Productive Policy Competition and Asymmetric Extremism\textsuperscript{1}

Alexander V. Hirsch\textsuperscript{2}

May 6, 2022

\textsuperscript{1}I thank Ken Shotts, Adam Meirovitz, Dan Kovenock, Ron Siegel, Leeat Yariv, Federico Echenique, Salvatore Nunnari, Betsy Sinclair, and seminar audiences at USC, the University of Utah (Eccles School), and the 2018 SAET conference for helpful comments and advice, as well as Joanna Huey for research assistance.

\textsuperscript{2}California Institute of Technology. Division of the Humanities and Social Sciences, MC 228-77, Pasadena, CA 91125. Phone: (626) 395-4216. Email: avhirsch@hss.caltech.edu
Abstract

Public policies do not simply appear out of thin air; they must be developed by individuals and groups with the expertise and willingness to do so. We analyze a model in which competing actors with differing ideologies and abilities develop policies for consideration by a decisionmaker. We show that the process will exhibit unequal participation, inefficiently unpredictable and extreme policies and outcomes, wasted effort, and an apparent advantage for extreme policies. These imbalances can arise both because of unequal intrinsic extremism or unequal ability at policy development, but in either case benefit the decisionmaker, in contrast to a large literature showing the benefits of balanced competition. Our model provides a rationale for why extreme actors may come to dominate policymaking that is rooted in the nature of productive policy development, and highlights the difficulty in assessing the normative implications of such dominance.
The reasonable man adapts himself to the world; the unreasonable one persists in trying to adapt the world to himself. Therefore all progress depends on the unreasonable man.

– George Bernard Shaw, *Man and Superman* (1903)

Ideological conflict is a key driver of political competition; politically-active citizens, politicians, parties, and interest groups compete through elections and the policy process to have policies enacted that reflect their ideological interests. Correspondingly, the political science literature has devoted substantial attention to these processes, in order to better understand why some policies become law and not others. However, the literature has generally paid less attention to the process through which public policies are *developed*; even though a policy that achieves an intended outcome cannot just be freely chosen from a “theoretical infinity of them.” Instead, as Kingdon (1984) puts it in his sweeping work on the policy process, “before a subject can attain a solid position on a decision agenda, a *viable* alternative [must be] available for decisionmakers to consider” (p142, emphasis added). In the words of one presidential staffer (p132),

Just attending to all the technical details of putting together a real proposal takes a lot of time. There’s tremendous detail in the work. It’s one thing to lay out a statement of principles or a general proposal, but its quite another thing to staff out all the technical work that is required to actually put a real detailed proposal together.

Who will “invest the resources – time, energy, reputation, and sometimes money” (Kingdon (1984), p122) – to develop concrete and viable policy alternatives? When and why do they do so? And how does competition in policy development influence which policies are ultimately adopted? To help answer these questions we build on the “competitive policy development” model of Hirsch and Shotts (2015). In this model, two identical policy developers on opposite sides of the ideological spectrum develop competing policies for consideration by a centrist decisionmaker. We extend this model to study the consequences of *asymmetries* between these policy developers; in their intrinsic ideological extremism, their ability at developing policies, or both. In so doing, we uncover new insights about the nature of competitive policy development, and surprising normative implications about the potential causes and consequences of extreme behavior and imbalanced political participation.
Competitive Policy Development  Before discussing our new results we first revisit the original model’s central elements and key insights. Two competing policy developers simultaneously craft policies consisting of both an ideological orientation and a level of “quality.” Quality is modelled as a policy-specific “public good” that all participants in the policy process value, and is intended to capture the sorts of “criteria for survival” that Kingdon identifies as necessary for public policies, such as technical and administrative feasibility, efficacy, equity, efficiency, and cost-effectiveness (Kingdon (1984), Ch. 6). The ideology of a policy may be chosen freely, but generating its quality requires a costly up-front investment. Once produced quality is “attached” to the policy, and cannot be preserved if its ideological orientation is altered. A centrist decisionmaker then chooses among the available policies, which includes those created by the developers as well as any preexisting alternatives.

Much like the “policy entrepreneurs” in Kingdon’s theory, the developers intrinsically value quality, but are predominantly motivated by ideology when developing their policies. Specifically, “they want to promote their values, or affect the shape of public policy” (Kingdon (1984), p123), and do so by developing policies that – through a combination of ideological concessions and quality investments – are sufficiently appealing to be chosen by a centrist decisionmaker. When crafting their policies, the developers therefore face a tradeoff between working on something more reflective of their ideological preferences – which must be higher quality to be appealing to a decisionmaker with differing preferences – vs. something more moderate – which may be lower quality and still remain competitive. Crucially, the developers also fear that should their policy fail to be adopted, they will instead have to live under a policy developed by an ideological competitor.

