both royalty percentage payments and cash payments are mostly increasing in the smaller firms length of experience, and the time of entry happens sooner for more experienced small firms.

Key words: Real Options; Joint Ventures; Optimal Contracts; Entry Time; Risk Sharing.
JEL classification: C61, G23.

^{*}Corresponding author. Caltech, Humanities and Social Sciences, MC 228-77, 1200 E. California Blvd. Pasadena, CA 91125. Ph: (626) 395-1784. E-mail: cvitanic@hss.caltech.edu. Research supported in part by NSF grants DMS 06-31298 and 10-08219 , and through the Programme "GUEST" of the National Foundation For Science, Higher Education and Technological Development of the Republic of Croatia. We are solely responsible for any remaining errors, and the opinions, findings and conclusions or suggestions in this article do not necessarily reflect anyone's opinions but the authors'.

[†]The Institute of Economics, Zagreb, Trg J. F. Kennedy 7, 10000 Zagreb, Croatia. E-mail: sradas@eizg.hr.

[‡]Department of Mathematics, University of Zagreb, Croatia. E-mail: hšikić@math.hr. Research supported in part by the MZOŠ grant 037-0372790-2799 of the Republic of Croatia

1 Introduction

Innovation is a crucial factor for a company's survival and success, and co-development partnerships are an increasingly utilized way of improving innovation effectiveness. These partnerships are working relationships between two or more partners with the goal of creating and delivering a new product, technology or service (Chesbrough and Schwartz, 2007). While the traditional business model centers on a company which develops a new product in-house (from own R&D) and then produces, markets and sells it using its own internal resources, the new model of open innovation includes co-development partnerships. In this way different partners' resources and capabilities can be optimally combined, thus creating significant reductions in R&D expense and time to market. According to Quinn (2000), using co-development "leading companies have lowered innovation costs and risks by 60% to 90%, while similarly decreasing cycle time and leveraging their internal investments by tens to hundreds of times".

In technology based industries incumbent firms frequently form strategic alliances with smaller firms and new entrants (Gulati, 1998; Hagedoorn, 1993). In pharmaceutical industry large firms with hefty R&D budgets and internal R&D capabilities have actively used the "market for knowhow" through contractual arrangements to acquire access to new technologies. On the other hand, small entrepreneurial firms seek alliances with large firms to avail themselves of the resources that are too costly, or too difficult to build internally.

In this paper we focus on a co-development alliance between a firm which is the originator of the project or the new product idea, called firm S (for "small") and a firm which provides research and other lacking resources necessary for product development, called firm L (for "large"). We model the decision to enter co-development using real options theory. In particular, we examine how the project entry time depends on the asymmetry of information and on the relative bargaining power.

Our paper relies on real options methodology in modeling interfirm alliances. Real options framework recognizes that investment opportunities are options on real assets, and as such is able to provide a way to apply the methods of pricing financial options to the problems related to firms investment decisions. Most of the literature considers the case of a single firm's R&D investment decision (Mitchell and Hamilton 1988; McGrath 1997; Folta 1998), as well as the timing of the investment (Dixit and Pindyck 1994; Sarkar 2000; Henderson and Hobson 2002; Lambrecht and Perraudin 2003; Henderson 2007; Miao and Wang 2007), the development of organizational capabilities (Kogut and Kulatilaka 2001), and entry decisions (Miller and Folta 2002). Real options have been used to model firm alliances such as joint ventures (Kogut 1991; Reuer and Tong 2005), acquisitions (Folta and Miller 2002), and university-firm contracts for commercializing technology (Ziedonis, 2007). An important paper by Habib and Mella-Barral (2007) studies incentives to form

joint ventures by detailed modeling of the benefits of acquiring knowhow. Unlike our paper, they focus on the time of dissolution of the venture rather than the time of entry, and their model is different from ours. The option to exit early is also studied in Savva and Scholtes (2007), where it is shown that it improves the efficiency of contracts.

In these alliances there is often an asymmetry of information which is then dealt with through contractual arrangements. Much of the economic modeling on company relationships is framed within an agency model (e.g. Bolton and Dewatripont 2005; Crama et al. 2007), where asymmetric information and risk aversion are studied as sources of inefficiency. Contractual arrangements in such alliances usually involve up-front payments plus royalties that protect prospective licensee from the risk; namely when the licensee estimates the risk to be high they can attempt to shift the balance of payments away from up-front fees toward future royalties on end sales, and thus transfer the project risk toward the licensor. Often milestone payments are used for successfully reaching certain stages in product development. Such milestone and royalty contracts arising from asymmetric information have been studied in the literature, dealing with issues of risk sharing between the two firms (Amit et al. 1990), as well as adverse selection and moral hazard (Gallini and Wright 1990; Crama et al. 2007).

1.1 Contributions

Our contributions consist of the following:

- (i) We add to the real options literature by modeling two companies deciding on entry time, instead of only one company (the existing real options literature mostly deals with the latter case). We consider three different contract designs. We first study the case of risk-sharing between the two firms, and we find the Pareto optimal contract, that is we maximize a linear combination of the two firms' objectives. This can be interpreted in two ways, as maximizing the joint welfare, but it is also the commonly accepted mechanism in contract theory for profit sharing between two economic agents with symmetric information. For a fixed value of a parameter representing the relative bargaining power, in addition to the optimal entry time, this procedure determines the optimal parameters of the linear contract, the slope and the intercept. Thus, the actual level of sharing depends on the bargaining power. This Pareto optimal contract design is not necessarily realistic, but it is the "first-best" benchmark case to which we compare the other two designs.

Next, we examine the contract design in which timing is incentive, i.e., the case in which the contract is constructed so that both firms would find the same entry time to be optimal. This case is used in a related paper Lambrecht (2004), as a reasonably realistic design for modeling friendly mergers between firms.

Finally, we consider the case with asymmetric contract decisions, in which one firm

decides on the initiation time, while the other firm decides on how to share the profits, while satisfying the participation constraint of the first firm. This design might be realistic for modeling hostile mergers, and joint ventures between asymmetric firms, the case we study in our dataset.

We find that the slope and the intercept of the optimal linear contract are much more sensitive to the model specifications than the optimal time of entry. We also find that the utility loss relative to the Pareto optimal case in the second and third design is not very large for most values of the bargaining power. In other words, as a practical matter it is of lesser importance which contract design is used (as long as it is feasible) than which contract parameter values are used.

- (ii) We model the risk attitudes in more general terms than is typical. That is, we assume that the firms are potentially risk averse. This is in contrast to Lambrecht (2004), who considers optimal timing of mergers between two risk-neutral firms. Unlike that paper, we allow for risk-aversion of the firms and for non-zero cash payments, and we also consider the effects of bargaining power. Allowing cash payments makes our results fundamentally different from Lambrecht (2004). In particular, one of our main theoretical results says that, with cash payments allowed, there is no difference between the three contract designs if the firms are risk-neutral. However, if there is risk aversion, the three designs are no longer equivalent, and the optimal contract parameters depend very much on what design is used. They also may change significantly with the level of risk aversion.

- (iii) Following the real options approach in modeling the decision to form a co-development alliance, methodologically, we use the theory of the optimal stopping of diffusion processes. Classical references of its applications in economics include McDonald and Siegel (1986) and the book Dixit and Pindyck (1994), where this theory was shown to be extremely useful for problems involving real options, and in particular for the option of entering and/or exiting a project. However, the standard results of the theory are not strong enough to enable us to incorporate all the cases we study. Among the approaches offered in the literature we found the recent very general mathematical treatment of Johnson and Zervos (2010) as the most useful for our purpose. However, their assumptions are not quite satisfied for all the models we consider. We extend some of the results of Johnson and Zervos (2010) in the main methodological theorem given in Appendix.

In Section 2 we set up the model, in Section 3 we solve for the optimal linear contract between the two firms, for the three different contract designs. We discuss comparative statics in Section 4, and in Section 5 we examine the agreement of those theoretical predictions with empirical facts implied from a dataset of real world alliances. Section 6 concludes. Appendix describes the underlying model in more mathematical detail and provides the methodological theorems.

2 The Model

There are two firms, S (for “small”) and L (for “large”). We think of firm S as the project originator, while firm L is the firm with complementary resources that enters into a co-development agreement with firm S . One example would be a biotech company (firm S) entering into a joint venture with a pharmaceutical company (firm L).

After entering the co-development project at time τ , they share the future profit/loss up to time $\tau + T$. Here, T is the time horizon, and all the results hold for $T = \infty$, too.¹ The profit/loss rate process P_t is the Brownian motion with the drift, i.e., it follows the Stochastic Differential Equation (SDE)

$$dP_t = bdt + \sigma dW_t$$

where b, σ are constants and W is a standard Brownian motion process.

The interpretation of process P_t is that it represents the future profit/loss rate, in the sense that the utility the firms get from it is accumulated over the time interval $[\tau, \tau + T]$ of pursuing the joint venture.

The profit/loss is shared according to a (adapted) contract process C_t . More precisely, the expected utility of firm L is given by

$$V_L := E \left[\mathbf{1}_{\{\tau < \infty\}} \int_{\tau}^{\tau+T} e^{-rt} U_L(P_t - C_t) dt \right]$$

and the expected utility of firm S is given by

$$V_S := E \left[\mathbf{1}_{\{\tau < \infty\}} \int_{\tau}^{\tau+T} e^{-rt} U_S(C_t) dt \right]$$

where r is a constant discount rate.

We remark that in much of the real options literature the firms are assumed to be risk-neutral (having linear utility). It can be argued that a small privately held firm may be more risk averse than a large firm, as its owners may be mostly invested in the firm, while the shareholders of a large firm are likely to be well diversified, and thus closer to risk neutral. This would create a need to allow for risk aversion of at least one firm. If both firms are privately owned, then they might both be risk averse. In addition, in practice there is evidence for difference in contracts when the contract decisions are asymmetric (see Lambrecht 2004), which would be in contradiction with risk-neutrality, as we show later below that there is no theoretical difference in various contract designs if the firms are risk-neutral. As we will also see below, it is important whether the firms are risk averse or not

¹We allow arbitrary values of T for the sake of generality, but, qualitatively, the results don't change much with T , as indicated in the section on comparative statics.

– the optimal contract parameters do change significantly with risk aversion (even though the optimal time of entry and the utility levels don't).

For tractability, we will assume that the firms have exponential utility functions:²

$$U_i(x) = k_i - l_i e^{\gamma_i x} \quad (2.1)$$

where $(-\gamma_i) > 0$ is the risk aversion, and $l_i > 0$. Parameters k_i and l_i serve to normalize the value of the overall expected utility and to model fixed costs or benefits from participating in the venture. In particular, if there is a fixed cost f_i , we can set $l_i = e^{-\gamma_i f_i}$ to be the utility of the loss $-f_i$ due to the cost.³ Note, however, that with risk-neutral, linear utility, this can be incorporated into the parameter k_i .⁴

As we will argue below, with exponential utility functions the contract C_t which optimizes the weighted joint welfare $V_L + \lambda V_S$ is linear, and we denote it as

$$C_t = aP_t + c \quad (2.2)$$

The interpretation of c and a is that they represent the future cash payments and the future royalty payments.⁵

We will consider only linear contracts in this paper, even when we are not maximizing the joint welfare.⁶

As we show in Appendix, and as is well known from the theory of optimal stopping and real options, the optimal time of entry is the first time process P_t reaches over a certain threshold x :

$$\tau = \tau_x = \min\{t : P_t \geq x\} \quad .$$

²The benchmark process used in the real options theory is the geometric Brownian motion, and it is usually interpreted as the firm's stock price, or the firm's value. We model here the profit/loss process, and not the stock/firm value, and, moreover, the joint venture may have negative present value. In such a framework it is customary to use the arithmetic Brownian motion for the state variable. However, it should be pointed out that, mathematically, using the arithmetic Brownian motion and exponential utility functions is equivalent to using the geometric Brownian motion and power utility functions.

