
Chemistry 24b **Lecture 21&22**
Spring Quarter 2004
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Two Dimensional NMR

The experiments and spectra so far described are one dimensional in the sense that the FID obtained is a function of a single time variable. It is possible to introduce a second time variable by including an incrementable delay in the pulse sequence. This incrementable delay is typically called t_1; the time period during which the FID is acquired is then known as t_2. We collect a series of FIDs corresponding to different values of t_1. This set of FIDs is a function of both time variables: $S(t_1,t_2)$. Fourier Transformation of these FIDS wrt t_2 and transposition of the data matrix yields a set of interferograms $G(t_1,\omega_2)$. Finally, FT wrt t_1 yields a 2D NMR spectrum $F(\omega_1,\omega_2)$. 2D spectra are typically displayed as contour plots.

The above description can be generalized to any number of dimensions by introducing additional incrementable delays in the pulse sequence. Practically, dimensionality greater than 4 is not useful. 3D and 4D experiments have become common since about 1990.

Homonuclear 2D-J-Resolved Spectroscopy

Good introduction to nD NMR, but no longer commonly used.

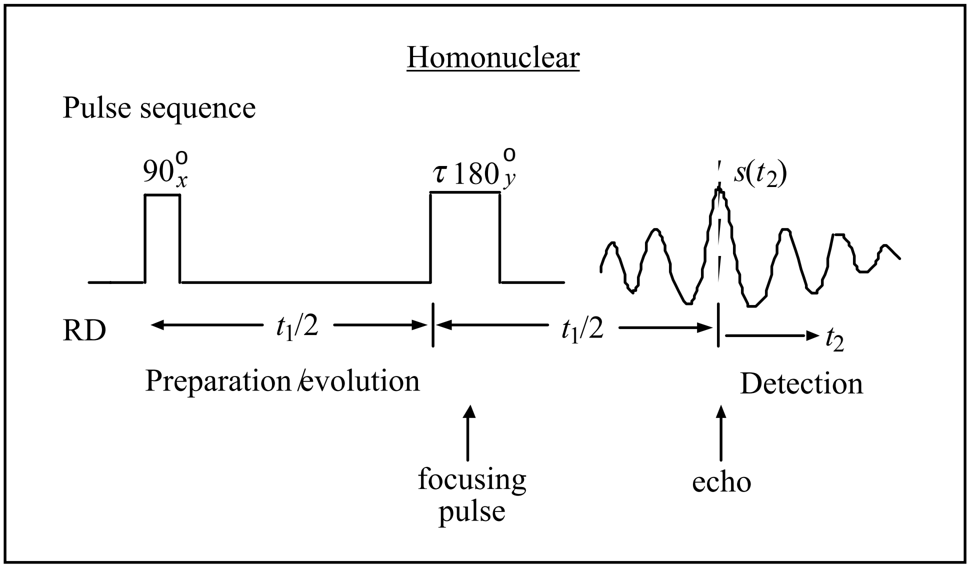
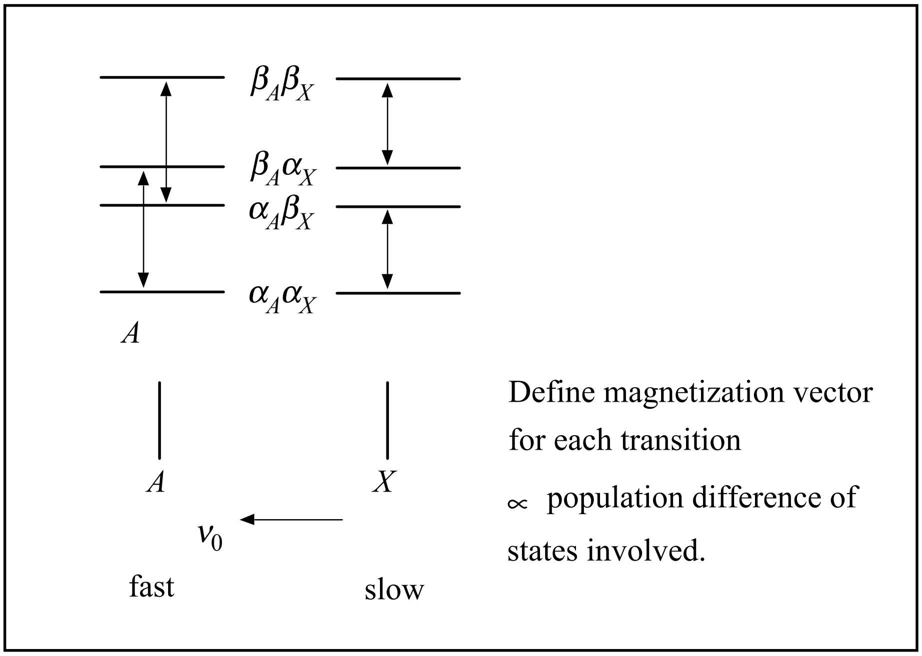


