Nuclear Energy: Fission & Fusion

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Reference materials

1. Book less advanced than course material:
   Oxtoby, Gillis, and Nachtrieb
   Principles of Modern Chemistry (5th Edition), Chapter 14
   (Ch1 textbook)

2. Book more advanced than course material:
   Lilley, John S.
   Nuclear physics : principles and applications
   ISBN 0-471-97936-8
   Publisher: J. Wiley, Chichester, New York, 2001
   2 copies in Sherman Fairchild Library under Ch/Aph2
   3 hours closed reserve, no overnight
   Library call numbers:
   QC776 .L45 2001
   QC776 .L45 2001 c.2

3. HyperPhysics Website http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

4. Nuclear Energy web links on class web site: Nuclear Energy WebLinks.docPhysics
   (links to documents on Economics, Policy, Enrichment, Chernobyl, Fusion)

5. Hungarian Energy Agency Chernobyl accident file posted on class web site:
   ReactorAccidents-Chernobil.html

6. List of world's nuclear reactors on class web site: World nuclear reactor list.htm

Fission/Fusion Module Synopsis

- First week
  - Basic nuclear physics, fission physics
- Second Week
  - Fission reactor design & issues
- Third week
  - Fusion physics, reactors, Caltech research

First Week

Basic nuclear physics, fission physics

- Alpha, beta, gamma radioactivity
- Implications of $E=mc^2$
- Binding energy: fission v. fusion
- Table of nuclides
- Decay chains
- Cross-sections
- Details of fission and fusion
- Neutron-rich property of large Z nuclides

Radioactivity, subatomic particles, history, penetration power

Radioactivity investigated

in late 19th, early 20th century

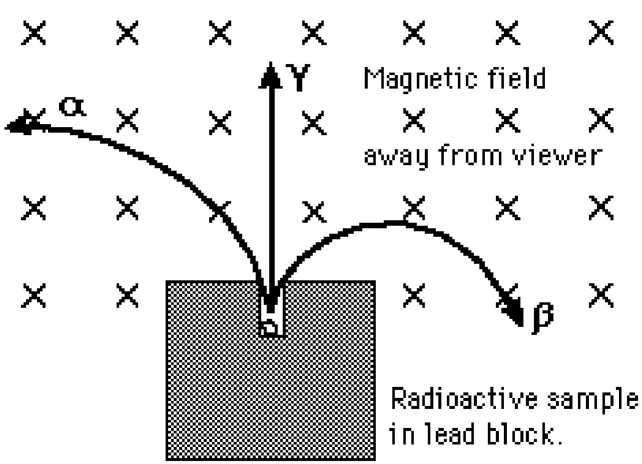
Electron discovered,

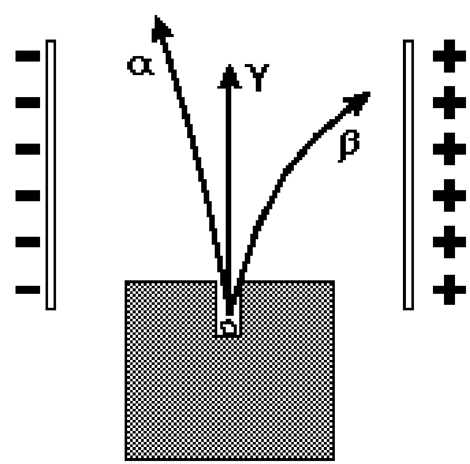
Bohr atom developed

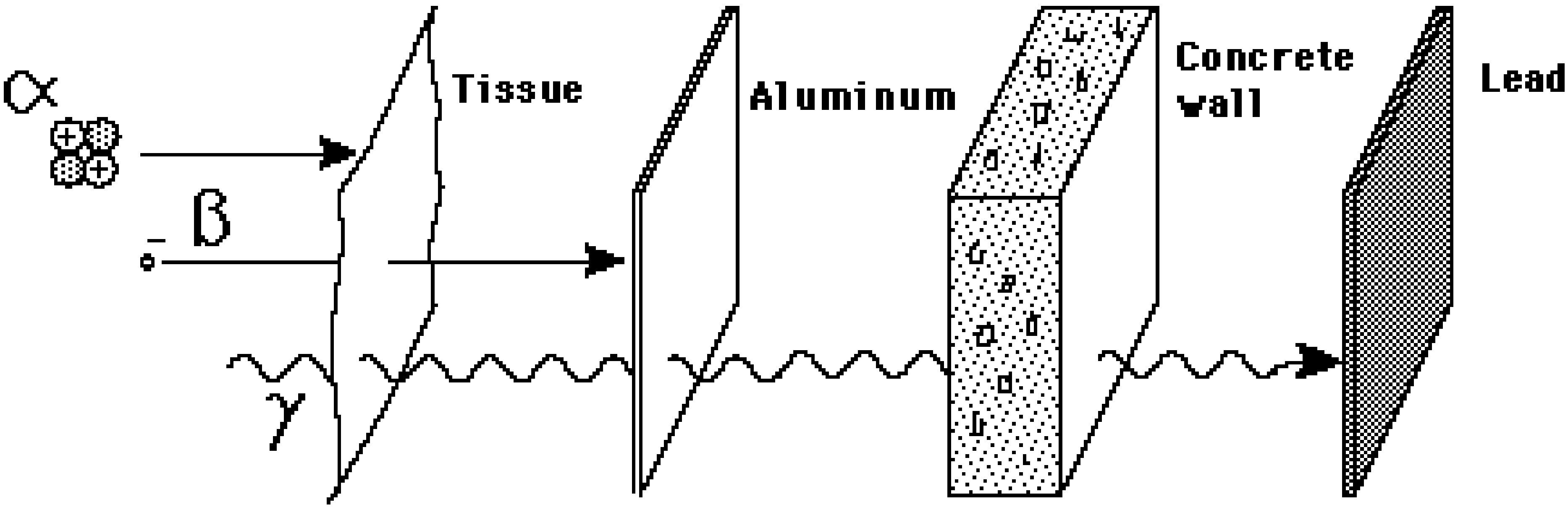
Radioactivity, subatomic particles, history, penetration power

Three types of radiation discovered,

labeled *alpha, beta, gamma* radiation (Rutherford)







Measurements show that:

1. The alpha particle is positive and heavy

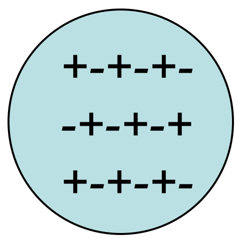
2. The beta particle is just an electron

3. The gamma "particle" is actually electromagnetic radiation in the MeV range

Early models

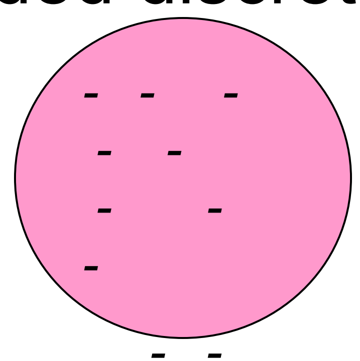
- Two early models of the atom:

1. Nucleus consisting of protons and electrons



nucleus

5. Plum pudding model, evenly distributed positive charge over whole atom with embedded discrete electrons



Atom (pink is positive charge)

The Neutron

- Both proton/electron nucleus and plum pudding were discarded with discovery in 1932 of the neutron (conjectured in 1920 by Rutherford)
- Neutron has slightly more mass than the proton
- Neutron is neutral, so not deflected by electric forces

- New model:
  - Nucleus consists of protons and neutrons
    (no electrons in nucleus)

Alpha particle

- Alpha particles are two protons and two neutrons
- This seems to be a *special, robust* combination
- The alpha particle is the nucleus of the Helium 4 isotope

Alpha particle

- Heavy nuclei can be thought of as being combinations of alpha particles plus zero, one, two, or three extra nucleons
- Alpha particle is a sort of “building block”

  A is atomic mass number:
- A=4n (n alphas)
- A=4n+1 (n alphas plus one nucleon)
- A=4n+2 (n alphas plus two nucleons)
- A=4n+3 (n alphas plus three nucleons)

Equivalence of mass & energy

- Einstein relation

$$E = mc^2$$

- m in kilograms, E in Joules
- Speed of light $c = 2.997925 \times 10^8$ meters per second