³Less obviously, there may be cases that require setting l_S higher. For example, Nicholson et al (2005) find that inexperienced biotech companies tend to sign the first deals with large pharma companies on terms that are less than optimal for them, but the deal itself acts as a signal to potential investors and the rest of the community about the quality of the project and the company. The discount in the deal can be considered as a payment to the pharma company for the evaluation that it performs.

⁴In the benchmark numerical case we will set $l_i = 1$, corresponding to zero fixed costs. We will set the value of k_i so as to make equal to zero the utility of zero profit. In particular, in the case in which the profit/loss process is always equal to zero ($P_t = 0$, for all t), the overall expected utility would be zero – the same as the value of never entering the venture.

⁵Here, the interpretation of a is that of a royalty percentage of profits, but only when the profit/loss rate process P is positive. When it is negative, that is, when the loss is being experienced, the payments reverse the direction, i.e., firm S pays a percentage of losses back to firm L during such periods.

⁶This is for tractability reasons – except for the joint welfare case, we do not know how to solve for the optimal contracts if we allow contracts outside the linear class.

Thus, we call a contract a triple (a, c, x) , where we require $0 \leq a \leq 1$. Denote the corresponding expected utility values by $V_i(a, c, x)$. We now compute these values for a fixed contract (a, c, x) . Denote

$$\beta := 1 + 2b/\sigma^2, \quad n := 1/2 - \beta/2 + \sqrt{(\beta/2 - 1/2)^2 + 2r/\sigma^2}, \quad \theta(\gamma_i) = r - \gamma_i^2\sigma^2/2 - \gamma_ib \quad .$$

We assume throughout the paper that

$$r > b - \sigma^2/2 \quad .$$

This condition implies $n > 1$, and guarantees that the problem of optimizing over τ does not explode when $T = \infty$. In case $T = \infty$ we also need the condition

$$\theta(\gamma_i) > 0, \quad i = L, S$$

in order to guarantee that the utility of owning the whole project is not negative infinity.

Next, denote

$$\begin{aligned} k_i(x) &= k_i \frac{1 - e^{-xT}}{x} \\ l_i(x) &:= l_i \frac{1 - e^{-xT}}{x} \\ g_S(a, c, x) &= k_S(r) - l_S(\theta(a\gamma_S))e^{\gamma_S c} e^{a\gamma_S x} \\ g_L(a, c, x) &= k_L(r) - l_L(\theta([1-a]\gamma_L))e^{-\gamma_L c} e^{[1-a]\gamma_L x} \end{aligned}$$

The first two functions are simply the integrals of the constants k_i, l_i discounted over time at rate x . As shown in Appendix, functions g_S, g_L correspond to the integrals, multiplied by $e^{r\tau}$, in the expected utilities V_S, V_L evaluated in the case the entry time is such that $P_\tau = x$ and the contract is of the form $aP + c$.

Then, we have

Proposition 2.1 *For contract (a, c, x) with $P_0 \leq x$, the expected utilities of firms S and L are given by*

$$\begin{aligned} V_S(a, c, x) &= g_S(a, c, x)e^{-n(x-P_0)} \\ V_L(a, c, x) &:= g_L(a, c, x)e^{-n(x-P_0)} \quad . \end{aligned}$$

Proof: From the standard results on Brownian hitting times, we get, for $P_0 \leq x$,

$$E [\mathbf{1}_{\{\tau < \infty\}} e^{-r\tau}] = e^{-n(x-P_0)} \quad . \quad (2.3)$$

The result then follows from (7.23) and Lemma 7.1 in Appendix.

■

As we can see, there is a tradeoff in expected utility between starting early (at low x), which increases the “discount” term $e^{-n(x-P_0)}$, and waiting for the potential venture profits to become higher (high x), which increases the terms of the form $g_i(a, c, x)$.

Denote

$$K_S(a) = \frac{k_S(r)n}{l_S(\theta(a\gamma_S))[n - a\gamma_S]}$$

$$K_L(a) = \frac{k_L(r)n}{l_L(\theta([1-a]\gamma_L))[n - (1-a)\gamma_L]} .$$

We have the following result for the optimal time of entry:

Proposition 2.2 *Fix a and c . The values x_i that maximize $V_i(a, c, x_i)$ are given by*

$$x_S = -c/a + \frac{1}{a\gamma_S} \log K_S(a) \quad (2.4)$$

$$x_L = c/(1-a) + \frac{1}{(1-a)\gamma_L} \log K_L(a) \quad (2.5)$$

respectively, provided $x_i \geq P_0$. If $x_i \leq P_0$, then it is optimal to enter immediately.

Proof: These equations are obtained by taking the derivative with respect to x of the values $V_i(a, c, x)$ obtained in the previous proposition. A general optimal stopping theorem in Appendix implies that these equations have unique solutions which are, indeed, the maxima.

■

3 Optimal contract and time of entry

The two firms have to decide on the time of entry, and on how to share the profits/losses. We will consider three contract designs: (i) the risk-sharing case of maximizing weighted joint welfare (the first-best, Pareto optimal case), henceforth called RSJW case (Risk-Sharing/Joint Welfare case); (ii) finding the optimal *timing-incentive contract* for which the optimal time of entry for the two firms is the same, henceforth called TI case (Timing-Incentive case); (iii) finding the contract which maximizes one firm’s utility given that the other firm decides on the time of entry; henceforth called ACD case (Asymmetric Contract Decisions case).

RSJW case is the benchmark (likely unattainable in reality) to which we compare the other two cases. TI case may be appropriate for modeling friendly mergers, for example, while ACD case may be appropriate for modeling hostile mergers and joint ventures between asymmetric firms.

In Section 4 below, we will consider the following question: given that firm S has to be paid at least a given reservation value in expected utility, how does the optimal linear contract vary across these three contract designs?

3.1 Risk-Sharing/Maximizing weighted joint welfare: RSJW case

With P_t being the total profit/loss rate to the two firms S and L , and U_L, U_S their utility functions, for a given $\lambda > 0$, the profit-sharing, or the risk-sharing problem is to maximize, over entry time τ and payment rate C_t from firm L to firm S , the value

$$V := V_L + \lambda V_S = E \left[\mathbf{1}_{\{\tau < \infty\}} \int_{\tau}^{\tau+T} e^{-rt} [U_L(P_t - C_t) + \lambda U_S(C_t)] dt \right] . \quad (3.6)$$

In other words, we maximize a weighted sum of the expected utilities of the two firms, where the weight λ is interpreted as the bargaining power of firm S relative to firm L . This is the standard approach for finding optimal risk-sharing contracts between two entities in the case of symmetric information.⁷ Note also that this risk-sharing formulation is equivalent to assuming that one firm decides both on the timing and on the payments, while providing the other firm with a given expected utility (“reservation wage”), determined by the level of the “Lagrange multiplier” λ .

By maximizing inside the integral with respect to C_t , we see that the first order condition for optimality of C_t is the classical Borch (1962) condition for risk sharing:

$$U'_L(P_t - C_t) = \lambda U'_S(C_t) .$$

The optimal payment rate C_t from firm L to firm S is the solution to this equation. In particular, when the utility functions are exponential as in (2.1), it is easy to verify that the optimal contract is linear, $C_t = aP_t + c$, with constants

$$a = \frac{\gamma_L}{\gamma_L + \gamma_S} , \quad c = -\frac{1}{\gamma_L + \gamma_S} \log\left(\lambda \frac{l_S \gamma_S}{l_L \gamma_L}\right) . \quad (3.7)$$

Note that the royalty payment a depends only on the risk aversion parameters of the two firms, while the cash payment c also depends on the level of bargaining power λ and the relative value $\frac{l_S}{l_L}$ of “fixed benefits/costs” of the two firms.

As in the previous section, the optimal threshold x of P_t determining the entry time can be found analytically. To wit, denote

$$\Gamma = \frac{\gamma_S \gamma_L}{\gamma_S + \gamma_L} .$$

We obtain, by direct maximization of $V_L(a, c, x) + \lambda V_S(a, c, x)$ over x , the following result:

Proposition 3.3 *Given optimal a, c as in (3.7), the optimal time of entry is the first time process P reaches over value x given by*

$$e^{\Gamma x} = nk_L(r) \left((n - \Gamma)(l_L(\theta([1 - a]\gamma_L))e^{-\gamma_L c} - \lambda l_S(\theta(a\gamma_S))e^{\gamma_S c}) \right)^{-1} .$$

We will use this expression for computing comparative statics in Section 4 below.

⁷See Bolton and Dewatripont (2005)

3.2 Timing-incentive contract: TI case

We call a contract (a, c, x) *timing-incentive* if the optimal entry thresholds for the two firms are equal, $x_S = x_L$. We can use Proposition 2.2 and set $x_S = x_L$ to give us an equation for the pairs (a, c) for which the contract is timing-incentive. We will vary values of a , for each given value of a we will solve the equation for c , and we will then compare thus obtained timing-incentive contracts to those from the other two cases.

This contracting framework would correspond in practice to the situation in which both firms insist on entering the venture at the time optimal for them, and then they decide on c and a consistent with that requirement and according to their relative bargaining power. In contrast, we consider next the case when one firm decides on the time of entry, and the other has no choice but to enter at that time, or to reject to enter the venture.

3.3 Asymmetric contract decisions: ACD case

We now assume that one firm decides on the starting time of the joint venture (the profit entry threshold x), while the other firm decides on the form of the contract payoff (the values of a and c). For example, in practice it may be the case that firm S decides when to "put itself on the market", while firm L has rules in place governing its required compensation for participating in joint ventures (given the values of b , σ and P_τ). As in Lambrecht (2004), this can be considered as a Stackelberg leader-follower game, in which the leader, firm L , credibly commits to the required compensation, and the follower, firm S , decides then when to start the joint venture. A possible theoretical rationale for this contract design is the following: suppose firm S has information about the quality of the product which has to be revealed to firm L when they start the joint venture. In return for its agreeing to reveal the information, firm S is given the option to decide when to start the venture.⁸

From another perspective, Lambrecht (2004) models hostile takeovers, where the acquiring firm initiates the takeover, while the target firm credibly commits to the conditions of the takeover, such as the compensation to the managers. In this interpretation, one would have firm L as initiating the venture, and firm S as having committed to required compensation, but this is just the change of notation. The main differences from Lambrecht (2004) are that we allow for risk aversion of the firms, for the cash part of the compensation c , and for varying bargaining power.

For concreteness, we assume that firm L is the one deciding on a, c and firm S decides on

⁸This is in the spirit of the "revelation principle" in Contract Theory, which says that it is sufficient to consider the contracts which induce truthfulness. However, the connection is only indirect, as we simply assume that firm S is obliged to present true information, perhaps by a legal clause in the contract. We do not know what the optimal truth-revealing contract is. Instead, we have here argued that the contract we consider is likely not to be too far in spirit from the optimal contract in the case of asymmetric information.

the time $\tau = \tau(a, c)$ of initiating the joint venture. In accordance with the above discussion, we suppose that both firms have full information about the value of P_τ and the parameters b and σ at entry time τ .⁹ Thus, the functional form of the optimal (for firm S) entry threshold $x_S = x_S(a, c)$ is known to both firms, and we suppose that firm L will choose (a, c) by maximizing its utility $V_L(a, c, x_S(a, c))$, under the *participation constraint*

$$V_S(x_S(a, c), a, c) \geq R$$

where R is a given *reservation utility* of firm S , and represents the level of its bargaining power.

Firm S maximizes, over stopping times τ ,

$$V_S(\tau) := E \left[\mathbf{1}_{\{\tau < \infty\}} \int_\tau^{\tau+T} e^{-rt} U_S(c + aP_t) dt \right] . \quad (3.8)$$

We again assume exponential utility functions as in (2.1) and linear contracts. Recall the notation $K_S(a)$ and that firm S will enter the venture optimally when the profit process P reaches over the threshold x_S determined by $e^{x_S} = e^{-c/a} K_S^{\frac{1}{a\gamma_S}}(a)$. Together with Proposition 2.1, the following result gives us the value of $V_S(\tau)$.