The Hirsch and Shotts (2015) model studies policy developers who are equally extreme and capable, and yields three main insights. First, whenever a developer chooses to craft a new policy, there will be a positive association between its extremism and its quality – the reason is that strategic developers know that more-extreme policies have no hope of adoption unless they are also higher-quality. Second, ideological polarization among the developers is not an unalloyed bad – competing desires to “affect the shape of public policy” driven by polarized ideologies can motivate costly investments in “good policy” that benefit the decisionmaker despite the greater polarization of the available policies. Finally, better technology at generating quality is not an unalloyed good; both developers will use it to produce higher quality policies, but also exploit it to develop ones that better reflect their ideological interests.
General Properties  The first contribution of our model – in which the developers may differ in both
their intrinsic ideological extremism and ability – is to identify six general properties of “policymaking
as a contest.” First, it is unpredictable – when crafting policies, the developers are uncertain about
what exactly their competitor will develop, whose policy will prevail, and what the final outcome will
be. This uncertainty is necessary to incentivize the developers’ participation; if one were certain that
his policy would fail to be adopted he would decline to develop it, and if he were certain it would be
adopted he would scale back his quality investment. Second, it is asymmetric – one developer always
crafts a new policy (i.e., is “active”), but the other sometimes declines to do so, anticipating that
he will be outmatched. This pattern arises naturally from asymmetries in the developers’ underlying
preferences and abilities. Third, it is divergent – when a developer crafts a new policy, its ideology
always diverges from the decisionmaker’s ideal; since one developer is always active, the final outcome
always diverges from the decisionmaker’s ideal as well.

Fourth, policymaking as a contest is inegalitarian, in that the benefits and costs are unequally
distributed among the participants. The decisionmaker always strictly benefits from competition in a
strong sense; she would never want to prohibit one developer from participating even if his preferences
are very extreme. In contrast, each developer is harmed by competition, in the sense that each would
prefer the other to be blocked from participation; this is true even when both are making substantial
quality investments. Fifth, it is inefficient. The expected ideological outcome generically differs from
both the decisionmaker’s ideal as well as the ideology that would maximize social welfare. There is
also substantial uncertainty about the ideological outcome which harms all participants. Moreover,
there is waste, in that the benefits of any quality developed for the losing policy are lost.

Finally, policymaking as a contest is biased toward extremism. Specifically, in equilibrium the deci-
sionmaker is more likely to choose more-extreme policies from a given developer than more-moderate
ones. This surprising behavior results from the developers’ strategic investments in quality. Specifi-
cally, when a developer chooses to craft a more extreme policy, he not only makes it higher quality to
preserve its appeal to the decisionmaker, but actually “overinvests” in the policy’s quality in order to
enhance that appeal.
Asymmetric Policy Competition  We next consider the consequences of both asymmetric extremism in the developers’ ideologies, and asymmetric ability at generating quality.

Asymmetric ideological preferences arise naturally as political parties respond to changing membership and internal organizational dynamics (Masket (2011)). It can also arise within institutions like legislatures, bureaucracies, and courts as a result of shifting electoral outcomes and political appointments that generate imbalances in the ideological alignment of senior leadership with internal factions (Lewis (2008)). When the developers’ ideologies are asymmetrically extreme, their participation is also (unsurprisingly) asymmetric; one always develops a new policy, while the other sometimes declines to do so. Intuition suggests that it is the more moderate policy developer who will be more active, since his preferences are better aligned with the decisionmaker. However, in fact the reverse is true; it is the more extreme developer who will be more active, because he has a stronger desire to “affect the shape of public policy.” His policies will be more extreme than those of his competitor (in a first order stochastic sense), but also higher quality to “compensate” for their greater extremism. In fact, they will be so much higher quality that they will also be better for the decisionmaker (again in a first order stochastic sense), and so the decisionmaker will appear to be biased toward the extremist.

We also consider how policies and outcomes change as one developer’s preferences become more extreme. A developer who becomes more extreme will become more active in policy development, and craft more extreme but also higher quality policies. In contrast, his opponent will become less active in policy development, and craft more moderate policies. The opponent becomes worse off even though the increasingly-extreme developer is crafting increasingly high-quality policies, because this quality is inadequate to “compensate” the opponent for their greater extremism. However, the decisionmaker actually becomes better off, even though increasingly-extreme developer crafts increasingly extreme policies, and also his opponent becomes increasingly inactive. Moreover, this effect is strong in a specific sense; any level of decisionmaker welfare can be achieved with a sufficient degree of unilateral extremism. Put more intuitively, unilateral extremism results in less observable competition, but a happier decisionmaker. From a theoretical standpoint this is very unusual; in most contest-like settings, a decisionmaker benefit from “competitive balance” because it also results in the broadest participation (Chowdhury, Esteve-Gonzalez and Mukherjee, 2020). In our model, however, competitive balance is not best for the decisionmaker because the developers care about the final policy even when
they lose, as befits a contest over policy outcomes rather than “particularistic rents.” As a result, an increasingly extreme developer also becomes increasingly fearful of losing to a centrist policy, which keeps him from “shirking” at policy development even as he comes to dominate the policy process.

Finally, we consider the consequences of asymmetric ability. Such asymmetries may arise as political parties fluctuate in their success at cultivating a community of policy experts (Rich (2004)); asymmetries in expertise and resources are also common feature of interest group politics (Berry and Wilcox (2015)) and formal rulemaking (Yackee and Yackee (2006)). Our main finding is asymmetric ability leads to a pattern of competition that is is observationally equivalent to asymmetric extremism. Specifically, a developer who is no more intrinsically extreme than his competitor, but who is more capable at producing quality, will also be strictly more active in policy development, develop policies that are more ideologically extreme than his competitor, but that are also higher quality and better for the decisionmaker. Similarly, as a developer becomes increasingly expert, his policies become increasingly extreme but also higher quality and better for the decisionmaker, and his competitor both moderates his policies and becomes increasingly unlikely to develop one. In the process, the competitor becomes increasingly worse off, and the decisionmaker becomes increasingly better off despite the increasing imbalance in political participation and extremism of the expert’s policies.