Einstein relation

Implications of $E = mc^2$

m in kilograms

$c = 2.997925 \times 10^8$ meters per second

E in Joules

A comparison:

KE of 2000 kg sport utility vehicle at 70 mph (31 m/s) is

$$K_{automobile} = \frac{1}{2}mv^2 \approx 1 \text{ MJ}$$

Einstein energy of 2.5 gram penny is

$$E = mc^2 = 2.2 \times 10^8 \text{ MJ}$$

$$1 \text{ MJ} = 10^6 \text{ J}$$

Rest mass energy
= 2x10^{8} Megajoules

Kinetic energy
= 1 Megajoule at 70 MPH

Mass Measurements

- Carbon 12 assigned mass of exactly 12 u
- This defines the atomic mass unit, u

$$1\,\text{u} = 1.66043 \times 10^{-27}\,\text{kg}$$

Energy units

- The Joule is too large a unit for use with individual particles
- Instead use "electron-volt" which is the energy change of an electron moving through a potential difference of 1 volt
- Charge on an electron is

$$e = 1.60210 \times 10^{-19}\ \text{Coulumb}$$

- Potential energy of charged particle is *qV*
- Change in electron energy when it falls through potential difference of 1 volt is

$$1\,\text{eV} = 1.60210 \times 10^{-19}\ \text{Joule}$$

Rest mass energy associated with 1 u

$E = mc^2$
$= 1.4923 \times 10^{-10}$ Joules
$= 931.46$ MeV

where 1 MeV $= 10^6$ eV

Rest mass energy associated with 1 u

$$E = mc^2$$
$$= 1.4923 \times 10^{-10} \text{ Joules}$$
$$= 931.46 \text{ MeV}$$

where 1 MeV $= 10^6$ eV

Rest mass and energy of a hydrogen atom and its ingredients

	mass (u)	energy (MeV)
electron	0.000549	0.5110
proton	1.007276	938.26
electron proton	1.007825	938.77
hydrogen atom	1.007825	938.77

So a hydrogen item weighs the sum of the an electron and an ion

Compare neutrons

	mass (u)	energy (MeV)
electron	0.000549	0.51101
proton	1.007276	938.26
neutron	1.008665	939.55
hydrogen atom	1.007825	938.77

Note:

Neutron weighs more than a hydrogen atom

A proposed way to construct the nuclides (elements)

- Z denotes the atomic number (number of protons, also the number of electrons)
- A denotes the number of mass units
- N denotes the number of neutrons

- An atom with Z electrons, Z protons, and N neutrons could be assembled from Z hydrogen atoms and N neutrons

Proposed formula

A is the number of nucleons (protons or neutrons)

$$A = Z + N$$

If an atom with Z protons, Z electrons, and N neutrons has a mass which is simply the sum of the mass of its ingredients, then expect

$$^{A}_{Z}m = Zm_{H} + Nm_{N}$$

Actual mass is less!

- Actual mass of a nuclide is less than sum of ingredients
- This indicates that assembly liberates energy, call this the binding energy *B*
- This energy would have to be added back to separate nucleus into H, n

