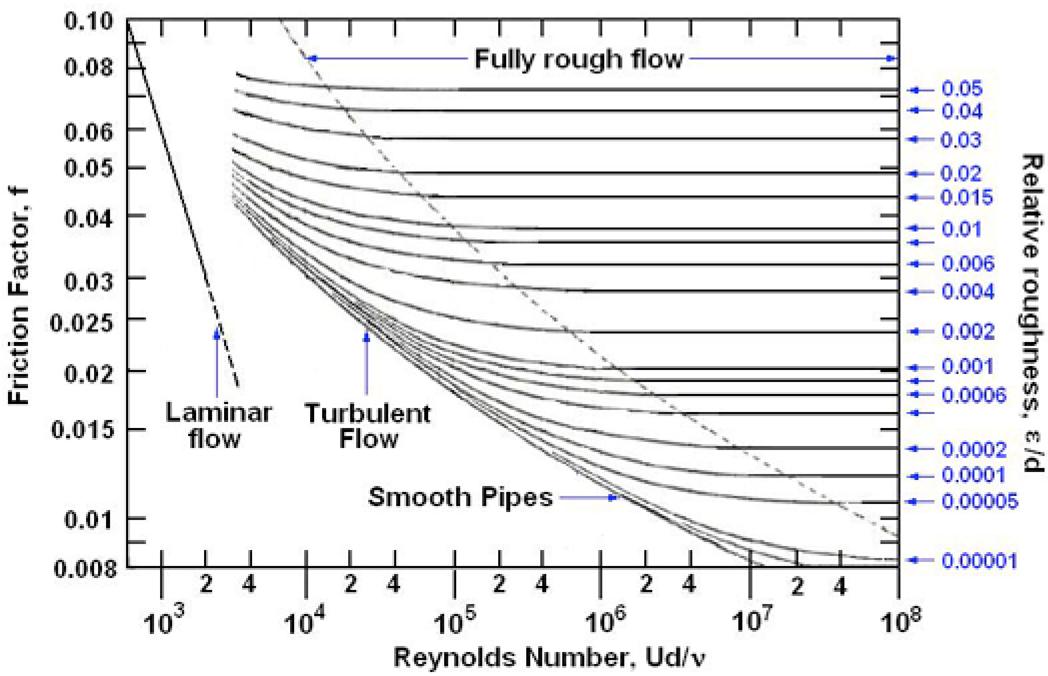
ME19b. **HOMEWORK.** **Feb. 22** **Due Mar. 1, 2007**

PROBLEM B16.

What is the relation between the friction factor, f, and the Reynolds number, Re, for turbulent pipe flow given the following approximate velocity profile?

$$u^* = 8.7(y^*)^{\frac{1}{7}}$$

Note that $u^* = \bar{u}/u_\tau$ where $u_\tau = (\tau_w/\rho)^{\frac{1}{2}}$ and $y^* = u_\tau y/\nu$.



At a Reynolds number of 10^6 how does the friction factor predicted by this relation compare with the value given by the above graph?

PROBLEM B17.

A long ventilation duct is used to transport air at normal temperatures (density, $\rho = 1.2\ kg/m^3$, kinematic viscosity, $\nu = 2.3 \times 10^{-6}\ m^2/s$). The duct has a smooth interior surface, a circular cross-section with a diameter of 0.5 m and is 50 m long. A pressure difference of 1 $kg/m\ s^2$ is applied between the two ends of the duct. Using the data in the graph below, find (by trial and error or other means) the average velocity of flow through the duct.

[Note that the friction factor, $f = d(-dp/dx)/\frac{1}{2}\rho U^2$, $Re = dU/\nu$ where d is the diameter and U is the volumetric average velocity of flow.]

PROBLEM B18.

The velocity profile in a turbulent boundary layer of incompressible fluid on a flat plate (U = constant) is to be approximated by the form:

$$u/U = (y/\delta)^{\frac{1}{7}}$$

[Disregard the fact that this does not exactly satisfy one of the constraints usually imposed on laminar boundary layer profiles namely that du/dy should tend to zero as y tends to δ]. Find the profile parameter α for this profile. If the wall shear stress, τ_w, for this turbulent profile is assumed to be given by the empirical formula

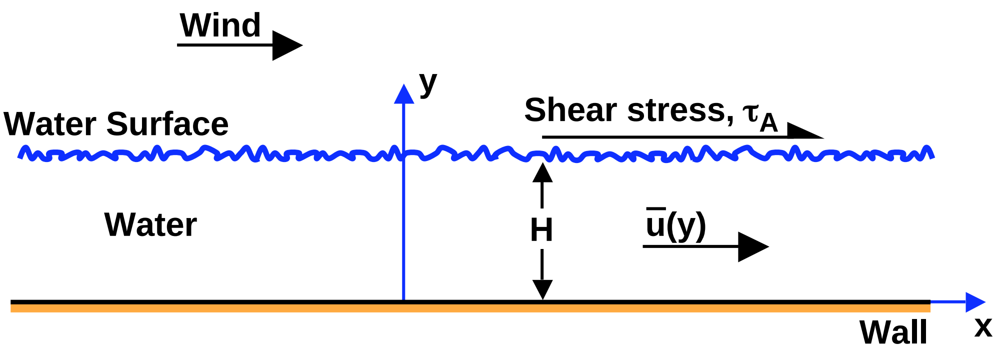
$$\tau_w = 0.023\rho U^2(\nu/\delta U)^{\frac{1}{4}}$$

where ρ and ν are the fluid density and kinematic viscosity, then solve the resulting Karman momentum integral equation to obtain an expression for the thickness of the boundary layer, δ, as a function of distance, x, along the plate. Assume that the layer first becomes turbulent at $x = x_0$ where the thickness is δ_0.

[Do not use $\tau_w = \mu(du/dy)_{y=0}$ which is inappropriate in turbulent boundary layer calculations.]

PROBLEM B19.

A high wind drives a film of water over a solid surface at such a speed that the flow in the film becomes turbulent. This occurs because the wind applies a shear stress, τ_A, to the surface of the water:



The thickness of the film, H, and the mean water velocity, $\bar{u}(y)$, are constant in time and with position, x. Using the assumptions listed below, find an expression for the mean velocity on the water surface, $\bar{u}(H)$, in terms of τ_A, H, ρ (the water density), ν (the kinematic viscosity of the water), and the Karman universal constant, κ. The assumptions:

- The laminar sublayer next to the solid surface (in which $u^* = y^*$) extends to $y^* = 5$ where the mean velocity is to be matched with that of the turbulent flow in the rest of the water film.
- Outside the laminar sublayer, the Reynolds stresses dominate and the viscous component of the shear stress can be neglected.
- Prandtl's mixing length theory is to be used with a Karman universal constant denoted by κ.

PROBLEM B20.

A utility company is laying a new water supply pipe with an internal diameter of 5 cm. The flow rate through this pipe will be 0.05 m^3/s. The kinematic viscosity of the water is 10^{-6} m^2/s.

When new the interior surface of the pipe has roughnesses which are typically 0.05 mm in size. However, as the pipe ages, the engineer estimates that the roughness could increase to 1.0 mm. Using the chart above find the *ratio* of the pressure difference needed to generate this flow when the pipe has aged to the pressure difference required when it is new.