Overall, our analysis provides a novel explanation for how apparent ideological extremism may come to dominate policymaking in a particular domain that is rooted in the nature of productive policy competition, rather than political dysfunction, capture, or some other systemic failure. The model illustrates how striking imbalances in political participation can arise naturally from differing motivations and abilities of participants in the policy process, and may even have beneficial effects since extreme preferences can motivate investments in “good policy.” It further shows how and why a centrist decisionmaker vs. ideologically divergent policy developers may come to hold very different views about the efficacy of the policy process at promoting public welfare. Finally, it highlights the empirical challenges associated with inferring the underlying preferences of political actors from their observable policy proposals and choices.
Preliminary Discussion

Theoretical Antecedents In the classical approach to studying policy development, a policy outcome results from the combination of a policy choice $y$ and an “unknown state of the world” $\omega$ that is (or can become known to) an “expert.” This framework has been widely applied to study the determinants of effective policymaking in many institutional environments, including legislatures and bureaucracies (see Gailmard and Patty (2012) for a review). The central tension in such models is that privately-informed experts worry that their expertise will be “expropriated” to implement policy outcomes that do not reflect their ideological interests.

Our model, in contrast, is part of a growing literature that captures an alternative conceptualization of policy development, in which actors can develop policy-specific expertise that they use to effectively achieve a particular ideological goal (e.g. Hirsch and Shotts (2012); Hitt, Volden and Wiseman (2017); Lax and Cameron (2007); Londregan (2000); Ting (2011); Turner (2017)). Borrowing from the elections literature (Stokes (1963)), such models typically posit that an expert can endow an ideological policy with “valence” that is valued by all participants in the process, but that is also attached exclusively to the policy in question. This conceptualization yields very different strategic tensions than the classical approach; rather than fear their information will be expropriated, experts “attempt to exploit their monopoly power over investments to compel decision makers to accept policies that promote their interests” (Hirsch and Shotts (2015)).1 Of particular note are works by Lax and Cameron (2007) and Hitt, Volden and Wiseman (2017), who also study competitive valence models, but in which the developers craft policies in a predetermined order. These models are better suited to settings like courts and legislatures where formal rules can privilege one developer, and capture very different strategic incentives that more closely resemble “entry deterrence” models of market competition. They thus yield very different equilibrium patterns of competition than our model. However, they also share many substantive insights about the distributional effects of extremism and ability.

In studying policy-specific expertise in a competitive environment, our model is also analytically related to a large literature studying “political contests” such as lobbying and electoral competition

---

1In between these approaches is work by Callander (2011) which models the “unknown state” governing the relationship between policies and outcomes as a function that is the realized path of a Brownian motion. In this approach, knowledge of a particular policy is more relevant for projecting the likely effects of “nearby” policies than “distant” ones.
(e.g. Ashworth and Bueno de Mesquita (2009); Epstein and Nitzan (2004); Herrera, Levine and Martinelli (2008); Munster (2006); Serra (2010); Wiseman (2006); Zakharov (2009)). In particular, foundational work by Tullock (1980) modelled lobbying as a process by which competing interest groups exert costly and wasteful effort to secure “politically-contestable rents.” Important follow-on work by Hillman and Riley (1989) studied “asymmetric” political contests in which competing interest groups valued policy control differentially, which would fall to the group exerting the most effort. This latter model is now known as the all pay contest due to its close relationship to the all-pay auction, and has been studied in depth and generality by economic theorists (e.g. Baye, Kovenock and de Vries (1996); Siegel (2009)). Our model is a successor to the asymmetric all pay contest; quality investments are “all pay” in the sense that developers make them before knowing whether their policy will be implemented, and the developers may differ in both their ideological extremism and abilities. However, there are three crucial differences that significantly complicate the analysis. First, quality investments are a public good that benefit all participants. Second, the developers choose both the quality and ideology of their policy, so their strategies are “multidimensional” (e.g. Che and Gale (2003)). Finally, the developers are policy motivated rather than “rent seeking,” in that they care about which policy wins, rather than whose policy wins. This last property is necessary for our novel finding that asymmetries may benefit the decisionmaker despite reducing participation.

Empirical Domain Our intended empirical domain is policymaking environments that are “healthy” in two particular senses – (1) there exists at least some common ground between competing ideological actors in the form of policy attributes that they all value (i.e., a shared notion of “good policy”), and (2) these actors are both able and willing to channel their ideological disagreements into productive investments in these attributes. A growing literature in political science applies a variety of models with fundamentals akin to our own to a range of policymaking environments, thereby implicitly or explicitly arguing that these environments (at least sometimes) exhibit these features. An early example is Londregan (2000), who posited that competing branches of the Chilean government “weigh policy alternatives in terms of ideology, about which they agree, and on the basis of shared public policy values, such as the desire for efficiency.” Subsequent research posited that such features are also present during bargaining within and across courts (Clark and Carrubba (2012); Lax and Cameron

2In the terminology of Baye, Kovenock and Vries (2012) the model features a rank order spillover.
(2007); Strayhorn (2020)) (with opinion attributes like “persuasiveness, clarity, and craftsmanship” valued by all judges); Congressional delegation to the bureaucracy (Huber and McCarty (2004); Ting (2011)) (with “effective implementation” in the sense of “whether regulations are enforced, revenues are collected, benefits are distributed, and programs are completed” valued by both legislators and bureaucrats); judicial oversight of the bureaucracy (Turner (2017)) (with “policy precision” valued by both risk-averse judges and bureaucrats); and legislative policymaking (Hirsch and Shotts (2012); Hitt, Volden and Wiseman (2017)) (with the “costs and benefits [of policies] across an array of societally valued criteria” being valued similarly by all legislators).