$$m(A,Z)c^2 + B = Zm_H c^2 + Nm_N c^2$$

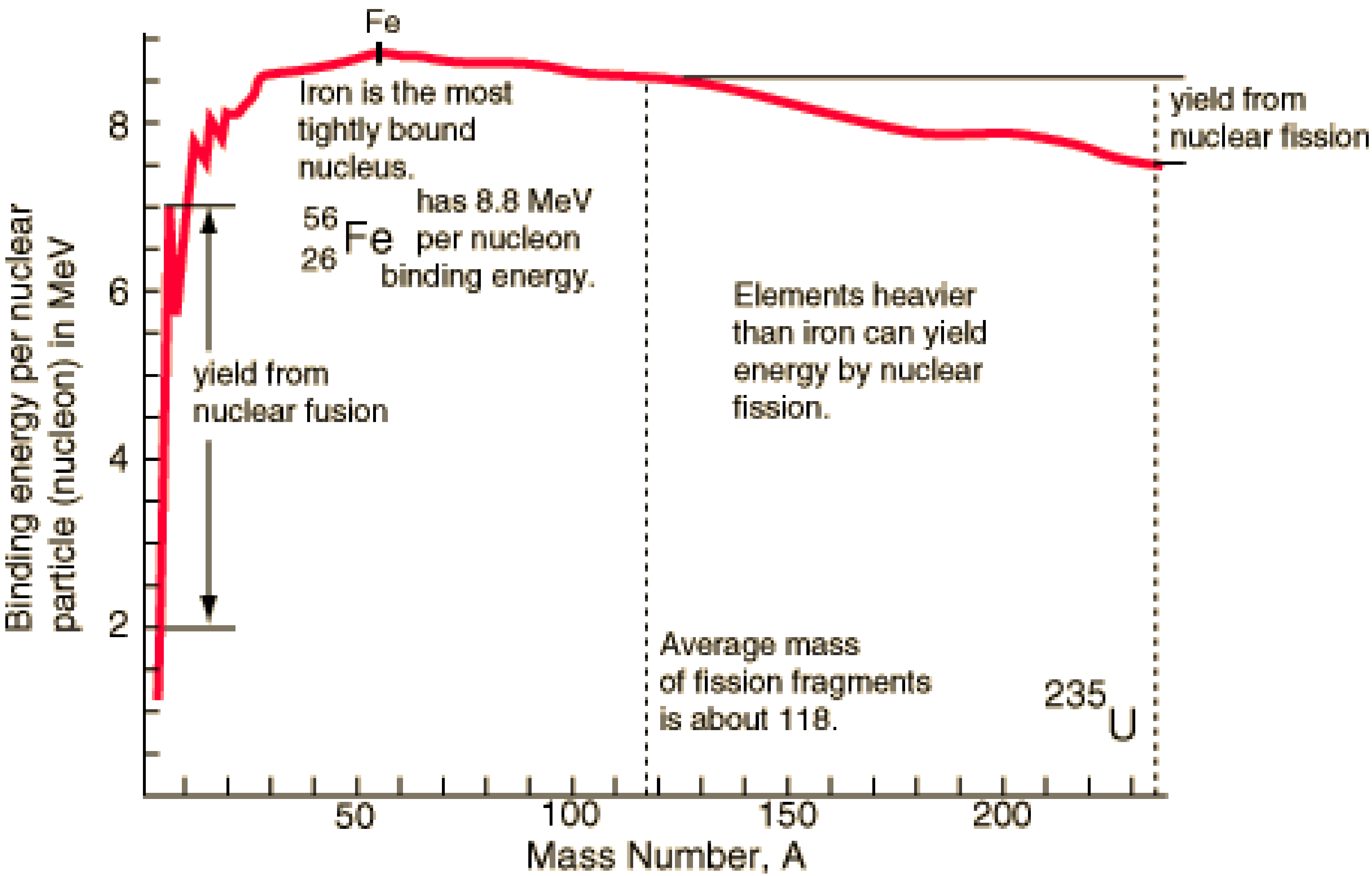
$$m(A,Z) = Zm_H + Nm_N - B/c^2$$

$$B = (Zm_H + Nm_N - m(A,Z))c^2$$

Binding energy/nucleon

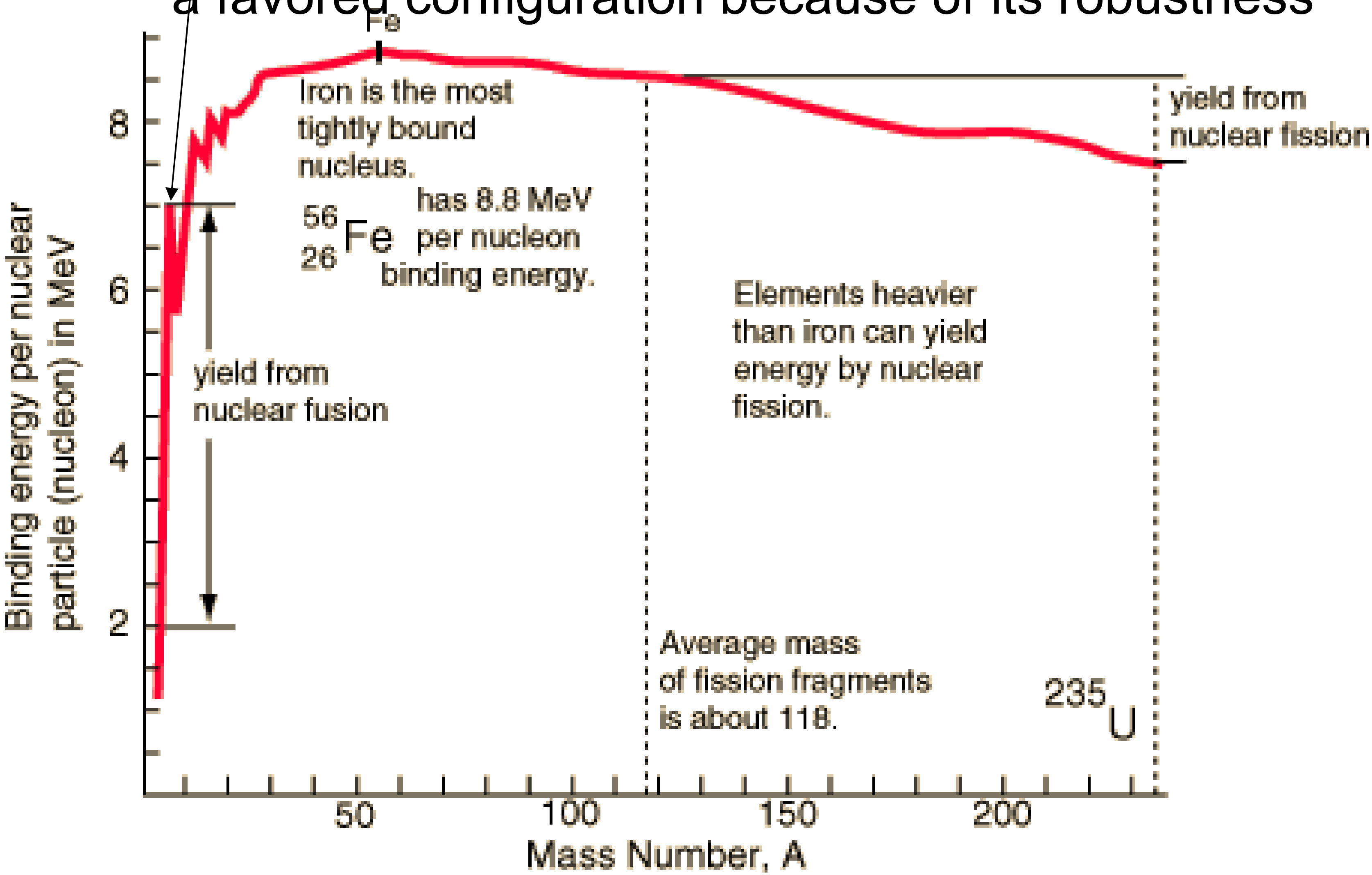
$$B/A = \frac{\left[Z m_H + N m_N - m(A,Z)\right]c^2}{A}$$

Larger B/A means that less energy/nucleon
is stored in nuclide
Plot this versus A

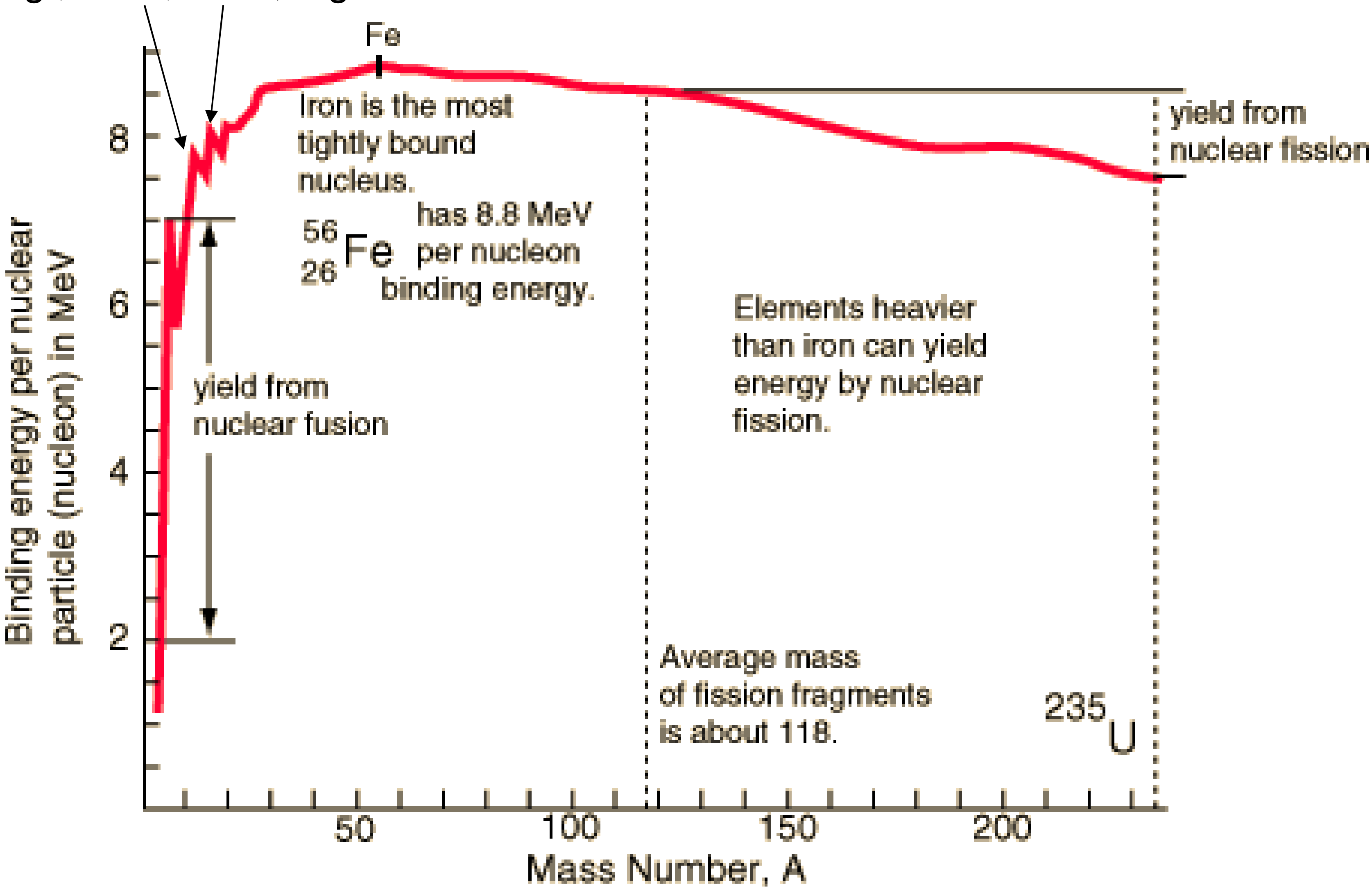


Fe
Iron is the most tightly bound nucleus.
56 26 Fe has 8.8 MeV per nucleon binding energy.
yield from nuclear fission
Elements heavier than iron can yield energy by nuclear fission.
yield from nuclear fusion
Average mass of fission fragments is about 118.
235 U
Binding energy per nuclear particle (nucleon) in MeV
8
6
4
2
50
100
150
200
Mass Number, A