Figure 11-1

Uncoupled Spins

Consider the effect of this pulse sequence on two spins A and X. Initially, assume that they are not J-coupled so we can see the effect of the pulse sequence on chemical shifts.



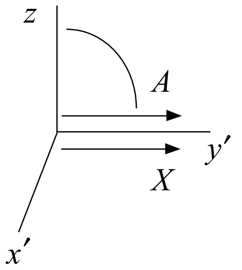
Define magnetization vector for each transition

$\propto$ population difference of states involved.

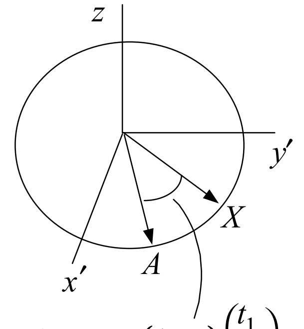
Figure 11-2

The effect of the $\left(\frac{\pi}{2}\right)_{x'}$ pulse is to bring both isochromats along the y' axis of the rotating frame.

Immediately after the $\left(\frac{\pi}{2}\right)$ pulse, A and X are in phase.

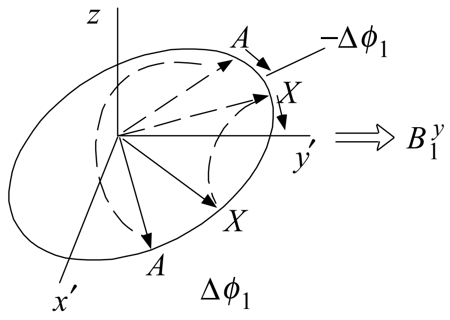


However, with time, the magnetization in the xy plane will evolve. Consider time $\frac{t_1}{2}$. Because of different precession frequencies of A vs X, there will be a phase difference between them.



$$\Delta\phi_1 = 2\pi\left(\Delta\nu_0\right)\left(\frac{t_1}{2}\right)$$

If at this time a π–pulse is applied along y' (by changing the transmitter phase), then



and $\Delta\phi_1(t = t_1) \;=\; -\Delta\phi_1 + \Delta\phi_2 \;=\; -\Delta\phi_1 + 2\pi(\Delta\nu_0)\left(\frac{t_1}{2}\right)$

$$= 0$$

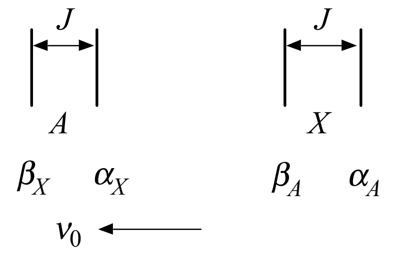
So at the end of the evolution period, i.e., $t = t_1$, the two magnetization vectors are refocused: the signal at time t_1 is exactly the same as after the first $\left(\frac{\pi}{2}\right)$ pulse (except for homogeneous effects. . . T_2 relaxation during the time period t_1). Any dispersion due to magnetic field inhomogeneity is refocused as well.

Result: chemical shifts are refocused by the pulse sequence <u>for any t_1</u>. . . *i.e.* chemical shifts are independent of t_1 and will not show up in the spectrum in ω_1.