In addition to these settings, we argue that our model and related ones may be fruitfully applied to a variety of other policymaking environments. One is decisionmaking in small groups like Presidential cabinets (see also Hirsch and Shotts (2018)); when facing a specific crisis or immediate policy dilemma, a chief executive typically solicits proposals for how to proceed from his or her cabinet secretaries. These secretaries have a vested interest in good policy and the success of the administration, but also competing personal beliefs and/or interests for their respective departments (Kearns (2005)). Another is competing government agencies pursuing a shared goal, such as branches of the military. A nation facing an immediate security crises may seek to achieve a specific military goal; the joint chairman of the various service branches and/or the civilian leadership can solicit proposals from them. The branches all value successful military outcomes, but also seek to further the resources and parochial interests of their respective branches (Zimmerman et al. (2019)). A third is policymaking in one-party states. An important principle of decisionmaking in communist regimes is “democratic centralism,” or the idea that diverse interests within the party should be free to make proposals for how to deal with a policy problem without fear of reprisal, but a central leader then has the authority to implement a final policy decision that all party members must obey (Angle (2005)).

Finally, clarifying the features of a policymaking environment that are associated with our model also helps to draw the boundaries of its empirical domain. For one, in some issue areas disagreement between competing actors may be so pathological that participants value the “quality” of ideologically-distant policies negatively. Consider for example the politics of reproductive rights; “pro-life” voters likely place an intrinsic negative value on attributes of a statute that “pro-choice” voters associate with “quality,” such as population coverage and cost-effectiveness. A simpler way of putting it is
that policy preferences in some issue domains may simply be “all ideology”; returning to the example of reproductive rights, the expertise needed to draft a statute legalizing abortion may be minimal, while the expertise needed to draft one that also substantially improves access at a low cost may be substantial, but both characteristics are “ideological” in the sense that actors on opposite sides of the political spectrum disagree strongly over their desirability. Clearly, the politics of policy competition in such settings will be very different than those characterized by our model.

In addition, there may be some issue domains where shared notions of quality are indeed present, but superseded by other (possibly strategic) considerations. For example, when policymaking is dynamic, implementing a high-quality policy “today” might improve one actor’s control over policymaking “tomorrow.” This can give a competing actor the incentive to “sabotage” the policy (in the sense of damaging quality that they genuinely value) to improve prospects for future control (Gieczewski and Li (2021); Hirsch and Kastellec (2022)). The applicability of our model requires that such destructive means of gaining political influence are absent, relatively costly to employ (as compared to the productive means we study), or prohibited by either formal rules or shared norms of governance. Thus, although many have argued that national parties in the United States presently exhibit asymmetric ideological extremism (e.g. McCarty (2015)), the applicability of our model to this phenomenon may be limited by the degree to which it has also been accompanied by increased electoral competitiveness (Lee (2016)), and a parallel “asymmetric” evolution of the parties’ relative commitments to ideological goals vs. “productive” governance (Grossmann and Hopkins (2016)).

The Model

Two developers, labelled $-1$ (left) and $1$ (right), develop policies for consideration by a decisionmaker (DM), labelled player 0. A policy $(\gamma, q)$ consists of an ideology $\gamma \in \mathbb{R}$ and a level of quality $q \in [0, \infty) = \mathbb{R}^+$. All players are purely policy-motivated, in the sense that their final policy payoffs depend only on the ideology and quality of the implemented policy. The utility of player $i \in N$ for a policy $(\gamma, q)$ is $U_i(\gamma, q) = \lambda q - (\gamma - i \cdot x_i)^2$. The expression $i \cdot x_i$ is player $i$’s ideological ideal point; the decisionmaker is located at 0, the left developer is distance $x_{-1}$ to her left, and the right developer is distance $x_1$ to her right. A developer’s distance $x_i$ from the decisionmaker reflects his ideological extremism. A
policy’s quality \( q \) is a public good that all players value at weight \( \lambda \); higher \( \lambda \) thus means that the players collectively care more about policy quality vs. ideology.

The game proceeds in two stages. In the first, the developers simultaneously select the ideology and quality their respective policies \((\gamma_i, q_i)\). Endowing a policy with quality \( q_i \) costs \( c_i (q_i) = a_i q_i \) up front, which reflects the initial time and energy needed to improve a policy’s quality. The parameter \( a_i \) is developer \( i \)'s marginal cost of endowing his policy with additional quality, and reflects his ability at doing so. \( \alpha_i = \frac{a_i}{\lambda} \) denotes the ratio of a developer’s marginal cost of quality to its marginal benefit, which captures the net cost of investing in quality once its intrinsic value is taken into consideration. We assume this is greater than 1 for both developers, implying that neither would invest in quality for its intrinsic value alone. In the second stage the DM chooses a policy to implement. This may be one of the two new policies created by the developers, or any other policy from a (possibly empty) set of outside options \( \emptyset \) of policies no better than the DM’s ideal point with 0-quality. This captures the idea that the DM has the power to freely choose policy, but not the capacity to develop it.