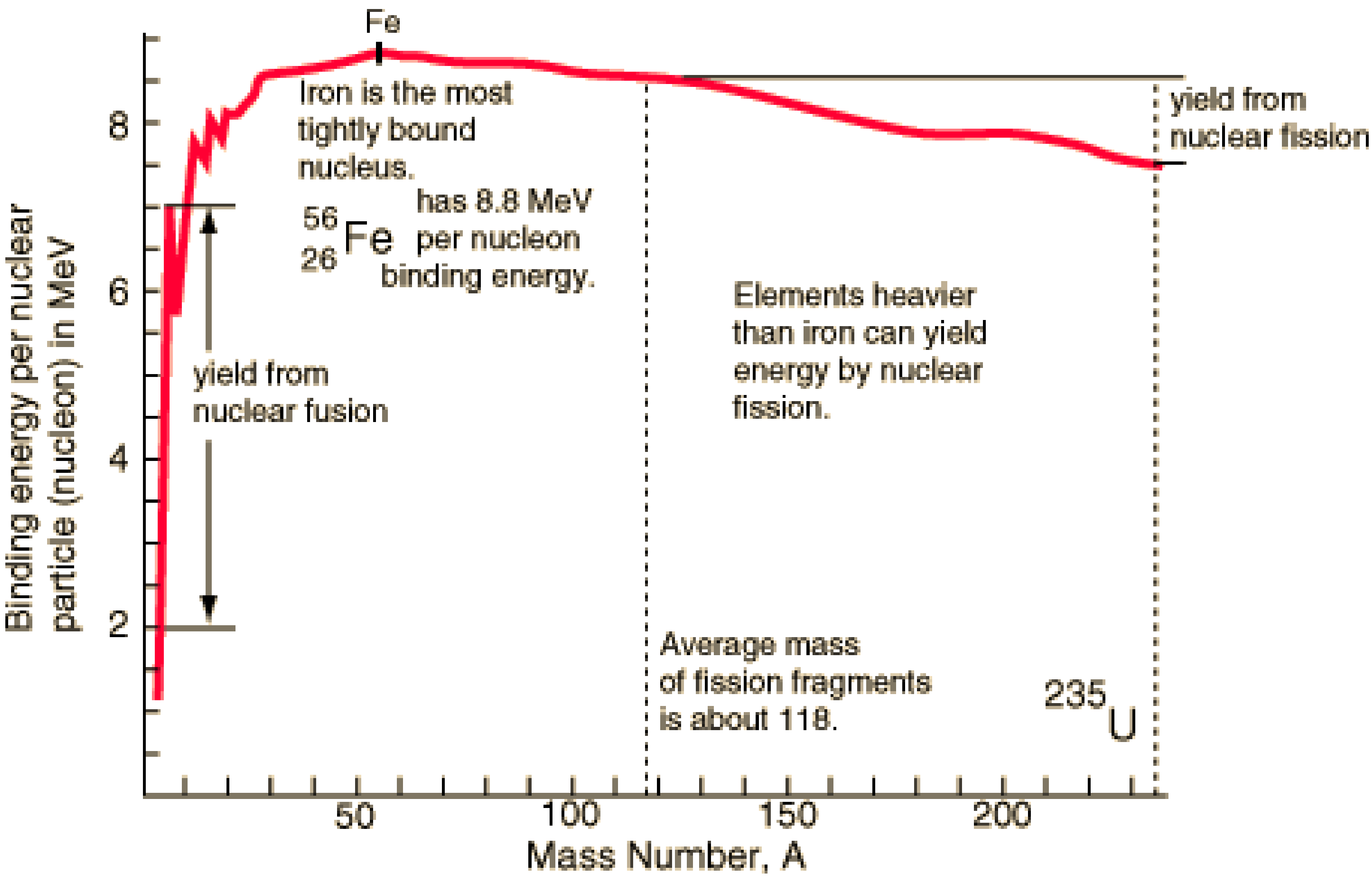
Helium (mass 4) is especially stable (local peak in binding energy); He-4 ion is an alpha particle, and is a favored configuration because of its robustness



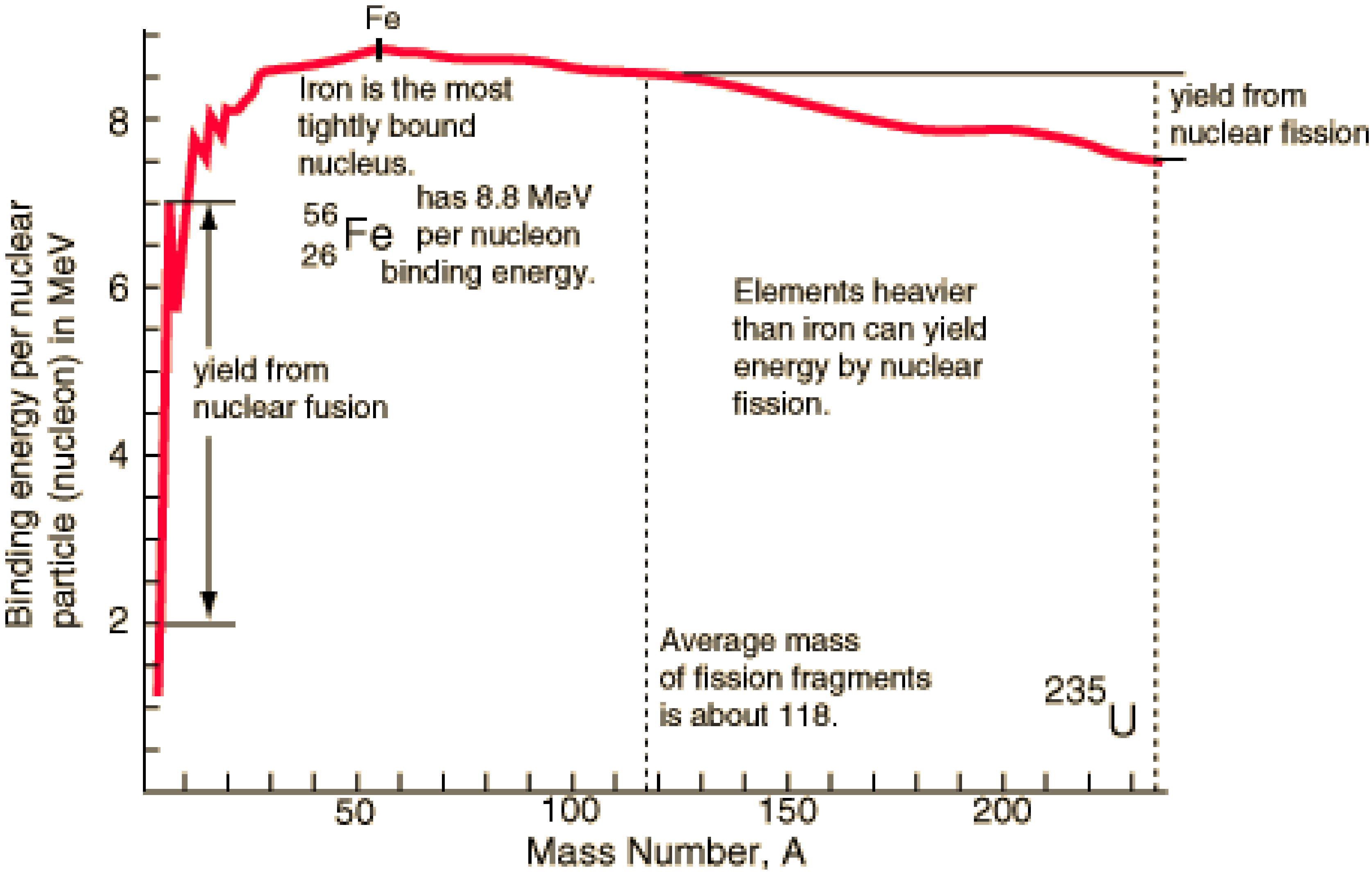
Multiples of mass 4 are also locally stable (if they exist), e.g., C-12, O-16, Mg-24