Proposition 3.4 *We have*

$$g_S(a, c, x_S) = g_S(a) = k_S(r) \left[1 + \frac{n}{a\gamma_S - n} \right] .$$

Moreover, if the firm S utility is fixed at a reservation value R , then we have

$$e^c = \left(\frac{R}{g_S(a)} \right)^{a/n} e^{-aP_0} K_S^{1/\gamma_S}(a)$$

and the firm L maximizes $R \frac{g_L(a)}{g_S(a)}$ with

$$\begin{aligned} g_L(a) &= k_L(r) - l_L^{\theta([1-a]\gamma_L)} K_S^{\frac{(1-a)\gamma_L}{a\gamma_S}}(a) e^{-\frac{c}{a}\gamma_L} \\ &= k_L(r) - l_L^{\theta([1-a]\gamma_L)} K_S^{-\frac{\gamma_L}{\gamma_S}}(a) e^{\gamma_L P_0} \left(\frac{g_S(a)}{R} \right)^{\frac{\gamma_L}{n}} . \end{aligned}$$

Proof: This follows by direct substitution of x_S into the expression for g_S , and by finding e^c from the constraint $V_S = R$.

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⁹Note that firm L does not need to know the model parameters before the entry time because it credibly commits to required compensation for all possible values of those parameters.

Remark 3.1 It can be shown that for the value of λ which satisfies

$$k_S \gamma_S \lambda^{\frac{\gamma_S}{\gamma_L + \gamma_S}} = k_L \gamma_L \lambda^{-\frac{\gamma_L}{\gamma_L + \gamma_S}}$$

the (optimal) contracts that give the same expected utility to firm S in all three cases are the same, and then the expected utility of firm L is also the same for all three cases. For example, if $k_S \gamma_S = k_L \gamma_L$, then for $\lambda = 1$ all three cases produce the same solutions, under the constraint that firm S gets the same expected utility as in the joint welfare case. In other words, under these conditions, the first-best solution of maximizing joint welfare is also the solution to the other two cases of incentive timing and of asymmetric contract decisions, and efficiency is reached with any of the three contract designs.

3.4 Risk-neutral firms

We consider now separately the risk-neutral case with

$$U_i(x) = k_i + x, \quad i = L, S.$$

We show that in this case the three contract designs, RSJW, TI and ACD all give the same solution. The following result will help us analyze such a case.

Proposition 3.5 *With given constants $B \neq 0$ and A , consider the problem of maximizing over $\tau_x = \inf\{t : P_t \geq x\}$, the value*

$$V(P_0) = E \left[\mathbf{1}_{\{\tau_x < \infty\}} \int_{\tau_x}^{\tau_x + T} e^{-rt} (A + BP_t) dt \right] = (C + Dx) e^{n(P_0 - x)}$$

where

$$C = A \frac{1 - e^{-rT}}{r} + \frac{Bb}{r^2} [1 - e^{-rT}(rT + 1)]$$

$$D = B \frac{1 - e^{-rT}}{r} .$$

The optimal threshold for this problem is given by

$$x = \frac{1}{n} - \frac{C}{D} . \tag{3.9}$$

If $P_0 < x$, the corresponding value is given by

$$V(P_0) = \frac{D}{n} e^{n(P_0 + C/D) - 1} .$$

In case $B = 0$, $A > 0$, $\tau \equiv 0$ is optimal. In case $B = 0$, $A < 0$, $\tau \equiv \infty$ is optimal.

Proof: This can be shown as a special case of the main theorem in Appendix. More directly, from Lemma 7.1 in Appendix we can compute that

$$V(P_0) = E [\mathbf{1}_{\{\tau < \infty\}} e^{-r\tau} (C + DP_\tau)] \quad . \quad (3.10)$$

The result then follows by replacing P_τ with x , using $E [\mathbf{1}_{\{\tau_x < \infty\}} e^{-r\tau_x}] = e^{n(P_0 - x)}$, and maximizing over x .

■

In this section we interpret RSJW problem as the problem of firm L maximizing its utility under the participation constraint that firm S receives reservation utility R . The following proposition shows that there is essentially no difference between the three cases with risk-neutral firms.

Theorem 3.1 *Let (a, c, x) be a TI contract that gives firm S expected utility R . Then the same contract is optimal for RSJW case and ACD case, under the constraint that firm S utility is at least R .*

Thus, *if the firms have no restrictions on the size of the cash payments, the Pareto optimal outcome can be attained with any of the three designs.* This stands in contrast with the results in Lambrecht's (2004) analysis of mergers, in which the firms can only share the total pie (that is, can choose a in our framework), but are not allowed to use side payments (c in our framework): in that paper the different contract designs lead to different outcomes.

The intuition on why the three designs are equivalent in our model in the risk-neutral case is as follows. Because of risk neutrality, in RSJW case it does not really matter what values of a and c constitute the contract, as long as the reservation value of firm S is attained. (In fact, as seen in the proof below, the optimal threshold does not depend on (a, c) .) Thus, a and c may be chosen so that the contract is also timing-incentive, which makes RSJW and TI cases equivalent. Moreover, since the optimal RSJW contract provides the best utility to firm L while guaranteeing the reservation value to firm S , firm L will choose this same contract also in ACD case. This argument does not work in the presence of risk aversion – in that case the optimal RSJW threshold depends on (a, c) , and (a, c) cannot be made timing-incentive. It also does not work if c is forced to be zero, as in Lambrecht (2004), because then a has to be chosen so as to satisfy the reservation value, but then it cannot be made timing-incentive.

In Figures 1-3 we assume that the two firms have the same risk aversion, and we present the values of x, c and a across a range of values of the common risk aversion. We see that a and c for TI and ACD designs converge to the same value as those risk aversions tend to zero, and so do the values of x for all three designs. The values of a and c for RSJW case happen to be different. This is because, as already mentioned above, in the risk neutral

case any value of a can be optimal, by finding c such that (a, c) satisfies the participation constraint of firm S , and here the value of a happens to be set at 0.5. We also see that the values of a, c and x for the three designs also become close to each other for very large values of risk aversion.

A rigorous proof of the theorem is given next.

Proof: We first consider RSJW case. Denote by C_S, D_S, C_L, D_L the values of C, D from Proposition 3.5 corresponding to the optimization problem of firm S and of firm L , respectively. The participation constraint implies that we need to have $(C_S + D_S x)e^{n(P_0 - x)} = R$, which can be written as

$$c = -k_S + R e^{-n(P_0 - x)} \frac{r}{1 - e^{-rT}} - ax - \frac{ab}{r} \times \frac{1 - e^{-rT}(rT + 1)}{1 - e^{-rT}} . \quad (3.11)$$

Plugging this back into $V_L = (C_L + D_L x)e^{n(P_0 - x)}$, we get that V_L does not depend on a , and that optimal x is given by

$$x = \frac{1}{n} - k_L - k_S - \frac{b}{r} \times \frac{1 - e^{-rT}(rT + 1)}{1 - e^{-rT}} . \quad (3.12)$$

Note also that we can set c to any value we want, and find a from the participation constraint (3.11).

For the timing-incentive case, we need to have $x_S = x_L$, which means, from (3.9) the definitions of C and D , and from

$$A_L = k_L - c , \quad B_L = (1 - a) , \quad A_S = k_S + c , \quad B_S = a$$

that we need to have $(k_L - c)/(1 - a) = (k_S + c)/a$, which gives us

$$c = ak_L - (1 - a)k_S . \quad (3.13)$$

Plugging back into the optimal threshold, we can check that we get the same value as in (3.12). Then, the value of a can be decided upon by giving a specific level of utility to firm S .

For the case of asymmetric contract decisions, we first note that the entry threshold is given by

$$x_S = \frac{1}{n} - \frac{k_S + c}{a} - \frac{b}{r} \times \frac{1 - e^{-rT}(rT + 1)}{1 - e^{-rT}} \quad (3.14)$$

and the firm L maximizes over a and c , under the constraint $V_S \geq R$, using (3.10),

$$V_L = e^{n(P_0 - x_S)} \left\{ (k_L - c) \frac{1 - e^{-rT}}{r} + \frac{(1 - a)b}{r^2} [1 - e^{-rT}(rT + 1)] + x_S(1 - a) \frac{1 - e^{-rT}}{r} \right\} .$$

It is readily seen that the timing-incentive choice of c as in (3.13) makes the optimal threshold for firm S in (3.14) the same as the one for RSJW case in (3.12), and, as mentioned

above, this is also the timing-incentive threshold. Since in RSJW case we are free to choose c or a , we can set c to this value. Since then (c, x) are the same for RSJW case and TI case, the value of a in those two cases also has to be the same in order for the utility of S to be the same. And since that is the best firm L can do while guaranteeing utility R to firm S , it is optimal for it to choose those same values also in ACD case.

■

3.5 Optimal exit time

We allow now for the possibility that the project is stopped at an optimal time, rather than at a fixed horizon. We assume again exponential utility functions, or the risk-neutral case.

We only consider the RSJW case, and the ACD case in which firm S decides on both entry and exit.¹⁰

For the optimal exit problem, the optimal time is the first time $\bar{\tau}$ at which process P crosses below a specific optimal level x^* , if $x^* < P_0$, otherwise it is optimal to stop immediately. We will need the fact that for a given $x < P_0$, the standard results from diffusion theory tell us that

$$E[e^{-r\bar{\tau}}] = e^{-\tilde{n}(x-P_0)}$$

where \tilde{n} is defined analogously to n , but with the minus sign in front of the square root. Thus, $\tilde{n} < 0$.

Consider first the ACD case in which firm S decides on both the entry and the exit time. The optimal exit time is determined by \tilde{x}_S given by the same expression as x_S in (2.4) except with n replaced by \tilde{n} . We then have the following result:

Proposition 3.6 *The optimal entry time for firm S is obtained by solving the problem*

$$\max_{x \geq \tilde{x}_S} \{ [k_S(r) - l_S(\theta(a\gamma_S))e^{\gamma_S(c+ax)} + v_S(x)]e^{-n(x-P_0)} \} \quad (3.15)$$

where $v_S(x)$ is defined by

$$v_S(x) = -k_S(r) \left(1 + \frac{\tilde{n}}{a\gamma_S - \tilde{n}} \right) e^{-\tilde{n}(\tilde{x}_S - x)} .$$

Next, denote by $x_S(a, c)$ the optimal entry point x obtained by solving the above problem for a given pair (a, c) . Then, the optimization problem of firm L is to maximize over (a, c) its

¹⁰The TI case here would consist in requiring that the two firms agree both on the entry and on the exit time, which would specify a and c uniquely no matter what the bargaining power, or there would be no a and c for which that is possible. One could also look at the case where both firms have the option to stop the project. This would make it a complex stochastic game of optimal stopping (in addition to finding the optimal entry time), and we do not consider it here.

expected utility given by

$$e^{-n(x_S(a,c)-P_0)} \times \left[k_L(r) - l_L(\theta((1-a)\gamma_L))e^{\gamma_L((1-a)x_S(a,c)-c)} \right. \\ \left. - e^{-\tilde{n}(\tilde{x}_S-x_S(a,c))} \left(k_L(r) - l_L(\theta((1-a)\gamma_L))e^{\gamma_L((1-a)\tilde{x}_S-c)} \right) \right] \quad (3.16)$$

over such (a, c) for which $x_S(a, c) > \tilde{x}_S$.

Proof: See Section 7.4 in Appendix. ■

We will use this proposition in the comparative statics section for computing numerically the optimal contract. As for RSJW problem with optimal exit and entry, the optimal a, c, x, \tilde{x} can be computed numerically by maximizing, over a, c, x, \tilde{x} the expected utility of firm L given by the expression in (3.16) (with x_S replaced by x and \tilde{x}_S replaced by \tilde{x}), under the constraint that the expected utility of firm S , given by an expression analogous to (3.16), is no less than the given reservation value.