**Preliminary Analysis**

**The Monopolist’s Problem** It is helpful to first analyze the model with only one policy developer, i.e. a “monopolist” (see also Hitt, Volden and Wiseman (2017) and Hirsch and Shotts (2018)). W.l.o.g. suppose he is the right developer \((i = 1)\). The “monopolist’s problem” is depicted in the left panel of Figure 1; ideology is on the x-axis and quality is on the y-axis. The shaded region depicts the set of new policies that the decisionmaker would be willing to implement in lieu of \((\gamma_0, q_0)\), which we use to denote the best policy she can implement without the developer’s help.3

The developer’s job is to choose both whether to develop a new policy that the decisionmaker is willing to implement (in the shaded region), and if so exactly which policy \((\gamma, q)\) to develop. This

---

3This exposition assumes that the decisionmaker’s best outside option \((\gamma_0, q_0)\) is at least as good for her as her ideal point with 0 quality, i.e. \(q_0 - \gamma_0^2 \geq 0\) (also assumed in Hirsch and Shotts (2018)). With a monopoly developer, this assumption is shorthand for an “open rule” procedure in which the decisionmaker can choose any 0-quality policy in lieu of the developer’s policy. The monopoly analysis herein is thus effectively a generalization allowing for positive-quality status quo of the “open rule” model studied in Hitt, Volden and Wiseman (2017) Prop. 3. Alternatively, when the rule is closed and there is a monopoly developer, there are additional cases to consider in which the developer proposes a 0-quality policy that the decisionmaker cannot access on her own in lieu of developing a new one; see Hitt, Volden and Wiseman (2017) Prop. 2. Finally, in the competitive model it is always “as if” the rule is open, since the developers have opposing ideologies, and one may always “develop” the decisionmaker’s ideal point with 0-quality.
problem can be understood using the inequality

$$\arg\max_{\{(q, y) : q - \gamma^2 \geq q_0 - \gamma^2\}} \left\{ \left( \lambda q - (\gamma - x_1)^2 \right) - a_1 q \right\} \geq \lambda q_0 - (\gamma_0 - x_1)^2$$

(1)

The policy \((q, \gamma)\) maximizing the left hand side is optimal if the developer chooses to act, and depends on both the developer’s ideology \(x_1\) and ability \(a_1\). Whether he will act in turn depends on whether the left hand side (his utility from the developing the optimal policy) exceeds the right hand side (his utility from the decisionmaker’s “outside option” \((\gamma_0, q_0)\)). The outside option appears on both sides of the inequality because it functions as both a constraint on and motive for policy development. It is a constraint because the developer must craft something at least as good as \((\gamma_0, q_0)\) for it to be adopted; the higher is the green curve (the decisionmaker’s indifference curve) passing through \((\gamma_0, q_0)\), the harder it is to “beat.” It is a motive because the developer must live with \((\gamma_0, q_0)\) if he doesn’t develop something else; outside options on the same green curve are equally difficult to “beat,” but those further to the left are worse for the developer, and more strongly incentivize policy development.

To solve this problem and aid in the subsequent analysis, it is helpful to rewrite policies \((\gamma, q)\) in terms of their ideology \(\gamma\) and the utility they give the decisionmaker – we henceforth call this a policy’s score, and denote it as \(s\). Observing that \(s = \lambda q - \gamma^2\) implies that a score-\(s\) policy with ideology \(\gamma\) must have quality \(q = \frac{s + \gamma^2}{\lambda}\). Substituting into the monopolist’s problem yields the revised problem:

$$\arg\max_{\{(s, \gamma) : s \geq s_0\}} \left\{ - (a_1 - 1) s + \frac{2x_1 \gamma - a_1 \gamma^2}{\alpha} \right\} \geq s_0 + 2\gamma_0 x_1,$$

(2)
where \( s \) and \( s_0 \) are the score of the developer’s new policy and the decisionmaker’s outside option, respectively. Now it is straightforward to see that if the developer develops a new policy, it will optimally be no better for the decisionmaker than her outside option \( (s^* = s_0) \), since quality is insufficiently valuable to the developer to create for its own sake \( (\alpha_1 > 1) \). The optimal ideology \( \gamma^* \) for the policy then trades off the ideological benefit \( 2\alpha_1 \gamma \) of a more extreme policy against the up-front cost \( \alpha_1 \gamma^2 \) of “compensating” the decisionmaker for it with additional quality, yielding \( \gamma^* = \frac{s_1}{\alpha_1} \).

**The Competitive Problem** When there is a second developer, the decisionmaker’s outside option \((s_0, \gamma_0)\) may no longer be some exogenous policy, but instead a policy \((s_{-1}, \gamma_{-1})\) developed by a competitor. The setup of the competitive problem is depicted in the right panel of Figure 1. A developer’s policy choice is a two-dimensional “bid” \((s_i, \gamma_i)\) consisting of the policy’s score \( s_i \) and ideology \( \gamma_i \). After seeing the two policies, the decisionmaker then chooses the one with the highest score (i.e., on the highest indifference curve in Figure 1), provided that it exceeds the score of her best outside option \((0, 0)\) (which is normalized to 0). The developers thus effectively compete in a “contest” over policy-development, where a policy’s likelihood of “winning” is determined exclusively by its score, but its ideology affects both its up-front development cost and its value to the developers.

