

PROBLEM B4.

Consider the laminar, viscous, planar flow of an incompressible fluid contained between two parallel plates distance H apart. The coordinates x and y are respectively measured parallel to and perpendicular to these plates. We shall take $y = 0$ at the static plate and $y = H$ at the moving plate for convenience. The plate at $y = H$ moves with a steady velocity, U , in the x direction. However, unlike simple Couette flow, a pressure gradient, dp/dx , is also applied to the fluid. Find:

- [1] The velocity distribution, $u(y)$, in the flow as a function of y , U , H , dp/dx and the viscosity of the fluid, μ .
- [2] The magnitude and direction of the particular pressure gradient for which there would be zero net volume flow in the x direction.

PROBLEM B5.

In cylindrical coordinates, (r, θ, z) , the Navier-Stokes equations of motion for an incompressible fluid of constant dynamic viscosity, μ , and density, ρ , are

$$\rho \left[\frac{Du_r}{Dt} - \frac{u_\theta^2}{r} \right] = -\frac{\partial p}{\partial r} + f_r + \mu \left[\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right]$$

$$\rho \left[\frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r} \right] = -\frac{1}{r} \frac{\partial p}{\partial \theta} + f_\theta + \mu \left[\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} \right]$$

$$\rho \frac{Du_z}{Dt} = -\frac{\partial p}{\partial z} + f_z + \mu \nabla^2 u_z$$

where u_r, u_θ, u_z are the velocities in the r, θ, z cylindrical coordinate directions, p is the pressure, f_r, f_θ, f_z are the body force components in the r, θ, z directions and the operators D/Dt and ∇^2 are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r} \frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Now consider the steady, planar, incompressible, viscous flow between two concentric cylinders. The inner cylinder has radius, a , and is rotating at an angular velocity, Ω (radians/second). The outer cylinder has radius, b , and is static. There is no flow in the direction parallel to the axis of the cylinders so only the velocity, u_θ , is non-zero. Body forces are to be neglected. The density of the fluid is denoted by ρ . Find:

- (a) The velocity distribution, $u_\theta(r)$, in the gap between the two cylinders.
- (b) The difference between the pressure on the outer surface of the inner cylinder and the pressure on the inner surface of the outer cylinder.

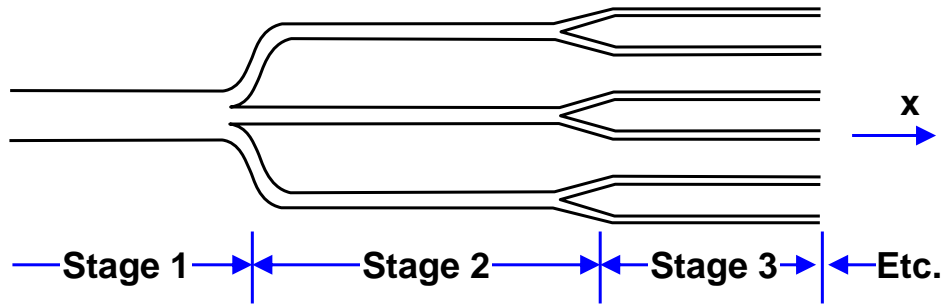
Note: The solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = 0 \quad \text{is} \quad y = A/x + Bx$$

where A and B are constants.

PROBLEM B6.

Both the mammalian respiration system and the mammalian blood circulation system are networks of tubes in which the flow from one large tube (respectively the trachea and the aorta) branches into parallel flows in tubes of smaller size. This branching continues through a number of stages:



If, for each stage, the number of tubes is denoted by n and the cross-sectional area for each and every tube in that stage is denoted by A_n , find the relation between A_n and n such that the pressure gradient, dp/dx , is the same for each stage. How does the average velocity depend on n ? Assume steady, fully-developed Poiseuille flow in all tubes even though this may not be the case in the actual systems.

If the diameter of the aorta is 3 cm and the diameter of the microcirculation (the smallest tubes) is 8×10^{-6} m, calculate the number of tubes at the microcirculation stage which would be present if the above property were to exist. The actual number is much smaller than this. Where, then, does most of the pressure drop occur in the blood circulation system?