In the risk-neutral case, the following analogue of Theorem 3.1 holds:

Theorem 3.2 *There is a contract (a, c, x, \tilde{x}) which is optimal both for RSJW case and ACD case, under the constraint that firm S utility is the same in the two cases. That is, there is a pair (a, c) such that in ACD case firm S will choose the same entry time x and the same exit time \tilde{x} that are optimal for RSJW case.*

Proof: We only give a sketch of the proof. It is similar to the proof of Theorem 3.1 – choosing (a, c) so that $c = ak_L - (1-a)k_S$, one can check by direct computation that not only the first order conditions become the same for the exit time \tilde{x}_{RSJW} in RSJW case and the exit time \tilde{x}_S in ACD case, but that also the first order conditions are the same for the entry time x_{RSJW} in RSJW case and the entry time x_S in ACD case. ■

4 Comparative Statics

Having developed the above framework, we can now compute the optimal entry points, the optimal contract and the expected utility for various parameters of the model, that we compute here numerically across different values of bargaining power of firm S .

4.1 The benchmark case

Our benchmark case has the following parameters, in annual terms:

$$P_0 = 0, \gamma_S = \gamma_L = -1, r = 0.1, b = 0.00875, \sigma = 0.35, T = 5, 1 = k_i = l_i .$$

In particular, the two firms have the same risk aversions in the computations in this case (but not zero risk aversion, the case already studied above).

The choice of parameters r, b, σ and T is in a rough agreement with real world examples. In particular, the annual discount rate of 10% is on a high side, but not that rare historically. The annual volatility of 35% is in the ballpark of the observed values for stocks of the firms whose risk is somewhat higher than average. Finally, the value of b is somewhat arbitrary, as its interpretation depends on the monetary units.¹¹

The choice of $P_0 = 0$ is a normalization. The choice of $l_i = 1$ corresponds to zero fixed costs and the values of k_i are chosen so that the utility of zero profit is zero. (In particular, this makes the overall expected utility of a zero profit/loss process equal to zero, the same as if never entering the venture.) The choice of the risk aversion parameters γ_i is quite arbitrary, but it turns out that the qualitative behavior of the optimal parameters as functions of bargaining power are not sensitive to the values of risk aversion, as reported below. On the other hand, we have already seen in Figures 1-3 how risk aversion affects the level of optimal parameters.

The figures here are obtained by varying across the x -axis the certainty equivalent of the values V_S of the guaranteed expected utility to firm S , requiring in all three cases that the expected utility is at least as much as the chosen value of V_S . More precisely, we have on the x -axis the values of $u_S^{-1}(V_S)$ where u_S is the (exponential) utility function of firm S . Thus, we can interpret the figures as showing the y -axis values across a range of values of the bargaining power of firm S .

Figure 4 shows the optimal threshold levels for the three contract designs we study. While the difference is not large, we see that for low values of the bargaining power, the entry occurs earlier in RSJW case than in the other two cases, while for high values entry occurs sooner in ACD case (because firm S dictates the terms of entry). In ACD case, which, as we recall, can be interpreted as the case of asymmetric information, firm S with moderately low bargaining power tends to wait relatively longer to enter the alliance. This leads us to predict that firms with moderately low experience (that should roughly correspond to firms with moderately low bargaining power) prefer to enter joint ventures later, a prediction we explore below in the empirical section.

Figure 5 shows the cash payments c . For low and moderate bargaining power c is close to zero for TI and ACD cases. In fact, for TI case it stays close to zero everywhere. It is increasing for the risk-sharing RSJW case, hitting zero for equal bargaining power. It is also increasing for ACD case, but much less steeply. Overall, we see that cash is useful for sharing risk if joint welfare is the consideration, but not needed much when timing is

¹¹Nevertheless, in a model in which P_t could be interpreted as the exponential rate or return (which is not the case in our model), noting that by Ito's rule the drift of $\exp(P_t)$ is equal to $b + \sigma^2/2 = 7\%$, the rate or return of 7% is not an unreasonable number.

incentive or when the firm with low bargaining power decides on time of entry. The negative values of cash for low bargaining power of firm S for RSJW case mean that firm S pays. For example, it could be required to invest a certain amount of cash in the co-development, for instance by paying salaries of additional employees.¹²

Figure 6 shows the optimal fraction a for the three cases. It is always equal to 50% in RSJW case (because the firms have the same risk aversion). It is approximately linearly increasing in TI case, and also in ACD case for low levels of S bargaining power, while for high values thereof ACD a still increases, but slower than linearly (because a part of the firm S reward comes from its decision to enter early, and a part comes from cash) .

4.2 Varying other parameters

An interesting phenomenon occurs when the participation constraint for ACD case is not necessarily binding. For example, that happens with the parameters as in our benchmark case, except we increase the value of risk aversions. An example with $\gamma_S = \gamma_L = -4$ is depicted in Figure 7. Now, in ACD case contracts with "sticky" royalties happen. By sticky royalties we mean those royalties that vary in a narrow range. More precisely, for not too high bargaining power of firm S the value of a in ACD case stays flat – the royalties offered by firm L will be almost the same for partner firms with different degrees of bargaining power, as long as the latter is not very high. In fact, for low and moderate values of bargaining power of firm S , firm L is willing to pay more than the reservation wage, by pushing up the value of a . Thus, the participation constraint is not actually binding in this range, and the optimum clusters around similar values of a . With high bargaining power, however, the reservation wage binds, and a has to be increased to provide higher utility to firm S .¹³

For completeness, we also report on the comparative statics we obtained when varying parameters b , T , σ and γ_i . We save on space and provide no figures here, the reason being that the effects of changing the parameters are either non-surprising or not very significant, especially qualitatively.

Varying b . We explore how project quality measured by b impacts the entry time. In the risk-neutral case, it can be seen, or computed from (3.12) that the time of entry may be either earlier or sooner with higher b . For much higher b it will be sooner, but for smaller

¹²Presumably, in practice firm S typically does not have higher bargaining power than firm L , which means that it would be required to make cash payments in RSJW case, but this is not directly observed in the data. This may be due either because a limited liability constraint is adopted, that is, the payment to firm S is required to be non-negative, or because the cash payments are indirect, or simply because RSJW model is not a good depiction of reality.

¹³We also have observed this same phenomenon in our numerical exercises in other cases when the participation constraint is not binding, for example for low values of bargaining power when the horizon T is long.

values of b it can be increasing in b because of the decrease of the value of n in b . That is, because of the decrease in the effect of discounting by rate r when b is increased, it may be better to enter later. This phenomenon, though, is not new – it occurs also in the classical real options theory with only one firm making the entry decision.

Varying σ . As could be expected, with low σ the firms enter sooner.

Varying time horizon T . The qualitative behavior of the optimal values does not change much when we vary T . The only changes we see in our numerical experiments are that the difference of the values across the three contract designs is more pronounced with larger T , and the optimal entry level x as a function of bargaining power may change its convexity/concavity properties in TI and ACD cases.

4.3 The case with optimal exiting

As in Section 3.5, we now consider RSJW case in which the exit time is also chosen optimally, and ACD case in which firm S decides on both entry and exit. We consider these problems with the same parameters as in our benchmark case, and under the constraint that the expected utility of firm S is no less than in the benchmark case. We compare it to the benchmark case in which the fixed venture interval is $T = 5$ (with corresponding figures discussed above), and also in which $T = \infty$ (figures not shown).

Figures 8 and 9 show the optimal cash c and the optimal profit percentage a . ACD cash c is now close to zero also for high values of firm S bargaining power, and RSJW cash c is still increasing, but lower. ACD profit percentage a is also lower. In other words, firm S gets paid less both in cash and in royalties, because of the additional option of exiting earlier. We also report (without figures) that the firms enter the project quite a bit earlier, again because of the possibility of choosing the exit time optimally. Moreover, while Savva and Scholtes (2007) find that the efficiency of ACD case is improved when exit time is optional, in agreement with that we find that the relative difference between the expected RSJW utility and ACD utility is smaller than in the case with fixed exit time $T = \infty$, but it is larger than in the case with fixed exit time $T = 5$.

5 Empirical results

In this section we check some qualitative features of our model against the real world data. We use the data on alliances from Recombinant Capital (www.recap.com). We choose the alliances classified as co-developments where the R&D originator is a biotech firm and the other partner is a pharmaceutical firm. Among the three designs we study, we think that ACD case is the most appropriate for such ventures, as the small firm decides when to start looking for a partner, while the large firm may have more say in the way the contract is

specified. If a deal is very transparent, with most information available to both firms, then RSJW case might also be appropriate.

We allow that the alliance, in addition to co-development, includes other activities such as licensing, co-marketing, etc. Our database spans the years 1984-2003. In the cases where the same molecule is being developed for multiple therapeutic applications (e.g. cancer, infectious diseases, cardiovascular diseases, etc.), each application is treated as a separate project. Let us note that we use the data not for exact empirical estimations, but, rather, to show that our model addresses realistic issues and gives consistent insights into alliance decisions.

The nature of our sample is the following: originally there were 325 co-development alliances that satisfied the above conditions. Out of those alliances 256 have reported either the size of the deal, the royalty or both, and 9 more alliances reported only royalty; see Table 1. The remaining alliances lacked that information and therefore were excluded from the sample. Table 2 shows the averages of the continuous variables used in this section, and the information at the stage of entering into the alliance.

Our theoretical results are expressed as a function of the bargaining power of the firm S (the biotech firm). To operationalize this concept, we use a proxy variable which we define as the number of prior alliances. The rationale behind this is that the more experienced the biotech firm is, the better it is able to negotiate and has higher bargaining power. On the other hand, for the inexperienced firms (such as the new entrants) we would expect that their negotiation leverage is lower, and that they have lower bargaining power. The average number of alliances in our sample is 19.18, with standard deviation of 18.26. We create a categorical variable “experience” in the following way: the firms with no prior experience are coded as 0, the firms with little experience (less than or equal to 5 alliances) are coded as 1, the firms with medium experience (more than 5 and less than or equal to 33 alliances) are coded as 2, and the firms with abundant experience (more than 33 alliances) are coded as 3. The frequency table of the alliances for these four groups is given in Table 3.

We first consider the time of entry for the firms. We find that the time of entry indeed differs with the experience of the project originator. Table 4 shows a frequency table of the alliances depending on the stage at which the alliance was entered. Data shows that firms with no prior experience (i.e., low bargaining power), tend to wait longer to enter the alliance. Due to the small number of alliances per cell for the stage BLA/NDA filed, we combined the first and the last two categories of experience and then ran the Pearson Chi-square test. The test came out significant, indicating that the firms with less experience tend to wait longer, while the firms with more experience tend to enter alliances sooner. In our model this was the case in the context of Figure 4 for moderately low values of bargaining power for ACD case, that is, with the contract parameters a and c decided by firm L and the entry time decided by firm S .

Next, we examine whether our theoretical findings regarding the size of the cash compensation is supported by the data. In our database we have information about the total size of the deal (and the royalties, which are not included in the total size). We subtract from it the amount of money used for equity payments to get to the cash amount paid in the alliance.¹⁴ To make sure that we do not overinflate the cash payments in the cases when we have several applications of one chemical compound, we count the payments only once (i.e., we count the cash per compound, not per application). Using ANOVA with cash as the dependent variable and experience as the categorical independent variable, we find that cash is significantly higher for firms that are classified as very experienced, with $F(3, 200) = 6.76, p = 0.0002$, the values provided in Table 5. This effect is shown in Figure 10 and is approximately in line with the theoretical findings from Figure 5 for ACD case. Obviously, it is only a rough agreement, as the dependence in Figure 10 is actually somewhat decreasing for low bargaining power.

As for the royalty percentage, an examination of our database shows that among all the co-development alliances between a biotech company in the role of a R&D originator and a partnering pharma company, the royalties that range between 40% and 60% appear in 51.5% of the contracts. In particular, the royalties of exactly 50% appear in 42.3% of the contracts. Recall that in our model royalties in this range occur for all three contract designs in the case of similar risk aversion for the firms and similar bargaining power (Figure 6), as well as in ACD case when the risk aversion is high (Figure 7).