Now recall that in the monopoly problem, a developer will choose to either develop no policy, or one no better than the decisionmaker’s outside option. Applying this insight to the competitive model straightforwardly implies that there is no pure strategy equilibrium. Were each developer \( i \) to expect a specific policy \((s_{-i}, \gamma_{-i})\) from her competitor \(-i\), each would treat the other’s policy like the outside option in the monopoly model, and equilibrium would require that the developers be crafting policies with the same score. But if both policies had the same score, then at least one developer would have a strict incentive to break the “tie,” either by developing a policy with a slightly higher score, or by dropping out of policy development entirely.

**Equilibrium**

The model features a unique equilibrium that is in mixed strategies; the developers randomize both over whether they develop a new policy, as well as the exact ideology and quality of the policy they develop. The unique equilibrium strategies are as follows; the derivation and all subsequent proofs are contained in the Online Appendix. All proofs are analytical.
Proposition 1. For each developer $i \in \{-1, 1\}$, define the strictly decreasing function

$$
\epsilon_i (p) = \int_p^1 \frac{x_i}{\alpha_i - q} \, dq = x_i \log \left( \frac{\alpha_i - p}{\alpha_i - 1} \right).
$$

Let $p_i(e) = \alpha_i - (\alpha_i - 1) e^{\frac{x_i}{e}}$ denote the well-defined inverse of $\epsilon_i (p)$, and let $k$ denote the developer with the smallest value of $\epsilon_i (0)$.

- **The probability developer $i$ develops a policy with ideology closer to the DM than distance $y$ is**

$$
F^Y_i (y) = p_{-i} \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right) = \alpha_{-i} - (\alpha_{-i} - 1) \left( \frac{x_i - y}{x_i - x_i/\alpha_i} \right)^{\frac{x_i}{x_i - \alpha_i}}
$$

- **When developing a policy whose ideology is distance $y$ from the DM, developer $i$ targets ideology**

$$
\gamma_i (y) = iy \text{ and invests quality } q^Y_i (y) = \frac{y^2 + s_i^Y (y)}{\lambda}, \text{ where }
$$

$$
s^Y_i (y) = 2 \int_{\epsilon_{i(y)}}^{\epsilon_{i(y)}} \left( \sum_{j \in \{-1, 1\}} \frac{x_j}{\alpha_j} p_j (\epsilon) \right) d\epsilon
$$

Although the equilibrium strategies are straightforward to express and compute, they are hard to interpret from the equations alone. We therefore describe the structure of equilibrium, alongside six key properties, using an example. Figure 2 depicts equilibrium strategies when the developers are equally skilled ($a_1 = a_{-1}$), but the right developer is more extreme ($x_1 > x_{-1}$). The left panel depicts the ideology and quality of the policies that the left (purple) and right (blue) developers randomize over. The decisionmaker’s indifference curves are in gray. The right panel depicts the probability distributions (PDFs) governing the ideology of the left (purple) and right (blue) developer’s policies. When a developer chooses to develop a new policy, that policy’s ideology is continuously distributed over an interval with the depicted density. In the example, the left developer also sometimes chooses to develop no new policy; this is depicted in the left panel by the purple dot at the origin (the DM’s ideal point with zero quality). The probability that this occurs is depicted in the right panel by the height of the thick purple segment. Finally, the density describing the ideology of the final policy chosen by the DM is depicted by the gray dashed lines.
Properties of Equilibrium

1. Uncertainty  Although the game is of complete and perfect information, in the unique equilibrium each developer is uncertain about exactly which ideology their competitor will target, how much they will invest in its quality, and whose policy will be chosen. This uncertainty remains regardless of how asymmetric the developers are – there is always some chance that the winning policy comes from either developer. This fundamental unpredictability arises from the need for both developers to remain competitive in the policy process when each can ensure victory over a known competing policy by investing sufficiently in quality and/or making enough ideological concessions.

2. Asymmetric Participation  The unique equilibrium generically exhibits asymmetric participation in the policy process. Specifically, one of the two developers (in the example the right developer) is always active, in the sense of developing a new policy with strictly positive quality. The DM will thus always receive at least one new policy. The other developer (in the example the left developer, and more generally developer $k$ defined in Proposition 1) is only sometimes active; with strictly positive probability he develops nothing. This asymmetry in participation arises from differences in the developers’ underlying extremism and/or ability at crafting high quality policies.

3. Ideological Divergence  The unique equilibrium exhibits ideological divergence in the policies developed. Specifically, when a developer chooses to develop a new policy, that policy’s ideology strictly diverges from the DM’s ideal. (In Figure 2, all positive-quality policies have divergent ideolo-
gies). Active participation in the policy process is thus always accompanied by an attempt to extract “ideological rents” in the form of a policy closer to one’s ideal than the DM ideal. This property arises because quality is insufficiently valuable to the developers to create it for its own sake.