Coupled Spins

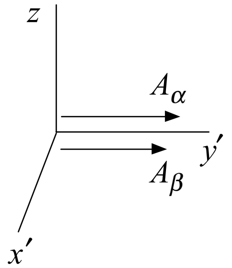
Now we want to look at how the J-coupling term of the Hamiltonian behaves under the pulse sequence. Assume A and X are now coupled, with coupling constant J:

$$J\check{I}_A \cdot \check{I}_X \qquad (A, X \text{ because } \delta >> J)$$

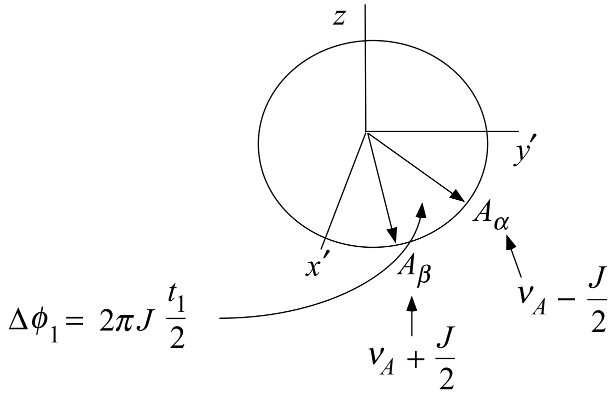


Consider the effect of the pulse sequence on A spins. Label the magnetization vectors A_α and A_β corresponding to whether the coupling partner (X spin) is α or β. Then

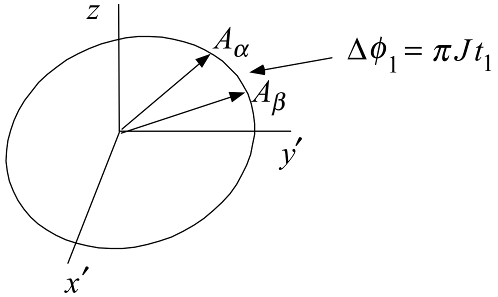
After $\left(\frac{\pi}{2}\right)_{x'}$ pulse



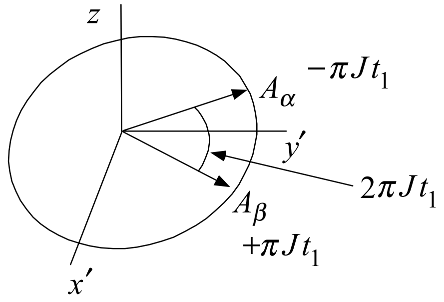
After evolution for time $\frac{t_1}{2}$



After $(\pi)_{y'}$ pulse



$(\pi)_{y'}$ pulse rotates the magnetization components A_α, A_β to mirror image position wrt the y' axis. In this case, the $(\pi)_{y'}$ pulse has an additional effect: the state of the coupled X is inverted, so the precession frequencies of the two A magnetization vectors are swapped.



Result: At $t=t_1$, the components of A magnetization are separated by $2\pi J t_1$. Thus, J-coupling is not refocused by spin echo. Instead, it causes the components of the A magnetization to oscillate as a function of t_1 at frequencies $-\pi J$ and $+\pi J$.

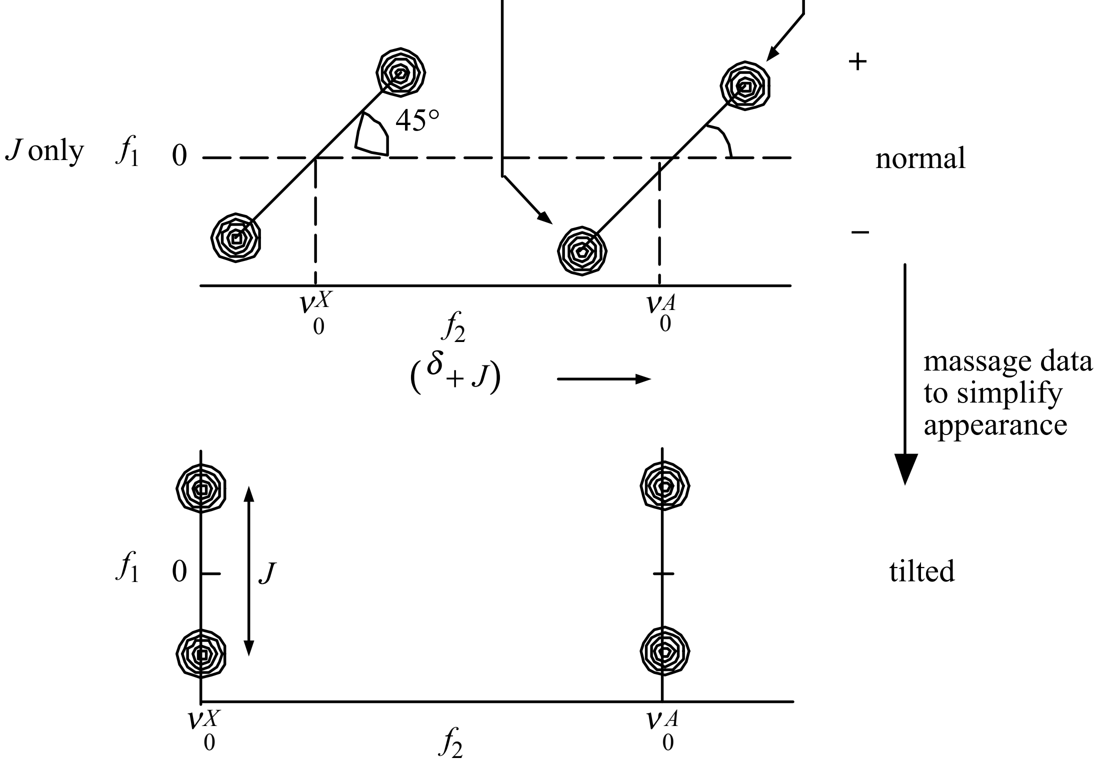
The time-domain NMR signal will be of the following form:

$$s(t_1,t_2) = \frac{M_0^A}{2}\left[e^{-t_1/T_2} e^{i\pi J t_1} e^{-t_2/T_2} e^{-i\left(\omega_0^A-\pi J\right)t_2} + e^{-t_1/T_2} e^{-i\pi J t_1} e^{-t_2/T_2} e^{-i\left(\omega_0^A+\pi J\right)t_2}\right]$$

In t_2, we detect both chemical shift and J-coupling ($\omega_0^A \pm \pi J$ terms above), while only the J-coupling shows up in t_1 terms.

$$F(\omega_1,\omega_2) = \int s(t_1,t_2) e^{-i\omega_1 t_1} e^{-i\omega_2 t_2}\, dt_1\, dt_2$$

Predict resonances at	ω_1	ω_2
	$+\pi J$	$w_0^A + \pi J$
	$-\pi J$	$w_0^A - \pi J$
or	f_1	f_2
	$+\frac{J}{2}$	$\nu_0^A + \frac{J}{2}$
	$-\frac{J}{2}$	$\nu_0^A - \frac{J}{2}$



spin-spin collapsed along f_2 axis, but oriented along f_1 axis

Figure 11-3

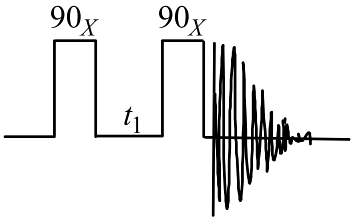
Correlation Experiments

The 2D-J-resolved experiment is an exception, in that it correlates one property of a particular spin with another property of the same spin. Most nD experiments are correlation experiments, correlating chemical shifts of pairs of spins which are related by *e.g.* J-coupling, dipole-dipole interaction, or chemical exchange. They may be homonuclear (all H's) or heteronuclear (^{13}C—^{1}H or ^{15}N—^{1}H).

Homonuclear:

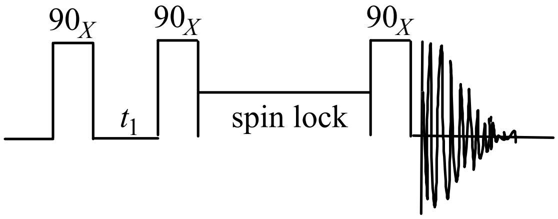
Correlation via J-coupling	Correlation via Cross-relaxation
COSY	NOESY
TOCSY	

COSY



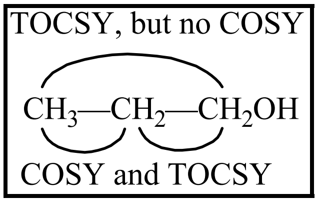
COSY gives crosspeaks between spins separated by two or three bounds only (vicinal, geminal protons).

TOCSY



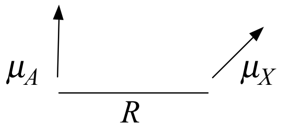
TOCSY allows mixing of all spins or a spin system. For example, you will see a TOCSY crosspeak between A and X in an AMX spin system where J_{AM}, $J_{MX} \neq 0$, $J_{AX} = 0$.