Finally, let us mention another finding, however of lesser statistical significance. We find some confirmation in the data that the inexperienced firms are paid smaller royalties: 34.2% of the inexperienced firms receive royalties of less than 20%, while that is true for only 17.4% of the experienced firms. That is, larger bargaining power tends to imply larger royalties, consistent with the theoretical implications (Figures 6 and 7). However, when we run a regression with royalty as the dependent variable and the number of prior alliances as the predictor, we do not get a significant relationship. This may be because most royalties are around 50% and therefore there is not enough variation for a significant relationship.

6 Conclusions

In this paper we use the real options theory to model the decision of two (potentially risk averse) firms, called S and L , to enter a co-development alliance, where firm S is the project originator. Methodologically, we use the theory of optimal stopping of diffusion processes. We also find the optimal sharing of profits between the two firms, among linear sharing rules.

¹⁴In our theoretical model, the cash is paid at a constant rate c , rather than as a lump-sum payment. However, this makes no difference for our analysis, as the amount of the lump-sum payment is simply the rate times the time horizon T .

We consider the case of risk-sharing between the two firms, the case of agreeing on the time to enter, and the case of asymmetric contract decisions. In the latter case we assume that firm S decides on the initiation time, while firm L decides on how to share the profits.

We find that allowing side cash payments makes all three contract designs equivalent if the firms are risk-neutral. However, for risk averse firms, the three contract designs may differ significantly in the slope and the intercept of the contract, but not much in the time of entry nor in the level of expected utility. We also find that while cash is useful for sharing risk if joint welfare is the consideration, it does not play an important role when timing is incentive or when the firm with low bargaining power decides on time of entry. The participation constraint of firm S is not always binding in ACD case. When it is not, the optimal percentage of profits becomes "sticky", i.e., firm L offers the same profit percentage to firms of various bargaining powers, as long as the latter is not very high, and for reasonable parameter values this percentage is in the mid-range. The optimality of royalties in the 40-60% range, the common royalty range in our data, also occurs theoretically in the case in which the firms have similar risk aversions and bargaining powers.

We also examine the sensitivity of the optimal contracts to the project quality, the project uncertainty, the length of time horizon and the difference in risk aversions. We find that in most cases the parameters of the optimal contract do not change their qualitative behavior as functions of bargaining power when those values vary. However, their quantitative levels may change significantly and their values may differ quite a bit across different contract designs.

In general, while our model is quite stylized and has few parameters, the qualitative conclusions we obtain are in a rough agreement with empirical data. Nevertheless, it would be of significant interest to extend our analysis to the following cases:

- Moral hazard risk coming from uncertainty about firm L 's commitment to the project. One way to model this is to replace the project quality parameter b with $b + e$, where effort e is controlled by firm L , but unobserved by firm S . This would mean solving a problem of optimal contracting with moral hazard and optional entry time, something which has not been done in general.¹⁵ Similarly, we could assume that firm S controls the size of the drift b .

- Including jumps into the profit/loss process, representing the sudden changes, for example due to the arrival of testing results for a new drug.

We leave the analysis of these ambitious modeling frameworks for future research.

¹⁵However, see Cvitanic, Wan and Zhang (2008) for moral hazard problems with optional exit time.

7 Appendix

7.1 Optimal entry time in the general case

Consider the process P following a Stochastic Differential Equation (SDE)

$$dP_t = bdt + \sigma dW_t$$

where b, σ are constants and W is a Brownian motion process. In some calculations it will be more elegant to present results in terms of the geometric Brownian motion $\exp(P)$. Since in the following lemma we use only the strong Markov property, the result is valid for both $X = P$ and $X = \exp(P)$.

Lemma 7.1 *We have*

$$E_x \left[\int_{\tau}^{\tau+T} e^{-rs} h(X_s) ds \right] = E_x \left[e^{-r\tau} E_{X_{\tau}} \int_0^T e^{-ru} h(X_u) du \right] .$$

Proof: Directly from

$$\begin{aligned} E_x \left[\int_{\tau}^{\tau+T} e^{-rs} h(X_s) ds \right] &= E_x \left[e^{-r\tau} E_{\mathcal{F}_{\tau}} \int_0^T e^{-ru} h(X_{\tau+u}) du \right] \\ &= E_x \left[e^{-r\tau} E_{X_{\tau}} \int_0^T e^{-ru} h(X_u) du \right] . \end{aligned}$$

■

Since the second expectation in the last line is a function $F(X_{\tau})$, it is sufficient for us to consider the following general stopping time problem:

$$\tilde{V}(p) = \sup_{\tau} E_x[\mathbf{1}_{\{\tau < \infty\}} e^{-r\tau} f(P_{\tau})] . \quad (7.17)$$

Assumptions needed below are such that the results are more naturally presented in terms of $X = \exp(P)$ (the singularity of the utility function is then positioned around zero instead of around $-\infty$). Because of this, we look at the process

$$dX_t = X_t[\tilde{b}dt + \sigma dW_t]$$

where $\tilde{b} = b + \sigma^2/2$. We assume $r > 0$ and $r > \tilde{b}$, and we denote $x = X(0) = e^{P(0)}$. We then rewrite our problem as

$$V(x) = \sup_{\tau} E_x[\mathbf{1}_{\{\tau < \infty\}} e^{-r\tau} g(X_{\tau})] . \quad (7.18)$$

Denote

$$\mathcal{L}y(x) = \frac{1}{2}\sigma^2 x^2 y''(x) + \tilde{b}y'(x) - ry(x)$$

and recall also the notation $\beta = 1 + 2b/\sigma^2$, $n = 1/2 - \beta/2 + \sqrt{(\beta/2 - 1/2)^2 + 2r/\sigma^2}$.

Assumption 7.1

- (i) $g \in C^2((0, \infty))$
- (ii) There exists a unique $x^* > 0$ such that $q(x^*) = 0$ where

$$q(x) := ng(x) - xg'(x) \quad .$$

Moreover, we have $q(0) < 0$.

- (iii) $\mathcal{L}g(x) \leq 0$, $x > x^*$.
- (iv) $[xg'(x)]^2 \leq C(1 + x^j)$, for some $j \geq 1$, $x > x^*$.
- (v) $E_x[\mathbf{1}_{\{\tau < \infty\}} e^{-r\tau} |g(X_\tau)|] < \infty$ for all stopping times τ and all $x > 0$.
- (vi) $\lim_{T \rightarrow \infty} E_x[e^{-rT} |g(X_T)|] = 0$.

Define

$$A := \frac{g(x^*)}{(x^*)^n}$$

and the function w by

$$w(x) = Ax^n , \quad x < x^*$$

$$w(x) = g(x) , \quad x \geq x^* \quad .$$

It is easily verified that $w \in C^1((0, \infty)) \cap C^2((0, \infty) \setminus \{x^*\})$.

Theorem 7.3 Under Assumption (7.1), we have

$$w(x) = V(x)$$

and the optimal stopping time is

$$\hat{\tau} = \inf\{t \geq 0 \mid X_t \geq x^*\} \quad .$$

Proof: Note that

$$\mathcal{L}w(x) \leq 0 , \quad x > 0 \quad .$$

We also want to show that

$$w(x) \geq g(x) , \quad x < x^* \quad .$$

By definition of w and A this is equivalent to

$$\frac{g(x^*)}{(x^*)^n} \geq \frac{g(x)}{(x)^n} , \quad x \leq x^* \quad . \tag{7.19}$$

We have

$$\frac{d}{dx} \left(\frac{g(x)}{(x)^n} \right) = - \frac{q(x)}{(x)^{2n}} \quad .$$

However, since x^* is the unique solution of $q(x) = 0$ and $q(0) < 0$, we see that the above derivative is positive for $x < x^*$, which proves (7.19).

Next, define

$$\tau_k = \inf\{t \geq 0 \mid X_t \leq 1/k\} .$$

Fix $T > 0$. By Ito's rule,

$$e^{-r(\tau \wedge \tau_k \wedge T)} w(X_{\tau \wedge \tau_k \wedge T}) = w(x) + \int_0^{\tau \wedge \tau_k \wedge T} e^{-rs} \mathcal{L}w(X_s) ds + M_\tau^{k,T} \quad (7.20)$$

where

$$M_t^{k,T} = \int_0^{t \wedge \tau_k \wedge T} e^{-rs} \sigma X_s w'(X_s) dW_s .$$

We have

$$\begin{aligned} E \left[\int_0^T [e^{-rs} \sigma X_s w'(X_s)]^2 \mathbf{1}_{\{s \leq \tau_k\}} ds \right] &\leq \sup_{x \in [1/k, x^*]} [\sigma x w'(x)]^2 T + E \left[\int_0^T [\sigma X_s w'(X_s)]^2 \mathbf{1}_{\{X_s > x^*\}} ds \right] \\ &\leq \sup_{x \in [1/k, x^*]} [\sigma x w'(x)]^2 T + C \left(T + \int_0^T E[X_t^j] dt \right) \\ &< \infty . \end{aligned}$$

This means that $M^{k,T}$ is a martingale, and that $E[M_\tau^{k,T}] = 0$. This implies, taking expectations in (7.20), using $\mathcal{L}w \leq 0$, $w \geq g$, that

$$E \left[e^{-r\tau} g(X_\tau) \mathbf{1}_{\{\tau \leq \tau_k \wedge T\}} \right] \leq w(x) - w(1/k) E \left[e^{-r\tau_k} \mathbf{1}_{\{\tau_k \leq T \leq \tau\}} \right] - E \left[e^{-rT} w(X_T) \mathbf{1}_{\{T < \tau_k < \tau\}} \right] . \quad (7.21)$$

Since $w(0) = 0$, the middle term on the right-hand side converges to zero as $k \rightarrow \infty$. Moreover, since $0 \leq w(x) \leq C(1 + |g(x)|)$, and because of Assumption 7.1 (vi), the last term converges to zero when $T \rightarrow \infty$. Finally, by Assumption 7.1 (v), we can use the dominated convergence theorem to conclude that the term on the left-hand side converges to $E \left[e^{-r\tau} g(X_\tau) \mathbf{1}_{\{\tau < \infty\}} \right]$. After taking expected value and the limits, we get $V(x) \leq w(x)$.

If we now repeat the above argument with $\tau = \hat{\tau}$, we will see that everything holds as equality, hence $V(x) = w(x)$ and $\hat{\tau}$ is optimal.

■

7.2 Verifying Assumption 7.1

Our method above applies to general utility functions. Let us now check that it works for exponential utility functions applied to P , or, equivalently, to power utility functions applied to $X = e^P$. We consider the function

$$g(x) = c_0 + \sum_{i=1}^I c_i x^{\gamma_i} / \gamma_i$$

for $x > 0$, with $c_i \geq 0$ and at least one $c_i > 0$ for $i \geq 1$. (In the body of the paper we have $I = 1$ or $I = 2$, depending on whether we are maximizing a single firm's objective or joint welfare.) For those $i \geq 1$ with $c_i > 0$ we also assume that all γ_i are of the same sign, that we have $\gamma_i c_0 < 0$, and that $0 < \theta(\gamma_i) := r - \gamma_i^2 \sigma^2 / 2 - \gamma_i b$.