4. Unequal Benefits and Costs In equilibrium, new policies always diverge from the DM’s ideal, and thus entail an ideological cost. It is therefore not obvious whether, and when, the DM actually benefits from them. Indeed, when there is only a single developer the DM does not benefit at all, since a monopolist extracts all the benefits of quality in the form of ideological rents. Competition, however, strictly benefits the DM – with certainty the DM will receive at least one new policy that is strictly better than her outside option. This holds regardless of the developers’ characteristics, and even when one developer is very unlikely to be active. This can be seen in Figure 2 by observing that all positive-quality policies are strictly above the DM’s indifference curve through the origin, combined with the fact that one developer always develops a new policy.

It is also not obvious whether the developers benefit from or are harmed by competition – a competitor develops ideologically-unappealing policies, but gains support for them by making productive investments in quality. Despite this, competition turns out to strictly harm the developers, in that each would strictly prefer to be the only one developing policy. The reason is that the developers invest in enough quality to compensate the DM for her ideological losses, but not enough to compensate their competitor.

5. Inefficiency Although competition benefits the decisionmaker, it is also inefficient in several ways. First, the expected ideology of the chosen policy generically differs from both the DM’s ideal, as well as the ideology that maximizes aggregate utility. Second, the ideology of the chosen policy is uncertain ex-ante, which harms all participants in the process due to risk aversion. (Its distribution is depicted by the dashed grey lines in the right panel of Figure 2 – any particular location has zero probability). Finally, because the developers must make quality investments before they know which policy will ultimately be chosen, all of the effort invested in the losing policy is wasted. These inefficiencies all arise from the fact that policy is developed and chosen via a “contest,” rather than an orderly process designed by the decisionmaker, with an ex-ante commitment to whose policy she will choose and under what conditions.
6. A Bias Toward Extremism  In equilibrium, the developers naturally invest more in quality when developing more extreme policies so they remain competitive \( q_i^y(y) > 0 \). More surprising, however, is that the developers actually invest so much more in the quality of more-extreme policies that they are also more appealing overall to the DM when they are developed, and therefore more likely to be chosen. (In the left panel of Figure 2, the developers’ quality functions are steeper than the DM’s indifference curves). A surprising implication is thus that the process will appear to be biased towards extremism, in the sense that more extreme policies will be more likely to be chosen than more moderate ones. A further consequence is that the ideology of the chosen policy (distributed according to the gray dashed line in the right panel of Figure 2) will be even more extreme than the developers’ initial policy proposals. The reason for this counterintuitive effect is that the developers are strategic when choosing to develop a more extreme policy. Specifically, when choosing their policy’s ideology, a developer trades off the benefits of winning with a more-extreme policy against the costs generating the quality necessary to make such a policy competitive. When a developer chooses to craft a policy that is better for the DM, ideological concessions effectively become a costlier way to gain her support relative to policy concessions, because the policy is more likely to actually become the final outcome. Reversing the statement, a policy that makes fewer concessions to the DM (i.e. that is more ideologically extreme) must also be more appealing to the DM and thus more likely to be chosen.

The Politics of Asymmetric Extremism

We now turn to the politics of asymmetric extremism. We begin by characterizing equilibrium when the developers are equally capable \( (\alpha_1 = \alpha_{-1}) \) but one is more ideologically extreme \( (x_i \neq x_{-i}) \). (We call the more extreme developer “the extremist” and the other “the moderate.”)

**Proposition 2.** If the developers are equally skilled but \( i \) is more extreme \( (x_i \geq x_{-i}, \alpha_i = \alpha_{-i}) \),

- the extremist always develops a new policy, while the moderate only sometimes does
- the extremist’s policies are first-order stochastically more extreme than the moderate’s policies, but also first-order stochastically higher quality and better for the decisionmaker
- the extremist’s policy is strictly more likely to be chosen
Recall that equilibrium strategies with asymmetric extremism are depicted in Figure 2.

Proposition 2 first states that asymmetric extremism results in imbalanced participation – the extremist always develops a new policy, but the moderate (who is better aligned with the decisionmaker) sometimes declines to do so. The extremist also develops first-order stochastically more extreme policies than the moderate. Surprisingly, however, these policies perform better in the contest because they are so much higher quality so as to be first-order stochastically better for the decisionmaker despite their greater extremism. What explains an ideological extremist’s dominance of the policy process despite his ideologically-extreme policy proposals? Simply put, it is because the extremist is also more motivated to craft policies that are sufficiently high quality to prevent his opponent’s policy from being chosen.

We next examine what happens to the developers’ policies when one becomes more extreme.

**Proposition 3.** If developer $i$ becomes more ideologically extreme (higher $x_i$), his own strategy and his opponent’s strategy are affected in the following ways:

**(Own strategy)**

- if he previously did not always develop a policy, he becomes strictly more likely to do so
- his policies become first-order stochastically more extreme
- his policies become first-order stochastically higher quality, better for the decisionmaker, and more likely to be chosen

**(Opponent’s strategy)**

- if he did not always develop a policy, he becomes strictly less likely to do so
- his policies become first-order stochastically more moderate

Although the above comparative statics apply to any configuration of preferences and costs, they are easiest to discuss in the special case of purely asymmetric extremism ($\alpha_i = \alpha_{-1}$).