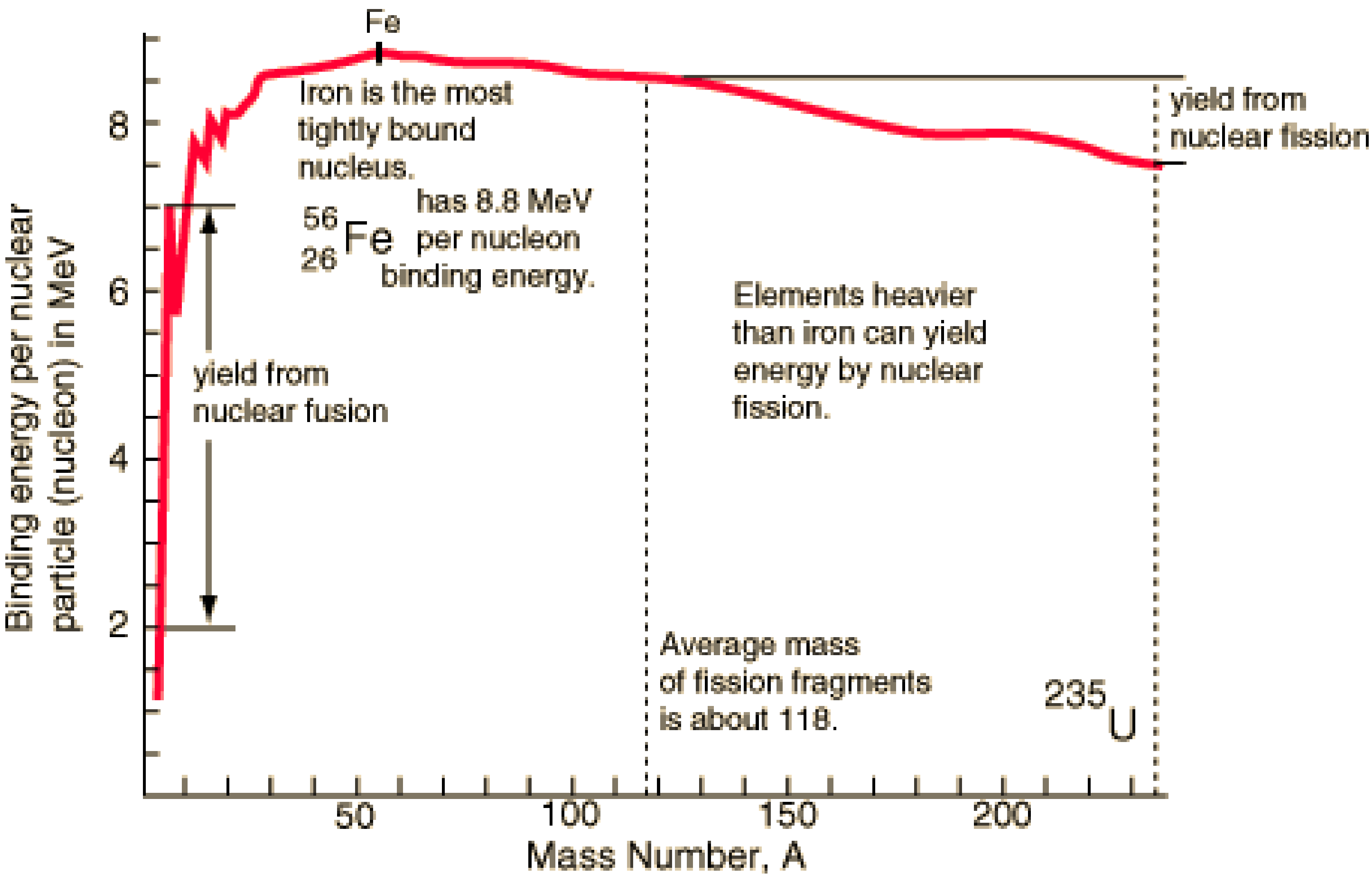
Interestingly, there are no stable isotopes with mass 5 or mass 8



Fission: If a nuclide has mass greater than 112 than it can be split into two daughters, each with mass exceeding Iron-56. Since *both* daughters have higher binding energy than the parent, energy is released.



Fusion: If two light nuclei combine, the product has higher binding energy than the Ingredients and so energy is released



Radioactivity measurement

- Nuclides (isotopes) are either stable or unstable
- Unstable isotopes decay, rate is characterized by a half-life

$$\frac{dN}{dt} = -\lambda N$$

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$t_{1/2} = \lambda^{-1} \ln 2$$

Activity λN is defined as disintegrations per second (Bq for Becquerel), and so half life is inversely proportional to activity

1 Curie $= 3.7 \times 10^{10}$ Bq is the activity of 1 gram of radium

Radioactivity

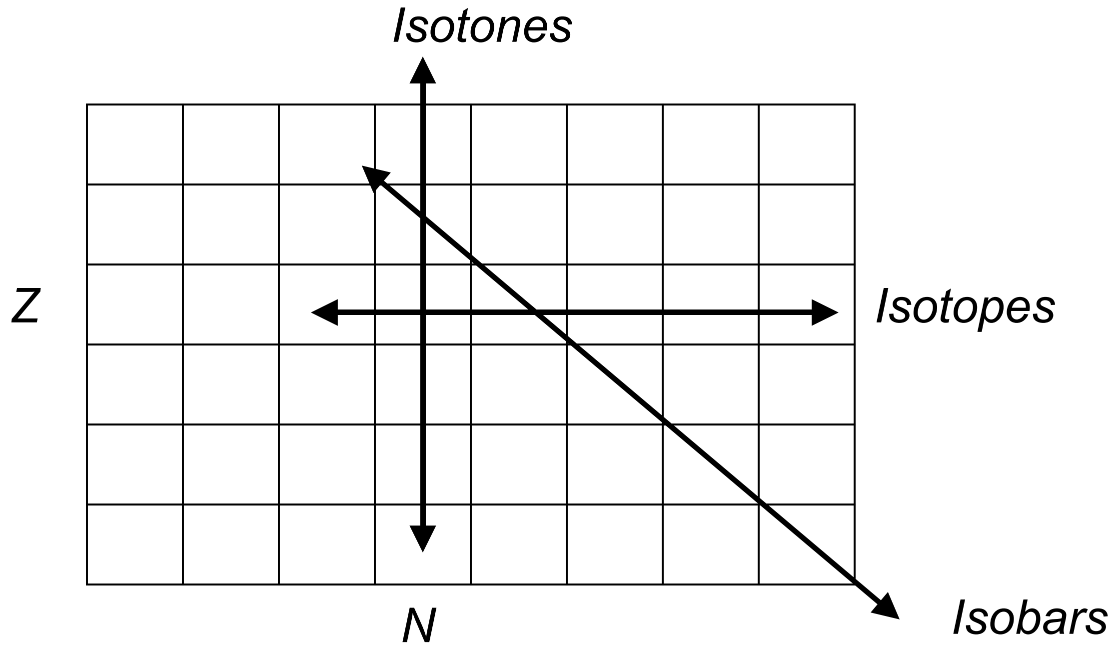
- Many processes:
  - Emission/absorption of an alpha particle
    - Z decreases/increases by 2,
    - N decreases/increases by 2
    - A decreases/increases by 4
  - Emission/absorption of beta- (electron)
    - Z increases/decreases by 1
    - A unchanged
  - Emission/absorption of beta+ (positron)
    - Z decreases/increases by 1
    - A unchanged
  - Emission/absorption of a neutron
    - N decreases/increases by 1
    - A decreases/increases by 1
    - Z unchanged
  - Gamma radiation is emission of a photon, corresponds to change in de-excitation of a nucleus from an excited state (analog to emission of a photon by dexcitation of an atom from an excited state)

Nuclide chart

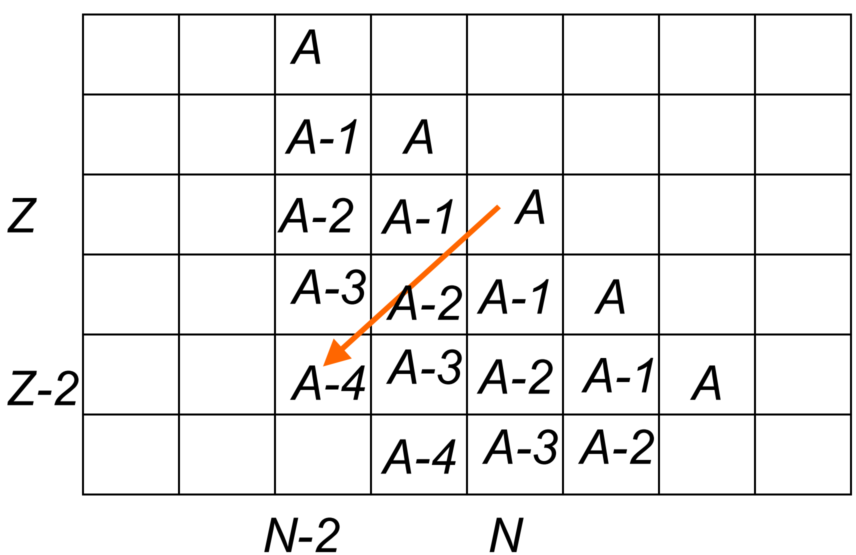
	Ca	**20**								
	K	**19**								
	Ar	**18**								
	Cl	**17**								
	S	**16**								
	P	**15**								
	Si	**14**								
	Al	**13**								
	Mg	**12**								
Z,	Na	**11**								
no of	Ne	**10**								
protons	F	**9**								
	O	**8**								
	N	**7**								
	C	**6**							12	13
	B	**5**						10	11	
	Be	**4**					8	9		
	Li	**3**			5	6	7			
	He	**2**		3	4					
	H	**1**	1	2	3					
	Chemical		**0**	**1**	**2**	**3**	**4**	**5**	6	7
	Symbol						N, number of neutrons			