Obviously, $g' > 0$, $g'' < 0$. We can compute

$$q' = (ng - xg')' = (n - 1)g' - xg'' > 0 \quad .$$

Also, we have $q(0) < 0$, $q(\infty) > 0$. Thus, there exists a unique x^* such that $q(x^*) = 0$. From this, and from

$$q(x) = nc_0 + \sum_i c_i x^{\gamma_i} (n/\gamma_i - 1)$$

we can compute

$$nc_0 = - \sum_i c_i (x^*)^{\gamma_i} (n/\gamma_i - 1) \quad .$$

We also have

$$L(x) := \mathcal{L}g(x) = - \sum_i c_i x^{\gamma_i} \theta(\gamma_i) / \gamma_i - rc_0 \quad .$$

Note that

$$L'(x) = - \sum_i c_i x^{\gamma_i - 1} \theta(\gamma_i) < 0 \quad .$$

So, in order to prove $L(x) \leq 0$ for $x \geq x^*$, it is sufficient to show $L(x^*) \leq 0$. From the above expressions for c_0 and $L(x)$ we get

$$\begin{aligned} L(x^*) &= \sum_i c_i (x^*)^{\gamma_i} \left[r \left(\frac{1}{\gamma_i} - \frac{1}{n} \right) - \theta(\gamma_i) / \gamma_i \right] \\ &= \sum_i c_i (x^*)^{\gamma_i} \left[-\frac{r}{n} + \frac{1}{2} (\gamma_i - 1) \sigma^2 + \tilde{b} \right] \quad . \end{aligned}$$

Using the notation $\beta = 2\tilde{b}/\sigma^2$ and that $\gamma_i < 1$, it is then sufficient to show

$$n\beta - 2r/\sigma^2 \leq 0 \quad .$$

Denote $\beta_r = 2r/\sigma^2$. Then, the above is equivalent to

$$\beta \sqrt{(\beta/2 - 1/2)^2 + \beta_r} < \beta_r + \beta^2/2 - \beta/2$$

or, after squaring

$$\beta^2(\beta^2/4 - \beta/2 + 1/4 + \beta_r) < \beta_r^2 + \beta^4/4 + \beta^2/4 + \beta^2\beta_r - \beta\beta_r - \beta^3/2 \quad .$$

After cancelations, this boils down to

$$0 < \beta_r^2 - \beta_r \beta = \beta_r(\beta_r - \beta)$$

which is true, and we are done with proving (iii).

Assumption (iv) is straightforward. Next, we can easily see that

$$e^{-rt} X_t^\gamma = C e^{-(r-\tilde{b})t} M_t$$

where $M_t = \exp\{\sigma^2 t/2 + \sigma W_t\}$ is a positive martingale with expectation equal to one. Then, (vi) follows immediately since $r > \tilde{b}$. Similarly, using Fatou's lemma and looking at a sequence $\tau \wedge N$ and letting $N \rightarrow \infty$, we also get (v).

7.3 Optimal entry time in the special case

We now turn to the particular case of the entry problem that interests us in this paper, that is, the problem of maximizing over stopping times τ the expression

$$E_x \left[\mathbf{1}_{\{\tau < \infty\}} \int_\tau^{\tau+T} e^{-rt} h(X_t) dt \right] \quad (7.22)$$

with

$$h(x) = \sum_{i=1}^I [k_i + l_i x^{\gamma_i} / \gamma_i]$$

where $\gamma_i < 0$, $l_i \geq 0$, at least one of l_i is strictly positive, and $\sum_{j=1}^I k_j \frac{1-e^{-rT}}{r} > 0$.

Denote

$$R_h(x) := E_x \int_0^T e^{-rs} h(X_s) ds .$$

We showed in Lemma 7.1 that the problem of maximizing (7.22) is equivalent to the maximization problem

$$w(x) := \sup_\tau E_x \left[\mathbf{1}_{\{\tau < \infty\}} e^{-r\tau} R_h(X_\tau) \right] . \quad (7.23)$$

We can compute

$$R_h(x) = \sum_i [k_i(r) + l_i(\theta(\gamma_i)) \frac{x^{\gamma_i}}{\gamma_i}]$$

where we recall that

$$k_i(r) = k_i \frac{1 - e^{-rT}}{r} , \quad l_i(\theta(\gamma_i)) = l_i \frac{1 - e^{-\theta(\gamma_i)T}}{\theta(\gamma_i)} .$$

According to Theorem 7.3 and subsection 7.2, the optimal time of entry is the first time process X hits the point \hat{x} that is determined from the equation

$$nR_h(\hat{x}) = \hat{x}R'_h(\hat{x}) \quad (7.24)$$

or

$$\sum_{i=1}^I k_i(r) = \sum_{i=1}^I l_i(\theta(\gamma_i)) \left(\frac{1}{n} - \frac{1}{\gamma_i} \right) \hat{x}^{\gamma_i} . \quad (7.25)$$

Remark 7.2 Let us emphasize that the method here allows for some flexibility in the choice of the state process vs. the choice of the utility functions. Suppose that our profit/loss rate process is a 1 - 1 function of the state process X , $P_t = F(X_t)$, and $X_t = F^{-1}(P_t)$. Hence, applying an utility function $u(\cdot)$ on P_t is equivalent to applying the utility function $u(F(\cdot))$ (assuming it is concave) on X_t . For example, as we have already mentioned above, mathematically the problem with the geometric Brownian motion and the power utility function is equivalent to the problem with the arithmetic Brownian motion and the exponential utility function. In this way we keep the convenience of both worlds; the applications based on the ordinary Brownian motion and the mathematical solutions of the problem based on the geometric Brownian motion.

7.4 Proof of Proposition 3.6

Consider the general entry/exit problem to maximize over the entry time τ_0 and duration time $\bar{\tau} \geq 0$ the quantity

$$\begin{aligned} & E_x \left[\mathbf{1}_{\{\tau_0 < \infty\}} \int_{\tau_0}^{\tau_0 + \bar{\tau}} e^{-rt} h(X_t) dt \right] \\ &= E_x \left[\mathbf{1}_{\{\tau_0 < \infty\}} \left(\int_{\tau_0}^{\infty} e^{-rt} h(X_t) dt - \int_{\tau_0 + \bar{\tau}}^{\infty} e^{-rt} h(X_t) dt \mathbf{1}_{\{\bar{\tau} < \infty\}} \right) \right] . \end{aligned}$$

Denote

$$R_h(x) = E_x \int_0^{\infty} e^{-rs} h(X_s) ds .$$

Then, similarly as in Lemma 7.1, the above is equivalent to the maximization problem

$$w_A := \sup_{\tau_0, \bar{\tau}} E_x \left[\mathbf{1}_{\{\tau_0 < \infty\}} e^{-r\tau_0} R_h(X_{\tau_0}) - \mathbf{1}_{\{\tau_0 < \infty\}} e^{-r\tau_0} E_{X_{\tau_0}} [e^{-r\bar{\tau}} R_h(X_{\bar{\tau}}) \mathbf{1}_{\{\bar{\tau} < \infty\}}] \right] .$$

Introduce the “exit value function”

$$v_E(x) := \sup_{\bar{\tau}} E_x [-e^{-r\bar{\tau}} R_h(X_{\bar{\tau}}) \mathbf{1}_{\{\bar{\tau} < \infty\}}] .$$

The mixed entry/exit problem can then be written as

$$w_A = \sup_{\tau_0} E_x [e^{-r\tau_0} [R_h(X_{\tau_0}) + v_A(X_{\tau_0})] \mathbf{1}_{\{\tau_0 < \infty\}}] .$$

Consider the ACD case of firm S deciding on both the entry and the exit. The optimal exit time is \tilde{x}_S given by the same expression as in (2.4) except with n replaced by \tilde{n} , where

\tilde{n} differs from n in the sign of the square-root term. As in Proposition 3.4, we then get, if $\tilde{x}_S < P_0$,

$$v_E(P_0) = -k_S(r) \left(1 + \frac{\tilde{n}}{a\gamma_S - \tilde{n}}\right) e^{-\tilde{n}(\tilde{x}_S - P_0)} .$$

If $\tilde{x}_S \geq P_0$ then it is optimal to stop immediately.

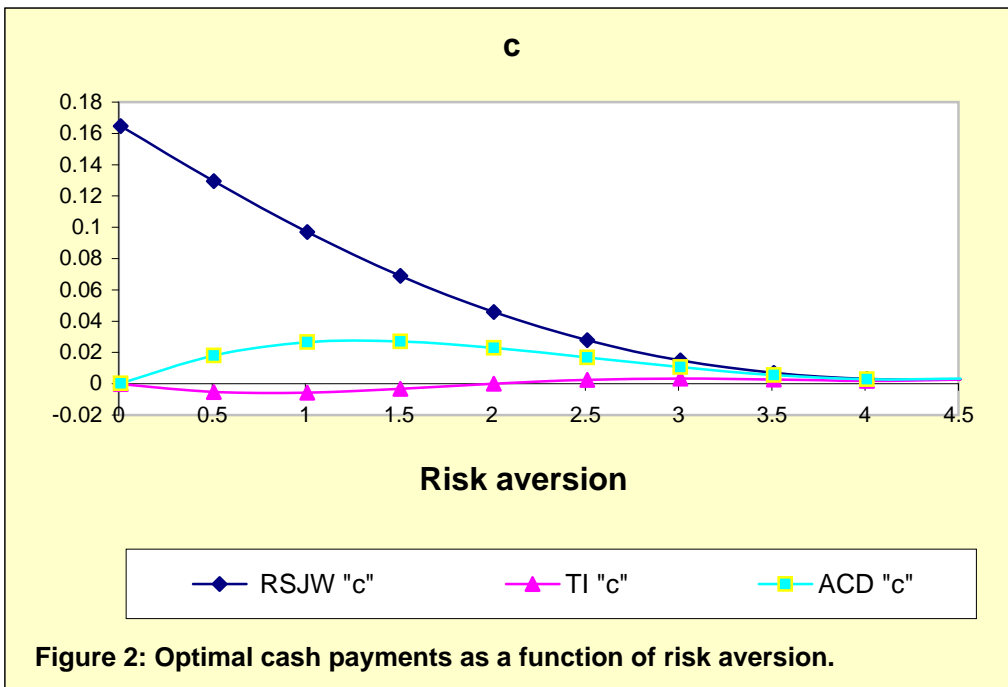
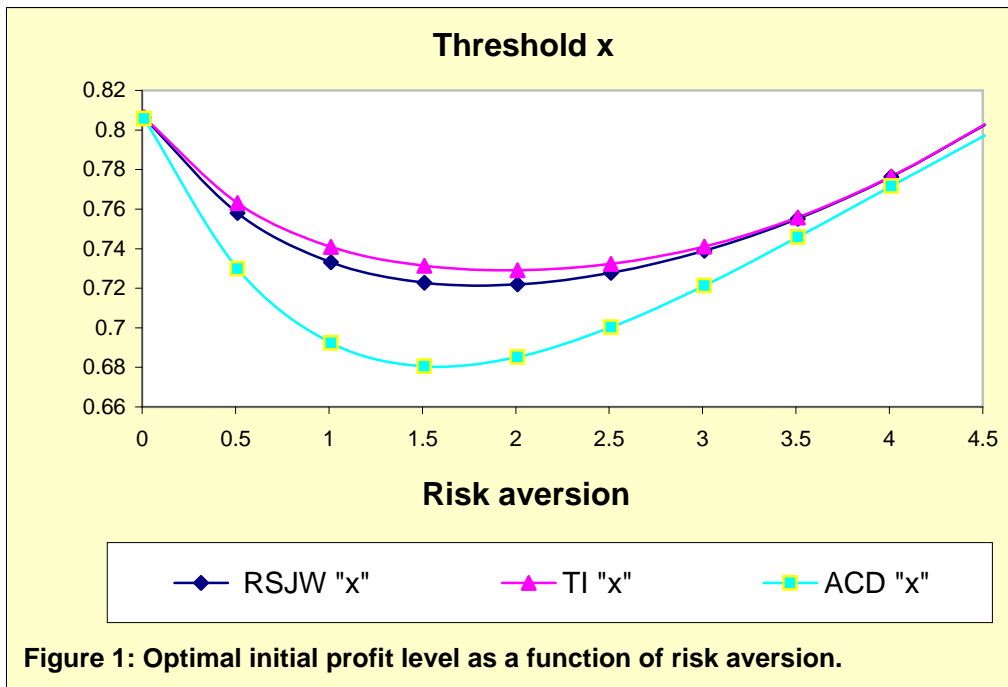
The results of Proposition 3.6 now follow from the above computations and the expressions for function h in RSJW and ACD problems.