An example:



Dipole-Dipole Interactions

Two magnetic dipoles will interact through space (contrast with the J-coupling interaction, which occurs only through the intervening electron density of bonds). Detection of this interaction is evidence for spatial proximity of the spins (measureable for distances ≤ 5 Å)



Let's call *A* spin *J* and *X* spin *S*. . . . conventional nomenclature for what follows.

$$E_{dd} = \frac{\check{\mu}_A \cdot \check{\mu}_X}{R^3} - \frac{3(\check{\mu}_A \cdot \check{R})(\check{R} \cdot \check{\mu}_X)}{R^5}$$

$$= \gamma^2 \hbar^2 \left[\frac{\check{I}_A \cdot \check{I}_X}{R^3} - \frac{3(\check{I}_A \cdot \check{R})(\check{R} \cdot \check{I}_X)}{R^5} \right] \quad (\text{for } \gamma_A = \gamma_X)$$

$$= \gamma_I \gamma_S \hbar^2 \left[\frac{\check{I} \cdot \check{S}}{R^3} - \frac{3(\check{I} \cdot \check{R})(\check{R} \cdot \check{S})}{R^5} \right] \quad (\text{generally})$$

This interaction provides the main means by which spins-$\frac{1}{2}$ exchange magnetization (T_2 processes — called cross-relaxation) or dissipate magnetization to the lattice in longitudinal relaxation.

The motion of *I* relative to *S* sets up local field fluctuations at *S* (or, equivalently, modulates *E* levels of *S*.). There exists a component of this modulation at *S* transition frequency, so the fluctuations can induce transitions.

Solomon Equations

The Solomon equations are a phenomenological description of the relaxation behavior of the Z-components of I, S magnetization in a two-spin dipolar-coupled system.

$$\frac{d}{dt}\langle I_Z\rangle = \frac{1}{T_1^{II}}\left(\langle I_Z\rangle - I_0\right) - \frac{1}{T_1^{IS}}\left(\langle S_Z\rangle - S_0\right)$$

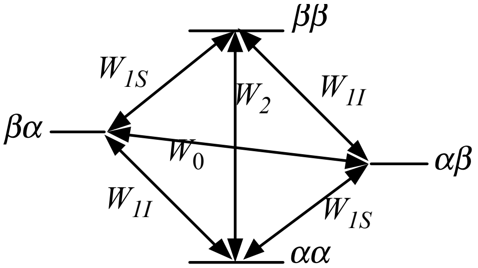
$$\frac{d}{dt}\langle S_Z\rangle = \frac{1}{T_1^{IS}}\left(\langle I_Z\rangle - I_0\right) - \frac{1}{T_1^{SS}}\left(\langle S_Z\rangle - S_0\right)$$

where

$$\frac{1}{T_1^{II}} = W_0 + 2W_{1I} + W_2$$

$$\frac{1}{T_1^{SS}} = W_0 + 2W_{1S} + W_2$$

$$\frac{1}{T_1^{IS}} = W_2 - W_0$$



Here W's are transition probabilities, related to molecular geometry and motion by time-dependent perturbation theory.

$$W_0 = \frac{1}{10}\,\frac{\hbar^2\gamma_I^2\gamma_S^2}{R^6}\,\frac{\tau_c}{1-\left(\omega_I-\omega_S\right)^2\tau_c^2}$$

$$W_{1I} = \frac{3}{20}\,\frac{\hbar^2\gamma_I^2\gamma_S^2}{R^6}\,\frac{\tau_c}{1+\omega_I^2\tau_c^2}$$

$$W_{1S} = \frac{3}{20}\,\frac{\hbar^2\gamma_I^2\gamma_S^2}{R^6}\,\frac{\tau_c}{1+\omega_S^2\tau_c^2}$$

$$W_Z = \frac{3}{5}\,\frac{\hbar^2\gamma_I^2\gamma_S^2}{R^6}\,\frac{\tau_c}{1+\left(\omega_I+\omega_S\right)^2\tau_c^2}$$

where $\tau_c =$ rotation correlation time for $\vec{R}$

$\left(\omega_I\tau_c \approx 1 \quad \text{for} \quad \tau_c \approx 10^{-9}\,S \quad \text{for } {}^1\text{H at 500 mHz}\right)$

Main Result If in a two spin system, there exists nonequilibrium Z-magnetization, there will be magnetization exchange between A and X.