Numbers show atomic mass of common isotopes

- Isotopes: same Z (same chemical behavior)
- Isotones: same N
- Isobars: same A (Z+N=A=constant, Z=A-N)



- Emission of an alpha particle
  - Z decreases by 2,
  - N decreases by 2
  - A decreases by 4



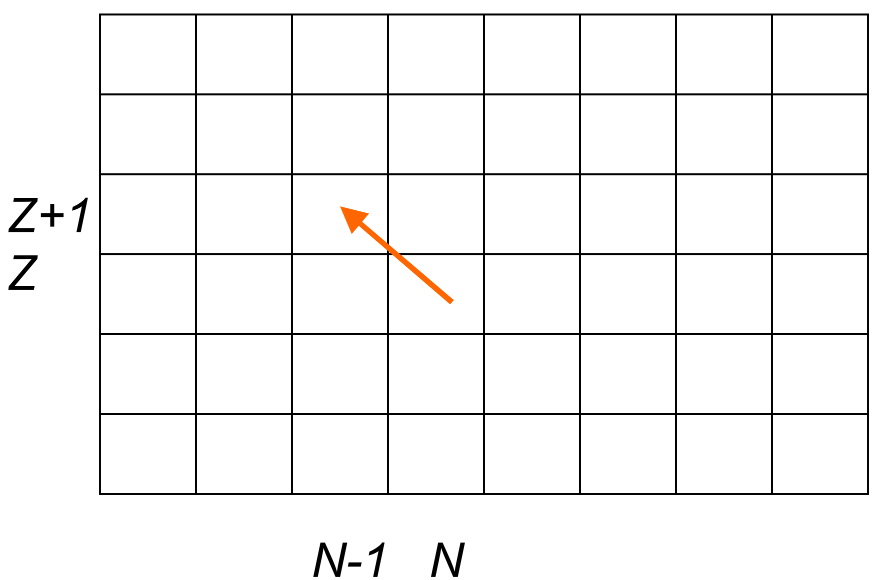
Beta decay and neutrinos

- Emitted beta particles (electrons/positrons) have range of energies which implies that some energy is going into another particle
- Missing energy goes into neutrino
  - Hard to detect since mass essentially zero
- Beta decay process involves “weak force”
- Nuclear force holding protons neutrons together is called “strong force”

$$n \rightarrow p + e^{-} + \bar{\nu}$$

electron　　anti-neutrino

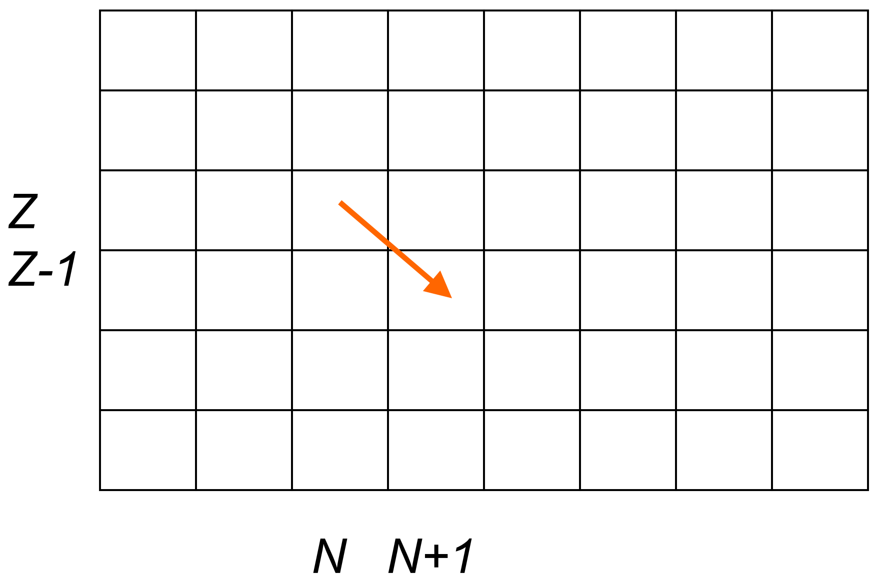
- Emission of beta- (electron)
  - Conversion of a neutron into a proton
  - Z increases by 1, N decreases by 1
  - A unchanged
    - So must stay on isobaric line



p ▯ n . ▯e . ▯ ▯

positron neutrino

– Emission of beta+ (positron)
  - Conversion of a proton into a neutron
  - N increases by 1, Z decreases by 1
  - A unchanged
    – So must stay on isobaric line



– Emission of a neutron

- N decreases by 1
- Z unchanged
- A decreases by 1

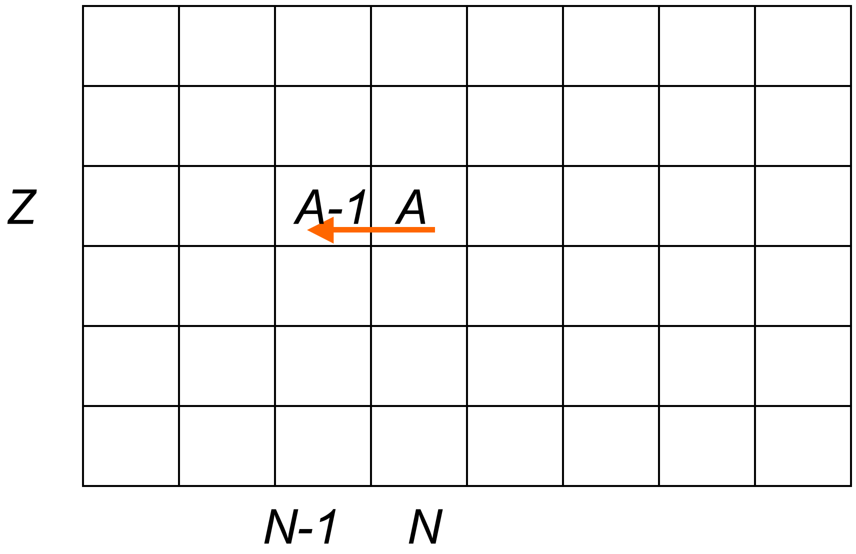
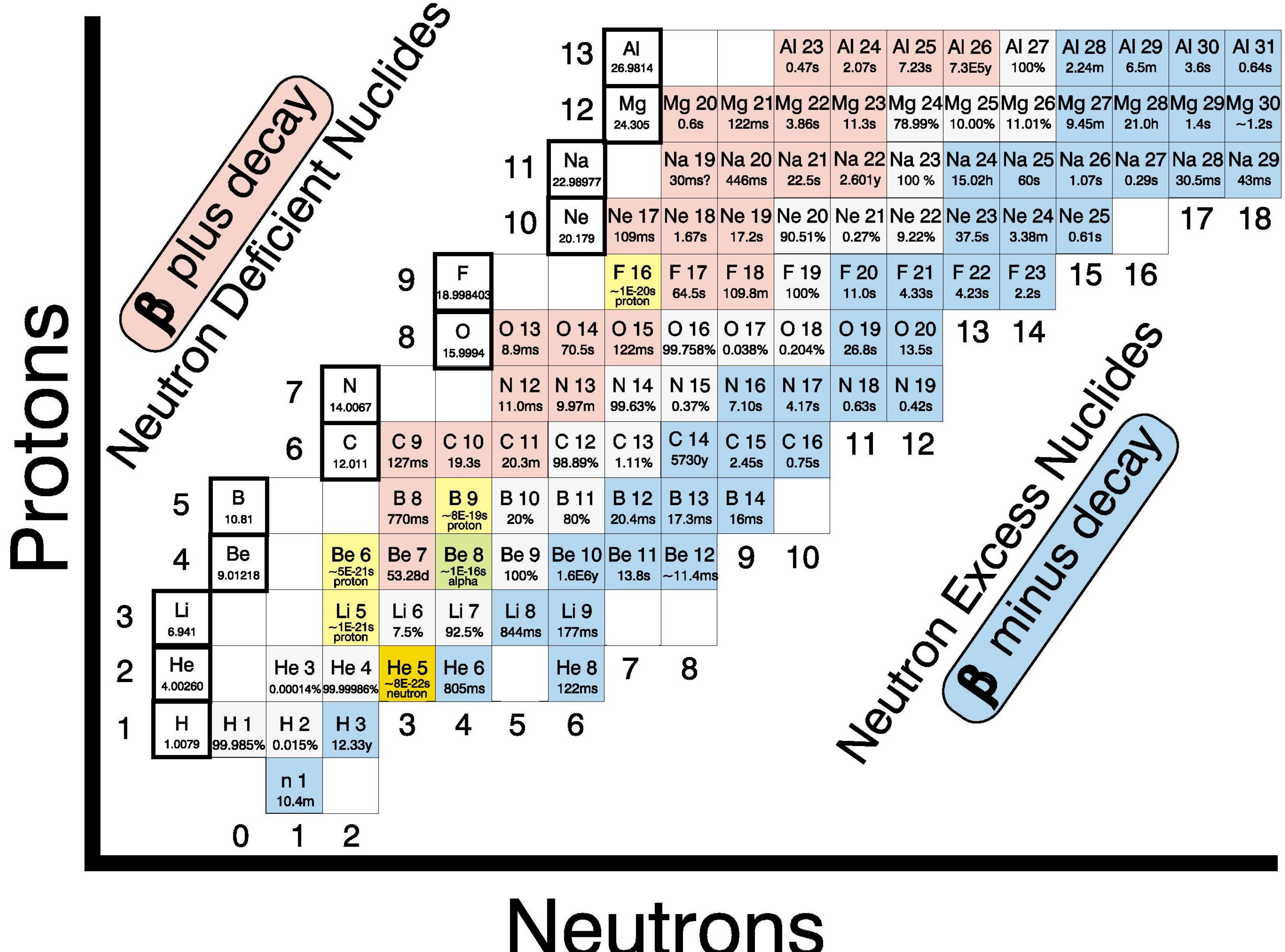