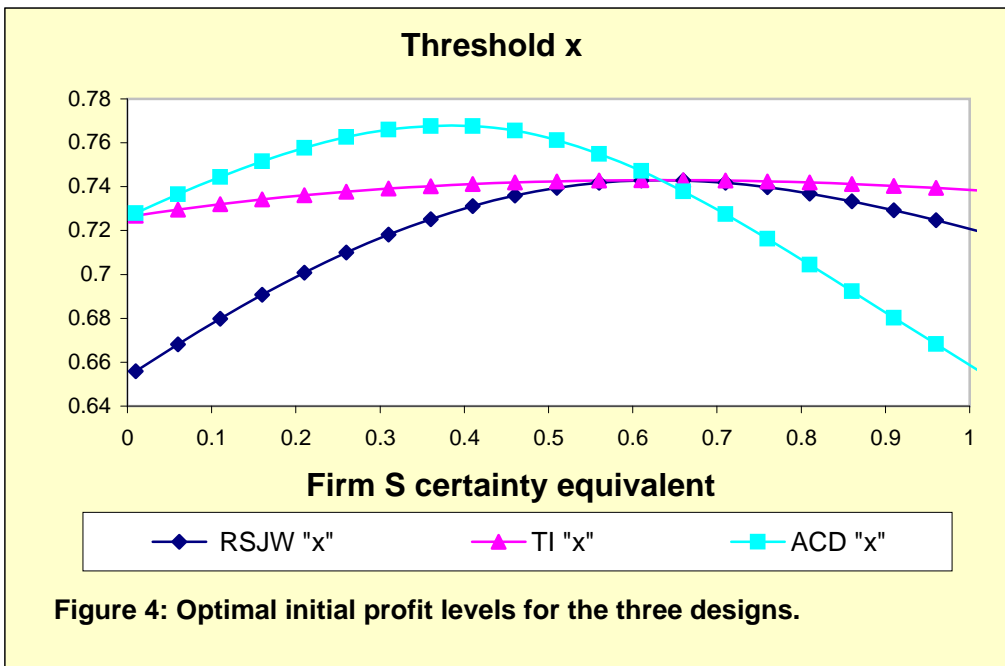
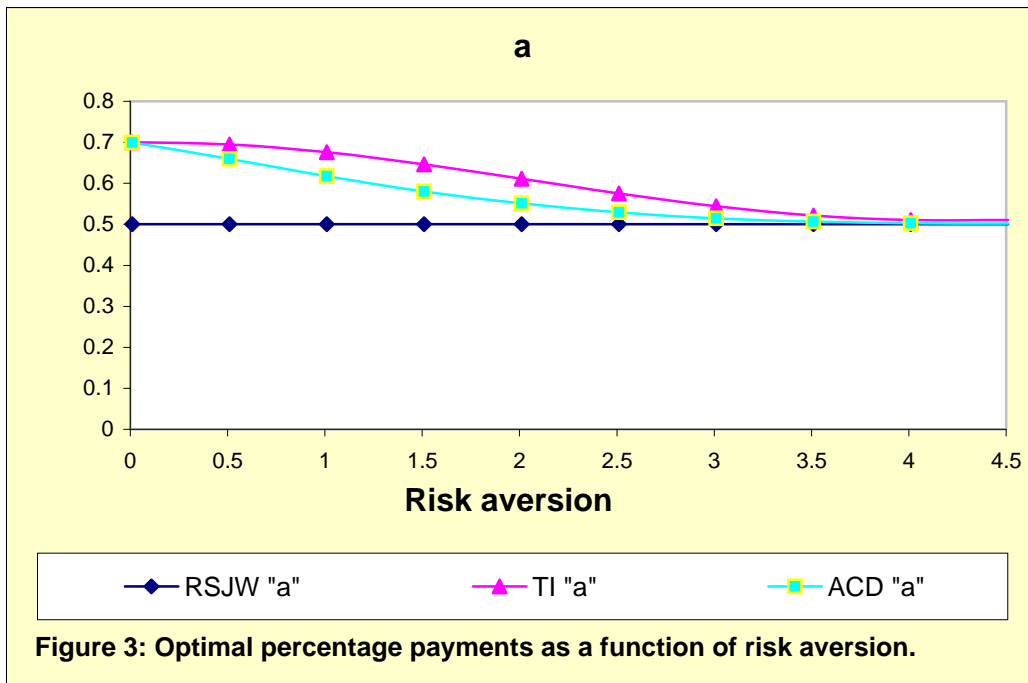
References

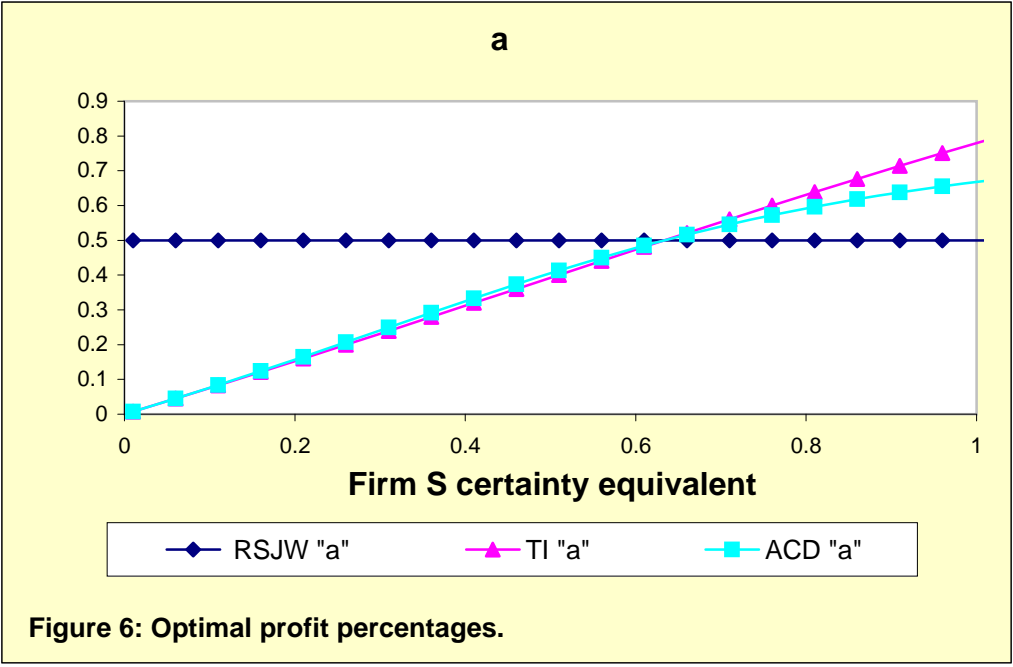
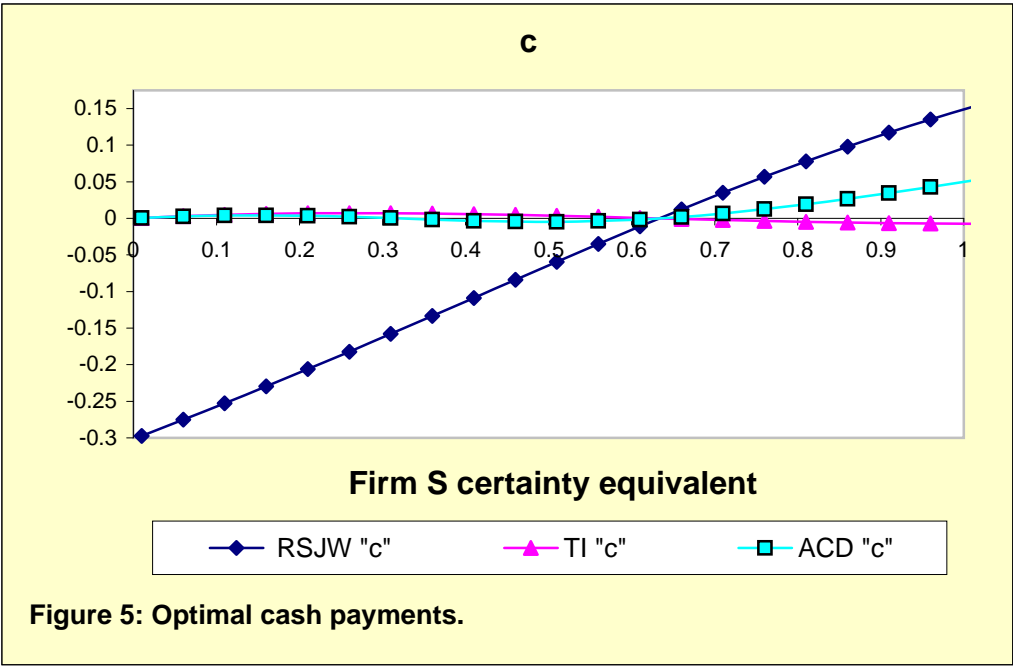
- [1] Amit, R., Glosten, L. and Muller, E. (1990) Entrepreneurial ability, venture investments, and risk sharing. *Management Science* 36, 1232–1245.
- [2] Bolton, P. and Dewatripont, M. (2005) *Contract Theory*. MIT Press.
- [3] Borch, K. (1962) Equilibrium in a reinsurance market. *Econometrica* 30, 424–444.
- [4] Chesbrough, H. and Schwartz, K. (2007) Innovating Business Models With Co-Development Partnerships. *Research Technology Management* 50, 55–59.
- [5] Crama, P., De Reyck, B. and Degraeve, Z. (2007). Milestone payments or royalties? Contract design for R&D licensing. Working paper.
- [6] Cvitanić, J., Wan, X. and Zhang, J. (2008) Principal-Agent Problems with Exit Options. *The B.E. Journal in Theoretical Economics* 8, www.bepress.com/bejte/vol8/iss1/art23 .
- [7] Dixit, A. K., and Pindyck, R. S. (1994) *Investment under Uncertainty*. Princeton University Press.
- [8] Folta, T. B. (1998) Governance and uncertainty: The trade-off between administrative control and commitment. *Strategic Management J.* 19, 1007–1028.
- [9] Folta, T. B. and Miller, K. (2002) Real options in equity partnerships. *Strategic Management J.* 23, 77–88.
- [10] Gallini, N. T. and Wright, B. D. (1990) Technology transfer under asymmetric information. *The RAND Journal of Economics* 21, 147–160.
- [11] Gulati, R. (1998) Alliances and networks. *Strategic Management Journal* 19, 293–317.
- [12] Habib, M. A. and Mella-Barral, P. (2007) The Role of Knowhow Acquisition in the Formation and Duration of Joint Ventures. *Review of Financial Studies* 20, 189–233.

