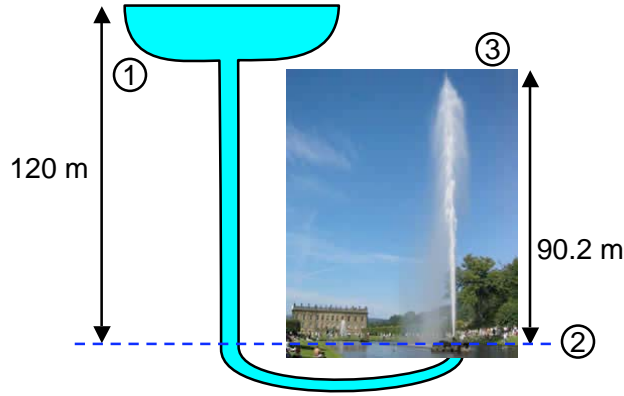


## PROBLEM B21.



1. Taking point 2 as our zero height, Bernoulli's equation shows that the difference in pressure between points 1 and 2 is

$$p_1 + \rho g h_1 = p_2 + \frac{1}{2} V_2^2$$

$$p_1 - p_2 = \frac{1}{2} \rho V_2^2 - \rho g h_1$$

where the velocity at point 2,  $V_2$  is an unknown velocity of the jet (not the velocity in the pipe). Since the pressure at points 2 and 3 are the same (atmospheric) Bernoulli's equation can be used to find the unknown velocity term.

$$p_2 + \frac{1}{2} V_2^2 = p_3 + \rho g h_3$$

$$\frac{1}{2} V_2^2 = \rho g h_3$$

Thus the pressure loss in the pipe is given by

$$p_1 - p_2 = \rho g (h_3 - h_1)$$

Assuming all of this loss occurs in the supply pipe it follows that the friction factor,  $f$ , in the pipe is

$$f = \frac{D \left( -\frac{dp}{dx} \right)}{\frac{1}{2} \rho V^2}$$

$$= \frac{D \left( \frac{\rho g (h_1 - h_3)}{L} \right)}{\frac{1}{2} \rho V^2}$$

$$= \frac{2 D g (h_1 - h_3)}{L V^2}$$

$$= \frac{2 (0.381 \text{ m}) (9.81 \text{ m/s}^2) (120 \text{ m} - 90.2 \text{ m})}{(800 \text{ m}) V^2}$$

where  $V$  is the velocity of the flow in the pipe. Since the pipe cross-sectional area is  $0.114 \text{ m}^2$ ,  $1500 \text{ l/min}$  means a velocity,  $V = 2.2 \text{ m/s}$ . Therefore  $f = 0.058$ .

2. The Reynolds number of the flow in the pipe is

$$\begin{aligned} Re_D &= \frac{VD}{\nu} \\ &= \frac{(2.2 \text{ m/s})(0.381 \text{ m})}{1.16 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 7.2 \times 10^5 \end{aligned}$$

3. For pipes, flows with a Reynolds number less than about 2000 are laminar and above 4000 are turbulent. With this Reynolds number, the flow is turbulent. Moreover, referring to the Moody chart, the pipe flow is also fully rough since, at this Reynolds number, the friction factor is well above that for smooth-walled turbulent flow.
4. Again, referring to the Moody chart it would appear that, at this Reynolds number, a friction factor of 0.058 will occur when the roughness has a typical height of  $0.03 \times D$  or 1.1 cm.
5. With the same friction factor but a pipe diameter of 0.19 m the head loss would be double that of the actual pipe. The maximum height of the fountain would have been  $120 \text{ m} - 2 \times 29.8 \text{ m}$  or  $60 \text{ m}$  – much less impressive.

### PROBLEM B22.

Calculate the drag on a streamlined, axisymmetric body as it moves in water.

- 1.) Find the form drag contribution,  $C_{DF}$ , to the total drag coefficient.

To find the form drag, we must examine the pressure distribution on the nose and the flat trailing portion of the body. Since we can consider the flow over the nose to be described by potential flow,  $u(\theta) = \frac{3}{2} \sin \theta$ , we can use Bernoulli's equation to get the corresponding pressure distribution over the surface.

$$\begin{aligned} p_\infty + \frac{1}{2}\rho U^2 &= p(\theta) + \frac{1}{2}\rho u^2(\theta) \\ \Rightarrow p(\theta) - p_\infty &= \frac{1}{2}\rho U^2 \left[ 1 - \frac{9}{4} \sin^2 \theta \right] \end{aligned}$$

This leads to a pressure coefficient on the nose that varies with  $\theta$

$$C_{p,N} = \frac{p(\theta) - p_\infty}{\frac{1}{2}\rho U^2} = 1 - \frac{9}{4} \sin^2 \theta$$

Since the flow separates at the sharp trailing edge, we can consider the pressure to be constant on the circular base with a constant pressure coefficient.

$$c_{p,T} = -0.5$$

Our goal is the drag coefficient due to form effects,  $C_{DF}$ .