Chart of the Nuclides

- Over 2300 known nuclides
- Over 50,000 reactions known
- Charted in Segre chart (Chart of Nuclides)
- Available at http://atom.kaeri.re.kr

CHART OF THE NUCLIDES



Radioactive series

- Since alpha particles are robust, typically get emission of alpha particles rather than parts of alpha particles
- A=4n decays to A=4(n-1)
- A=4n+1 decays to A=4(n-1)+1, etc.
- There are thus four distinct radiactive decay series

Radioactivity - ^{238}U radioactive decay series

The Decay Path of 4*n* + 2 or ^{238}U Family

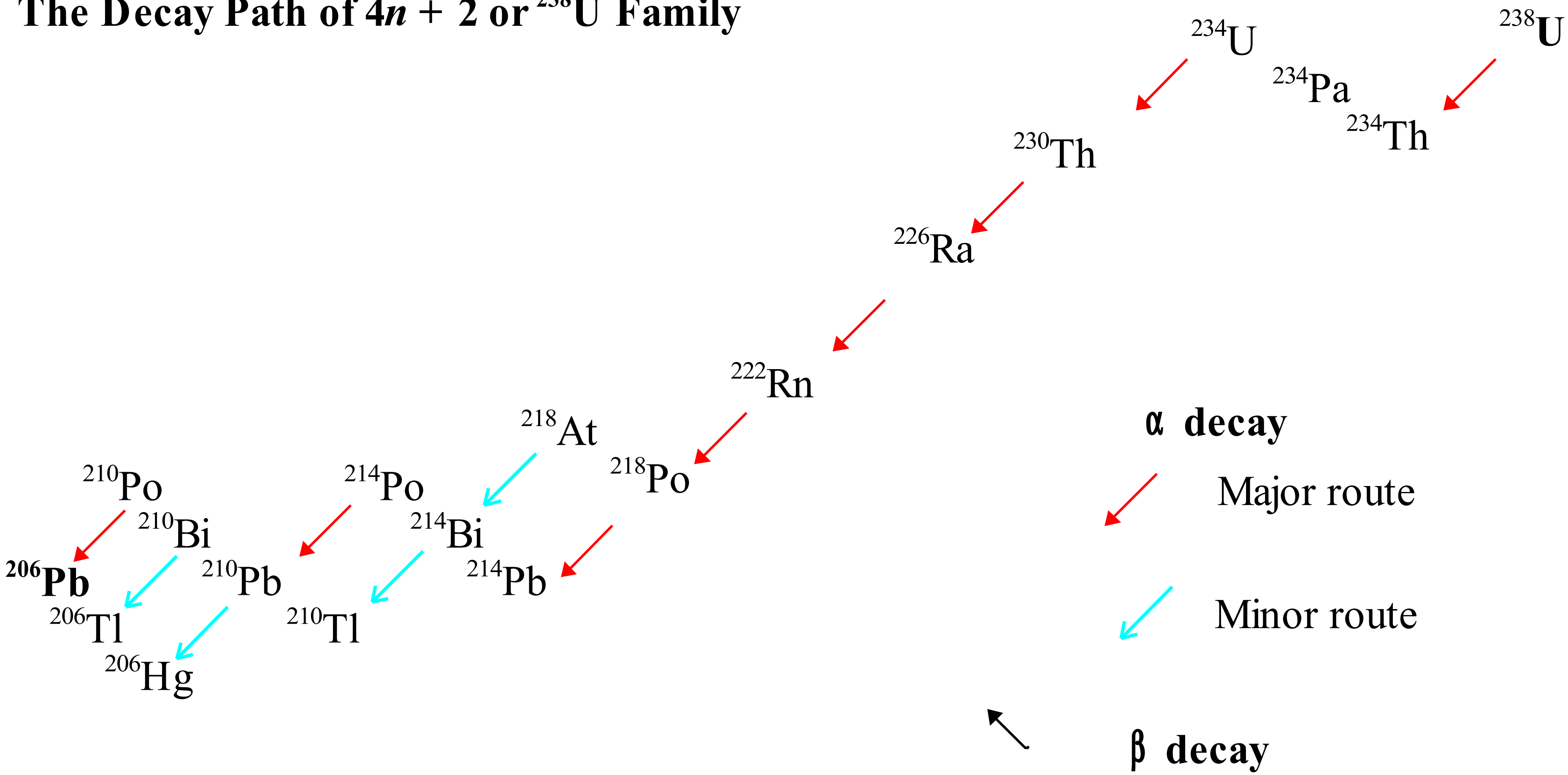


Chart from University of Waterloo website

Natural radioactive decay series (from Bodansky)

Type	Name	Parent	Parent $\tau_{1/2}$ 10^9 years	stable end nucleus	α decays	β^- decays
$A = 4n$	Thorium	^{232}Th	14.05	^{208}Pb	6	4
$A = 4n + 2$	Uranium	^{238}U	4.47	^{206}Pb	8	6
$A = 4n + 3$	Actinium	^{235}U	0.704	^{207}Pb	7	4