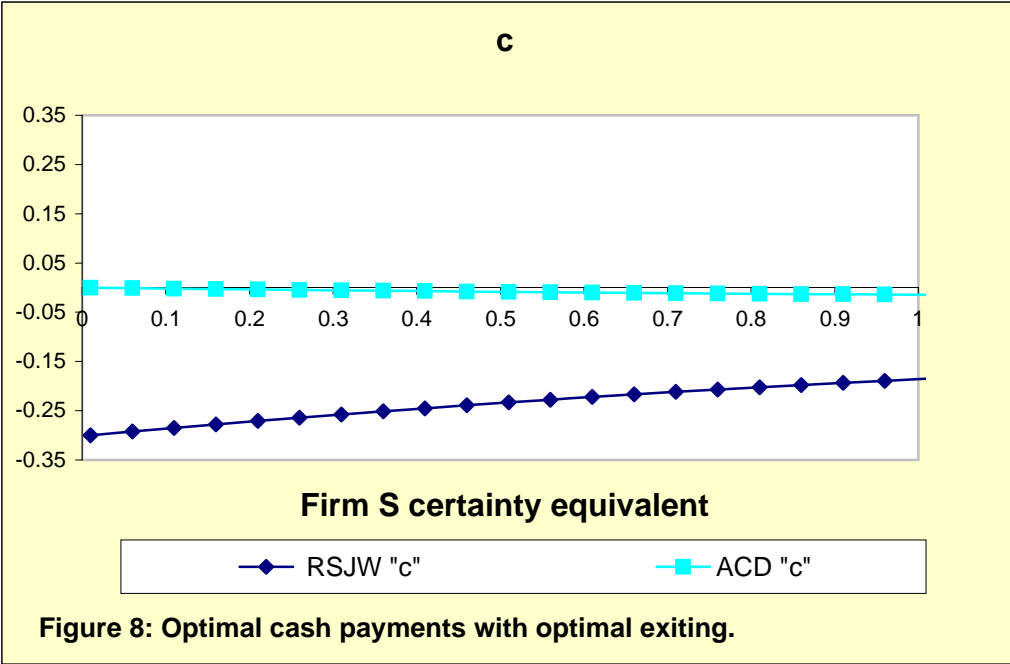
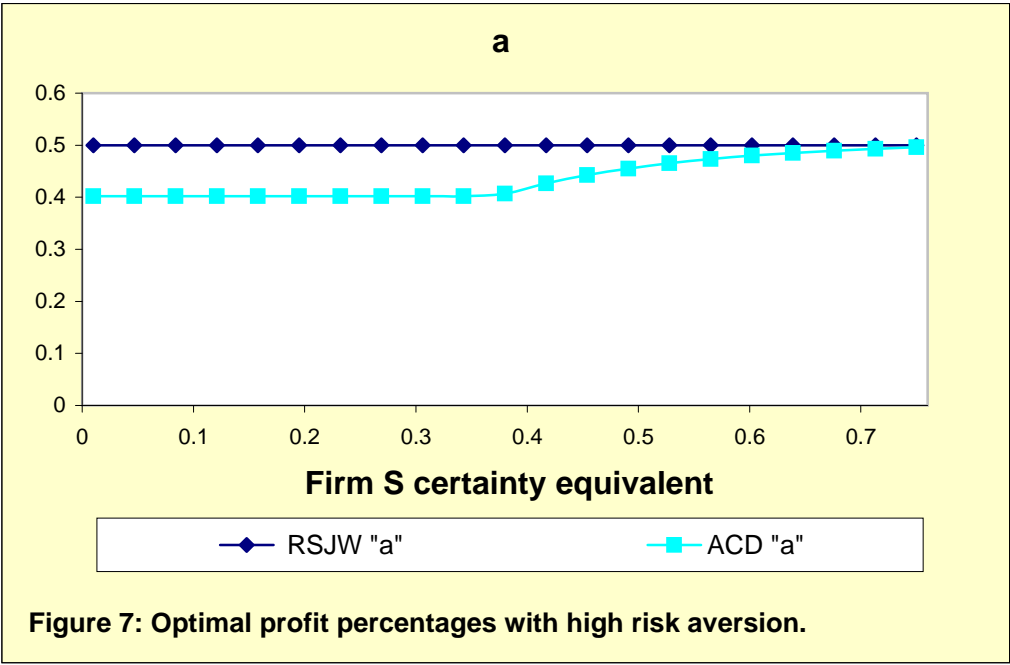
- [13] Hagedoorn, J. (1993) Understanding the rationale for strategic technology partnering: interorganizational models of co-operation and sectoral differences. *Strategic Management Journal* 14, 371–386.
- [14] Henderson, V. and Hobson, D.G. (2002) *Journal of Economic Dynamics and Control* 27, 329–355.
- [15] Henderson, V. (2007) Valuing the Option to Invest in an Incomplete Market, *Mathematics and Financial Economics* 1, 103–128.
- [16] Johnson, T.C. and Zervos, M. (2010) The explicit solution to a sequential switching problem with non-smooth data. *Stochastics: an international journal of probability and stochastic processes* 82, 69–109.
- [17] Kogut, B. (1991) Joint Ventures and the Option to Expand and Acquire. *Management Science* 37, 19–33.
- [18] Kogut, B. and Kulatilaka, N. (2001) Capabilities as real options. *Organization Science* 12, 744–758.
- [19] Lambrecht B. M. (2004) The timing and terms of mergers motivated by economies of scale. *Journal of Financial Economics* 72, 41–62.
- [20] Lambrecht, B. and Perraudin, W. (2003) Real options and preemption under incomplete information. *Journal of Economic Dynamics and Control* 27, 619–643.
- [21] McDonald, R. and Siegel, D. (1986) The Value of Waiting to Invest. *The Quarterly Journal of Economics* 101, 707–728.
- [22] McGrath, R. G. (1997) A real options logic for initiating technology positioning investments. *Acad. Management Rev.* 22, 974–996.
- [23] Miao, J. and Wang, N. (2007) Investment, consumption, and hedging under incomplete markets. *Journal of Financial Economics* 86, 608–642.
- [24] Miller, K. D. and Folta, T. B. (2002) Option value and entry timing. *Strategic Management J.* 23, 655–665.
- [25] Mitchell, G. R. and Hamilton, W. F. (1988) Managing R&D as a strategic option. *Res.-Tech. Management* 31, 15–22.
- [26] Nicholson, S., Danzon P.M. and McCullough, J. (2005) Biotech-Pharmaceutical Alliances as a Signal of Asset and Firm Quality. *Journal of Business* 78, 1433–1464.

- [27] Quinn, J.B. (2000) Outsourcing innovation: the new engine of growth. *Sloan Management Review* 41, 13–28.
- [28] Reuer, J. J. and Tong, T. W. (2005) Real options in international joint ventures. *J. Management* 31, 403–423.
- [29] Sarkar, S. (2000) On the investment-uncertainty relationship in a real options model. *Journal of Economic Dynamics and Control* 24, 219–225.
- [30] Savva, N. and Scholtes, S. (2007) Double or quits: New Product Co-development with Opt-Out Options. Working paper.
- [31] Ziedonis, A.A. (2007) Real Options in Technology Licensing. *Management Science* 53, 1618–1633.









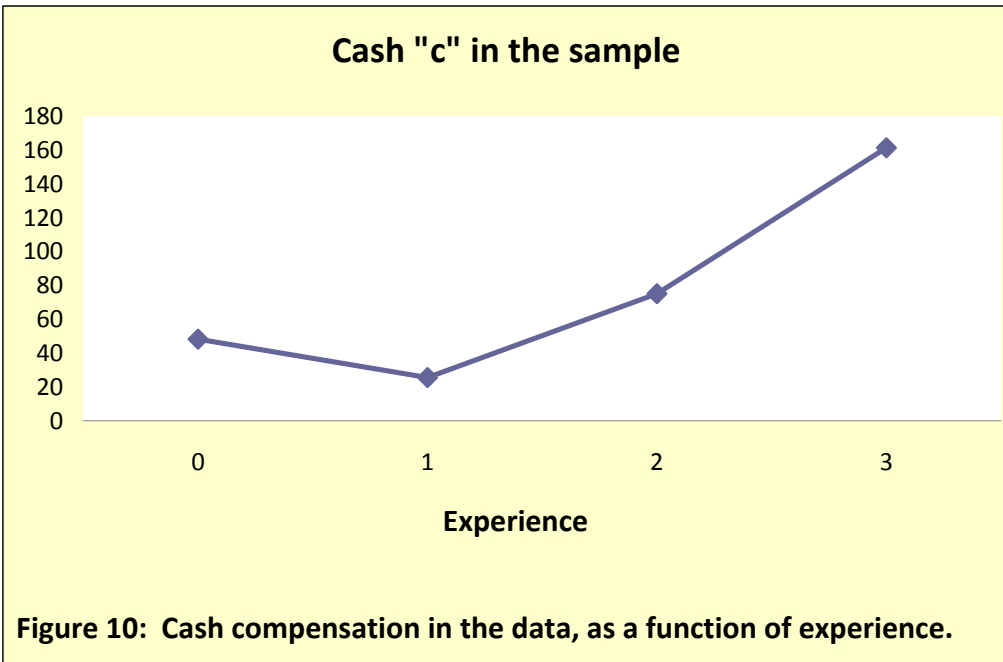
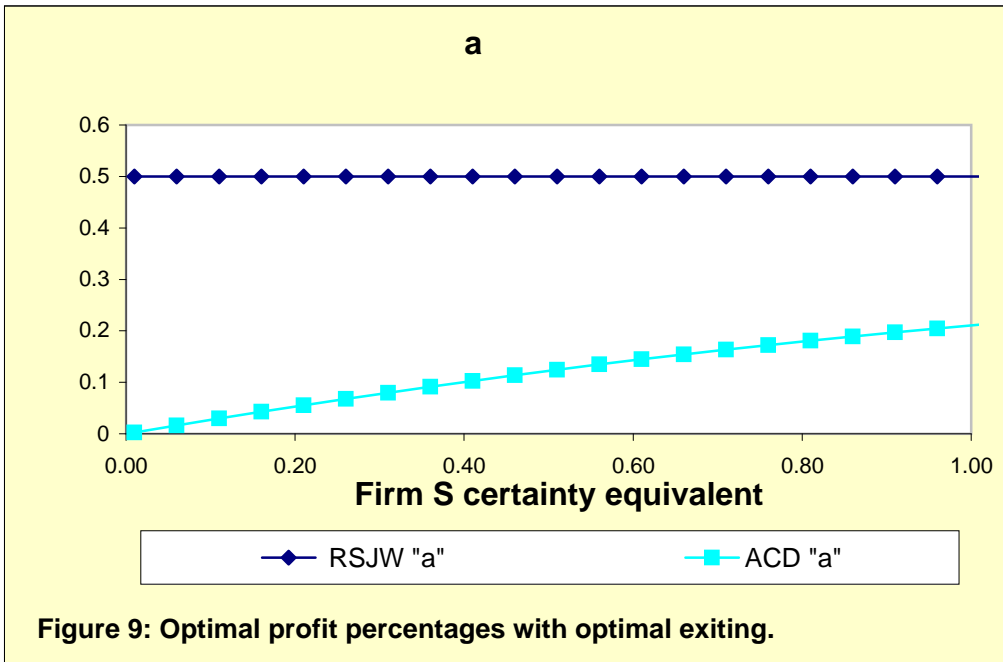


Table 1. Descriptions of the sample

		Royalty		Total
		Reported	Non-reported	
Size of the deal	Reported	188	68	256
	Non-reported	9	0	9
Total		197	68	265

Table 2. Additional descriptions of the sample

Variable	Average	Minimum	Maximum	St.Dev.	
Number of previous alliances	19.18	0.00	91	18.26	
Royalty (in percentages)	0.35	0.01	0.75	0.19	
Size of the deal (in millions of dollars)	129.32	0.00	1658.00	287.41	
Length of alliance (in years)	4.13	0.00	10.00	2.43	
	Preclinical/ discovery	Phase I	Phase II	Phase III	BLA/ NDA filed
Number of alliances per stage of signing	138	22	29	44	22

Table 3. Distribution of alliances dependent on experience

	Variable coding	Number of alliances per cell
Totally without experience (no prior alliances)	0	56
Small experience (less than or equal to 5 alliances)	1	46
Medium experience (more than 5 and less than or equal to 33 alliances)	2	120
Abundant experience (more than 35 alliances)	3	43

Table 4. Time of entry and co-development experience

Phase of signing	Number of alliances per experience				Row total
	Totally without experience	Small experience	Medium experience	Abundant experience	
Preclinical/discovery	26 (46.43%)	28 (66.67%)	61 (53.04%)	23 (54.76%)	138
Phase I	2 (3.57%)	4 (9.52%)	11 (9.57%)	5 (11.9%)	22
Phase II	4 (7.14%)	5 (11.9%)	16 (13.91%)	4 (9.52%)	29
Phase III	9 (16.07%)	3 (7.14%)	23 (20.0%)	9 (21.43%)	44
BLA/NDA filed	15 (26.79%)	2 (4.76%)	4 (3.48%)	1 (2.38%)	22
TOTAL					255
	Limited experience		Larger experience		
Preclinical/discovery	54 (55.1%)		84 (53.5%)		138
Phase I	6 (6.12%)		16 (10.19%)		22
Phase II	9 (9.18%)		20 (12.74%)		29
Phase III	12 (12.24%)		32 (20.38%)		44
BLA/NDA filed	17 (17.35%)		5 (3.18%)		22
TOTAL					255

For the second model: Pearson Chi-square: 18.1993, df=4, p=.001129

Table 5. Cash as a function of experience: ANOVA results

	N	Cash		
		Mean	Std.Dev.	Std.Err
Total	204	71.42	128.85	9.021
Totally without experience	39	47.67	44.75	7.17
Small experience	40	25.51	28.31	4.48
Medium experience	100	76.62	135.15	13.51
Abundant experience	25	161.14	218.38	43.68