$$C_{DF} = \frac{D_F}{\frac{1}{2}\rho U^2 \pi R^2}$$

where  $D_F$  is the form drag. We calculate this drag by finding the difference between the pressure integrated over the nose and the base of the streamlined body.

$$D_F = \int p_N dA - \int p_T dA = \int (p_N - p_\infty) dA - \int (p_T - p_\infty) dA$$

Dividing this equation by  $\frac{1}{2}\rho U^2 \pi R^2$  gives us the form drag coefficient in terms of integrals of the pressure coefficients over the nose and tail.

$$C_{DF} = \frac{\int C_{p,N} dA}{\pi R^2} - \frac{\int C_{p,T} dA}{\pi R^2}$$

The second integral is trivial since the pressure coefficient is constant.

$$\frac{\int C_{p,T} dA}{\pi R^2} = \frac{-0.5\pi R^2}{\pi R^2} = -0.5$$

The integral over the nose is slightly more involved.

$$\begin{aligned} \frac{\int C_{p,N} dA}{\pi R^2} &= \frac{\int \left(1 - \frac{9}{4} \sin^2 \theta\right)}{\pi R^2} \\ &= \frac{1}{\pi R^2} \int_{-\pi/2}^{\pi/2} \int_0^\pi \left(1 - \frac{9}{4} \sin^2 \theta\right) R^2 \sin \theta \cos \theta d\phi d\theta \\ &= 2 \int_0^{\pi/2} \left(1 - \frac{9}{4} \sin^2 \theta\right) \sin \theta \cos \theta d\theta \\ &= 2 \int_0^1 \left(\sin \theta - \frac{9}{4} \sin^3 \theta\right) d(\sin \theta) \\ &= 2 \left[ \frac{1}{2} \sin^2 \theta - \frac{9}{16} \sin^4 \theta \right]_0^1 = -\frac{1}{8} \end{aligned}$$

The form drag coefficient is the difference between these integrals.

$$C_{DF} = \frac{\int C_{p,N} dA}{\pi R^2} - \frac{\int C_{p,T} dA}{\pi R^2} = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$$

Note: The form drag coefficient could also have been evaluated as a single integral, more akin to what was done in class. As was shown in class, it is sufficient to integrate over the projected area.

2.) Find the skin friction (on the cylindrical surface) contribution,  $C_{DS}$ , to the total drag coefficient.

Since we assume that the boundary layer remains laminar, we can use the Blasius solution to calculate the skin friction drag,  $D_S$ , on the cylindrical surface.

$$\begin{aligned} D_S &= \int \tau_0 dA \\ &= \int_0^L \frac{1}{2} \rho U^2 c_f 2\pi R dx \\ &= \frac{1}{2} \rho U^2 2\pi R \int_0^L 0.664 \sqrt{\frac{\nu}{Ux}} dx \\ &= \frac{1}{2} \rho U^2 2\pi R L \cdot 1.328 \sqrt{\frac{\nu}{UL}} \end{aligned}$$

This leads to a drag coefficient due to skin drag of:

$$C_{DS} = \frac{D_S}{\frac{1}{2}\rho U^2 \pi R^2} = \frac{2L}{R} \cdot 1.328 \sqrt{\frac{1}{Re_L}}$$

where  $Re_L = UL/\nu$  is the Reynolds number based on the length of the body. The skin friction drag coefficient can be rewritten in terms of a radius-based Reynolds number,  $Re_R = U2R/\nu$  to give:

$$C_{DS} = 1.328 \cdot \frac{2L}{R} \sqrt{\frac{2R\nu}{2RUL}} = 1.328 \cdot 2\sqrt{2} \sqrt{\frac{L}{R}} \sqrt{\frac{1}{Re_R}}$$

Note: Here we used the original free stream velocity in the Blasius calculation of the shear stress. One may argue that it may be more appropriate to use the accelerated value,  $\frac{3}{2}U$  for free stream. The flow after the nose can no longer be considered potential and the higher free stream value will decrease back to  $U$  with distance along the body. Thus, there are likely regimes, based on the length of the body, in which one assumption for the velocity is preferable to the other. But, after all, we only desire an estimate of the contribution of the skin friction drag so either choice is fine.

3.) Calculate the aspect ratio,  $L/R$ , at  $Re_D = 10,000$  for which the total drag is composed of equal parts form and skin friction drag.