There is no natural $A = 4n + 1$ series;

the longest lived isotope is ^{237}Np which has half-life of 2.1×10^6 years

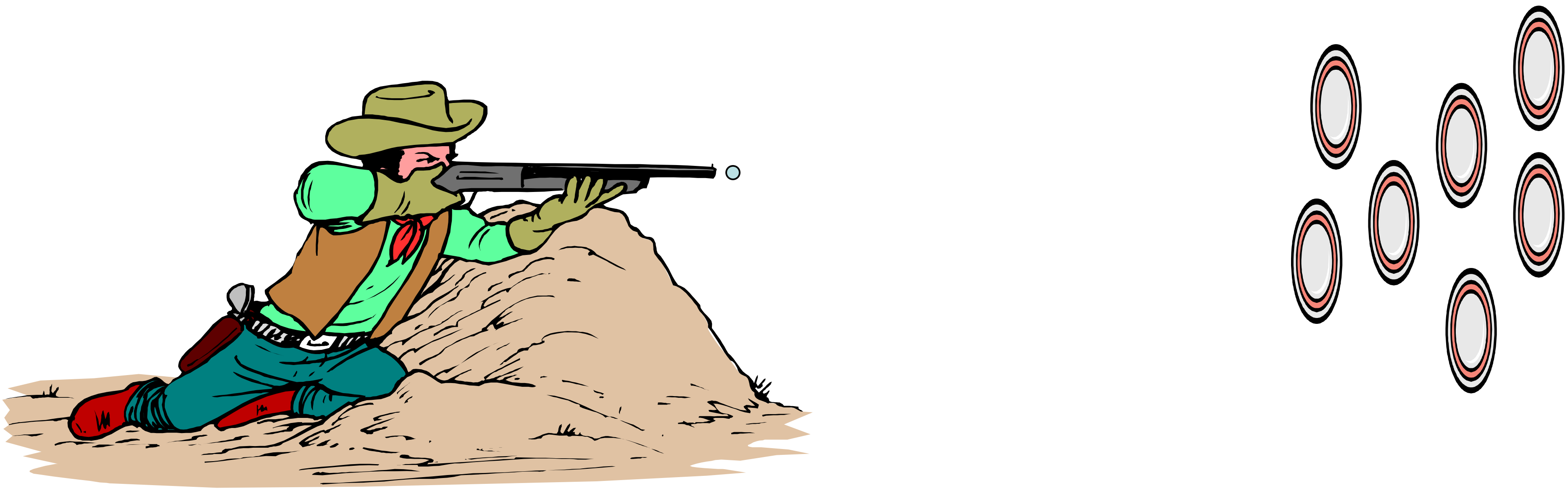
Energy of nuclear reactions

- Typical nuclear energy reactions are MeV
- Compare to typical chemical reactions which are in eV

- Certain reactions are most feasible for energy production
- Reactions are characterized by their cross-section

Cross-section

- Suppose you shoot R_0 pellets per second at a large number of dinner plates arrayed in front of you



R_0 pellets per second

Cross-section

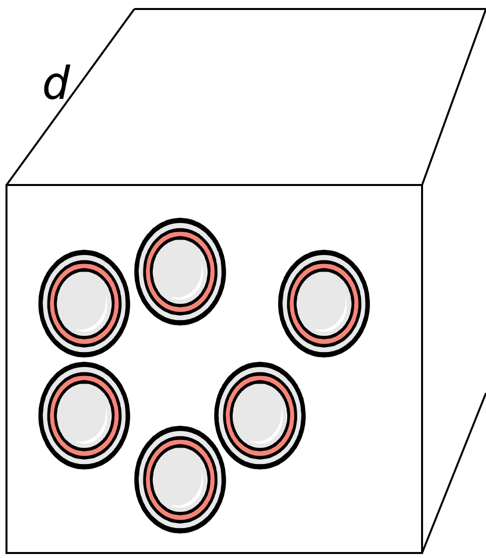
- Shoot R_0 pellets per second
- The rate per second R at which pellets hit plates is

$$\frac{R}{R_0} = \text{fractional area subtended by plates}$$

- If the density of plates (targets) is small, then plates do not block each other from view and so the area of the plates is all the plates you can see
- Suppose n is the density of plates per cubic meter and sigma is the area of each plate

Cross-section, cont'd

- Consider a target region that has area A and depth d
- The number of plates is N=nAd
- Let σ be the area of one plate
- Total area of all the plates = nAdσ
- Fractional area of all plates = ndσ



Front area=A

$$\frac{R}{R_0} = n\sigma d$$

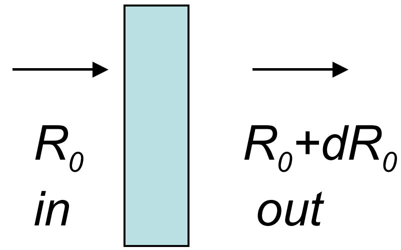
Cross-section

- This formula can also be used to find the cross-sectional area of targets

$$\sigma = \frac{R}{R_0}\frac{1}{nd}$$

Beam attenuation length

dx =width of collision region



$R=R_0 n\sigma\, dx$
=amount lost due to collisions

so $dR_0= - R_0 n\sigma\, dx$

$$\frac{dR_0}{R_0} = -n\sigma\, dx$$

$$d\ln R_0 = -n\sigma\, dx$$

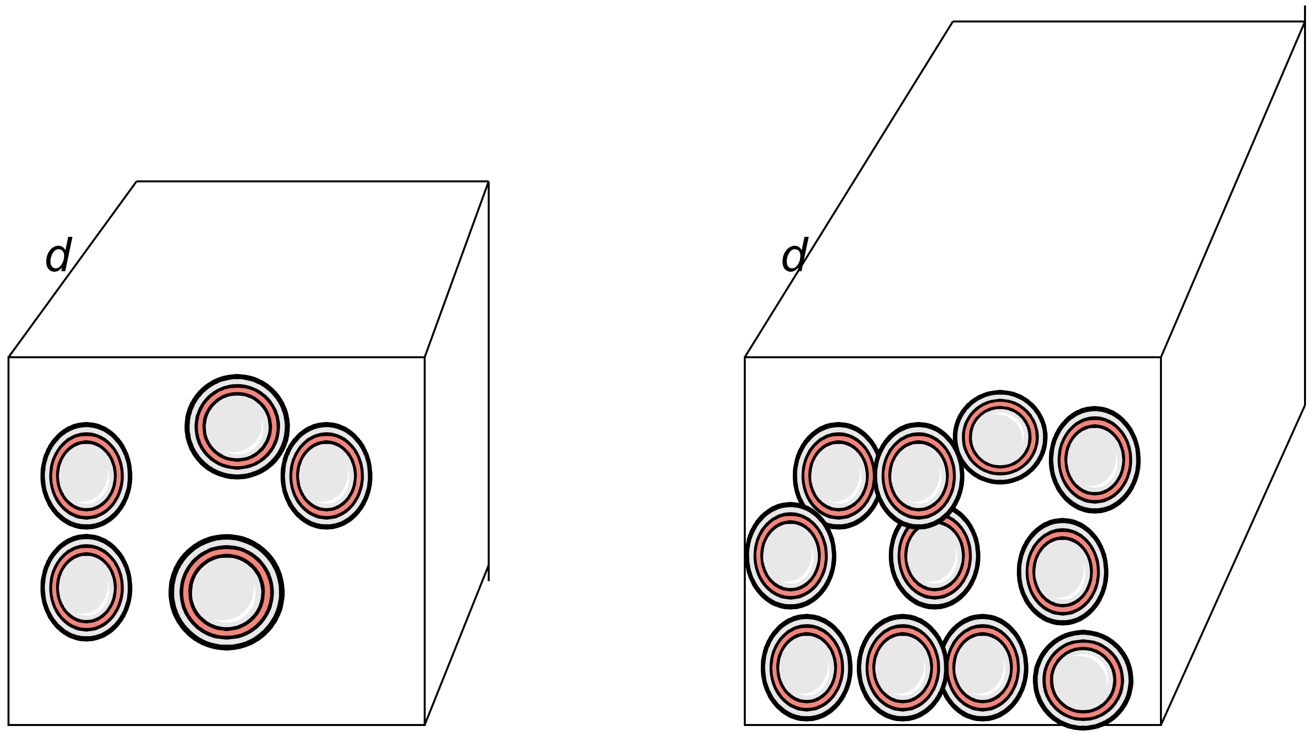
$$R_0(x) = R_0(0)\exp(-n\sigma x)$$

$$= R_0(0)\exp(-x/L)$$

where $L = 1/n\sigma$ is length for beam intensity to go down by factor of e

Mean free path: How far does a pellet have to go to have a 100% probability of hitting a plate?

- As *d,* the target depth, is increased, the total area subtended by the plates increases
- When the total area subtended by the plates equals the box frontal area, then a pellet cannot traverse box without hitting a plate
- This occurs when $N\sigma=A$ or $nAd\sigma=A$
  - i.e. when $d=1/n\sigma$
- So $L=1/n\sigma$ is also the mean free path, the nominal distance a typical pellet travels until hitting a target



Front area=A

Front area=A

When projected area equals actual area then get 100% probability of pellet hitting a plate