Here we equate the two drag coefficients for the given Reynolds number.

$$C_{DF} = \frac{3}{8} = 1.328 \cdot 2\sqrt{2} \sqrt{\frac{L}{R}} \sqrt{\frac{1}{10000}} = C_{DS}$$

$$\Rightarrow \sqrt{\frac{L}{R}} = \frac{3}{8} \frac{100}{1.328 \cdot 2\sqrt{2}} = 10$$

$$\frac{L}{R} = 100$$

### PROBLEM B23.

At takeoff:

$$W = \frac{1}{2} \rho V^2 A C_L$$

$$V = \left( \frac{2W}{\rho A C_L} \right)^{\frac{1}{2}}$$

$$= \left( \frac{2(3 \times 10^6 \text{ kg m/s}^2)}{(1.2 \text{ kg/m}^3)(550 \text{ m}^2)(1.6)} \right)^{\frac{1}{2}}$$

$$= 75.38 \text{ m/s}$$

Neglecting drag,  $Thrust(T) = Mass(M) \times Acceleration(a)$ , then:

$$a = \frac{T}{M}$$

$$= \frac{4(2 \times 10^5 \text{ kg m/s}^2)}{3 \times 10^6 \text{ kg m/s}^2} \times g$$

$$= 2.61 \text{ m/s}^2$$

Then, takeoff distance is:

$$L = \frac{V^2}{2a} = 1086 \text{ m}$$

Drag(D) at takeoff is the lift divided by 22, where the lift is equal to the weight(G). So  $D = G/22 = 136364 \text{ kg m/s}^2$ . So the net thrust is:

$$T_{net} = (8 \times 10^5 - 1.36 \times 10^5) \text{ kg m/s}^2 = 6.64 \times 10^5 \text{ kg m/s}^2$$

So, the acceleration is:

$$a \frac{T_{net}}{T} = 2.17 \text{ m/s}^2$$

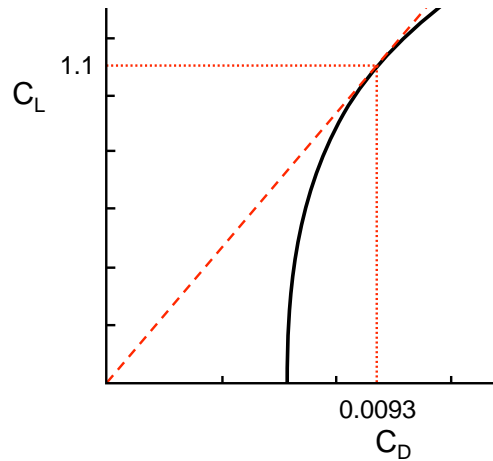
### PROBLEM B24.

- From the graph, stall occurs at about  $C_L = 1.72$ . For horizontal flight, the lift must be balanced by the weight.

$$\begin{aligned} \frac{1}{2}\rho U^2 AC_L &= 2000 \text{ kg m/s}^2 \\ u &= \sqrt{\frac{2(2000 \text{ kg m/s}^2)}{\rho AC_L}} \\ &= \sqrt{\frac{2(2000 \text{ kg m/s}^2)}{(1 \text{ kg/m}^3)(10 \text{ m}^2)(1.72)}} \\ &= 15.2 \text{ m/s} \end{aligned}$$

- The glide angle,

$$\theta = \tan^{-1}\left(\frac{C_D}{C_L}\right)$$



has a minimum met by the conditions at the point where  $C_L = 1.1$ ,  $C_D = 0.0093$ . Therefore the minimum glide angle is approximately  $0.48^\circ$ . [Note: If the fuselage drag was included, this angle would be much larger.] In gliding,

$$\begin{aligned} L &= W \times \cos \theta \\ \frac{1}{2}\rho u_T^2 AC_L &= (2000 \text{ kg m/s}^2) \cos(0.48^\circ) \\ u_T &= 19 \text{ m/s} \end{aligned}$$

where  $u_T$  is the total velocity. Therefore, the horizontal velocity is  $u_T \cos(0.48^\circ) = 19.1 \text{ m/s}$

### PROBLEM B25.

Consider a simplified view of the propulsion of a paddle steamer:

- Find a relation between the propulsion velocity,  $U$  and the relative paddle velocity,  $V$ . Since the steamer isn't accelerating, the forces must match. The drag force,  $D = C_D(\frac{1}{2}\rho U^2)A$ . Thus,

$$\begin{aligned} C_{DP}\frac{1}{2}\rho(V-U)^2 A_P &= C_{DH}\frac{1}{2}\rho U^2 A_H \\ V &= \left(1 + \sqrt{\frac{C_{DH}A_H}{C_{DP}A_P}}\right)U \end{aligned}$$

or

$$U = \frac{V}{1 + \sqrt{\frac{C_{DH}A_H}{C_{DP}A_P}}}$$

2. The efficiency is the ratio of useful work done to total work done.

$$\begin{aligned}\eta &= \frac{C_{DH}\frac{1}{2}\rho U^3 A_H}{C_{DH}\frac{1}{2}\rho U^3 A_H + C_{DP}\frac{1}{2}\rho(V-U)^3 A_P} \\ &= \frac{1}{\sqrt{\frac{C_{DH}A_H}{C_{DP}A_P}} + 1}\end{aligned}$$

If  $C_{DH}$  and  $C_{DP}$  are both of order unity, to obtain reasonable efficiencies  $\frac{A_H}{A_P}$  should be made as small as possible. As it's difficult to make  $A_H$  small with traditional ships,  $A_P$  should be made as large as possible. If one looks at the development of paddle steamers, the size of the paddles was indeed made larger and larger through time. As the paddle cannot be made infinitely large, the efficiency for these vessels was never very big and they were eventually overtaken by propeller boats.