The effects of developer $i$ becoming more extreme on his own behavior are quite natural. If he is initially the moderate, then he becomes strictly more likely to be active; the reason is that greater
competitive balance results in greater overall participation. Regardless of whether he is initially the moderate or the extremist, his policies also become (first-order stochastically) more extreme, but also higher quality and better for the decisionmaker. This reflects his greater motivation to both win, and to win with more extreme policies. The effects of developer $i$ becoming more extreme on his opponent’s behavior are more subtle. If $i$ is initially the extremist, then his opponent becomes strictly less likely to be active; the reason is that less competitive balance results in less overall participation. Regardless of whether $i$ is initially the moderate or the extremist, his opponent also moderates his policies. This moderation is not motivated by the opponent’s desire to make his policies more competitive, but rather by his acceptance of the fact that he is less likely to win against an increasingly extreme developer, and therefore has a lower “return” on investing in quality for ideological gain.

We last examine the effect of a developer $i$ becoming unilaterally more extreme on the participants’ welfare, beginning with his competitor.

**Proposition 4.** If developer $i$ becomes more ideologically extreme (higher $x_i$), the equilibrium utility of his competitor $\neg i$ decreases

The effect of unilateral extremism on a competitor’s welfare is thus unambiguous – despite the greater quality of the extremist’s policies, the competitor is harmed. The reason is that the extremist’s greater investments in quality are insufficient to “compensate” his competitor for his greater extremism. However, the effects of unilateral extremism on the decisionmaker’s welfare are quite different.

**Proposition 5.** Unilateral changes in extremism have the following effects on the decisionmaker.

- **If the developers begin symmetric ($x_i = x_{\neg i}$, $\alpha_i = \alpha_{\neg i}$) and developer $i$ becomes more ideologically extreme, the decisionmaker’s utility locally increases**

- **As a developer becomes increasingly extreme ($x_i \to \infty$), the competitor’s probability of developing a policy approaches 0, but the decisionmaker’s utility approaches infinity**

While characterizing the precise effect of unilateral extremism on the decisionmaker’s welfare is difficult at any particular set of parameters, the general relationship is both simple and striking; the decisionmaker strongly benefits. Specifically, if the developers begin symmetric and one becomes unilaterally more extreme, the decisionmaker unambiguously benefits despite the resulting imbalance in
participation. Moreover, as one developer becomes increasingly extreme, his opponent becomes increasingly unlikely to participate, but the decisionmaker also becomes increasingly – and unboundedly – better off. Unilateral extremism thus strongly benefits the decisionmaker, even as it also decreases political participation. These results are a striking contrast from both standard models of political contests, and models of contests in general. For example, the all-pay contest – in which two bidders make costly up-front bids to win a fixed “prize” – has been widely applied to study political competition (e.g. Hillman and Riley (1989); Meirowitz (2008)). In that model, however, asymmetries in the developers’ desire to win always harm the decisionmaker because they cause the less-motivated developer to participate less, a widely-studied phenomenon known as the “discouragement effect” (Chowdhury, Esteve-Gonzalez and Mukherjee, 2020). In our contest over policy development, in contrast, the effect is reversed because the developers care about policy outcomes even when they lose. Specifically, an increasingly extreme developer also becomes increasingly fearful of his opponent’s potential policy, which preserves his willingness to invest in quality even though he becomes vanishingly unlikely to face any actual competition.

The Politics of Asymmetric Ability

We last examine the politics of asymmetric ability. We begin by characterizing equilibrium when the developers are equally extreme ($x_i = x_{-i}$), but one is more skilled at producing quality ($\alpha_i < \alpha_{-i}$). (We call the more capable developer “the expert” and the other “the amateur.”)

**Proposition 6.** If the developers are equally extreme but $i$ is more skilled ($x_i = x_{-i}, \alpha_i < \alpha_{-i}$), the equilibrium pattern of competition is identical to the pattern described in Proposition 2 when the developers are equally capable but $i$ is more extreme ($x_i > x_{-i}, \alpha_i = \alpha_{-i}$).

Surprisingly then, asymmetric ability and extremism turn out to be observationally equivalent. Equilibrium strategies with asymmetric ability are depicted in Figure 3. The expert exploits his greater ability at developing high quality policies to craft more competitive policies that also better reflect his ideological interests, consistent with the finding in Hitt, Volden and Wiseman (2017) that “more effective lawmakers” (i.e., with lower $\alpha_i$) “are more likely to offer successful proposals.”

\footnote{Specifically, if our model is altered so that each developer receives a fixed payoff from losing, the result vanishes.}
contrast, the amateur reacts by both disengaging from the process, and by moderating his own policy. The empirical implication is that observably-extreme behavior by one political faction may not actually reflect greater underlying extremism, but rather greater ability at crafting “good policies” that are appealing on non-ideological grounds. This observational equivalence between asymmetric extremism and ability extends to the consequences of one developer becoming more skilled.

**Proposition 7.** If developer i becomes more skilled (lower $\alpha_i$), his own strategy and his opponent’s strategy are affected in the same ways as when he becomes more ideologically extreme (higher $x_i$).

Thus, a developer i becoming more skilled (a) increases his own activity (if he was the amateur) and makes his policies more extreme, higher quality, and better for the decisionmaker, and (b) decreases his opponent’s activity (if he was the amateur) and makes his policies more moderate.

As before, we next examine the effect of unilaterally greater ability on an opponent’s welfare.

**Proposition 8.** If developer i becomes more skilled, the equilibrium utility of his competitor decreases.

The isomorphism between greater skill and extremism thus extends to this welfare effect. While anticipated by the preceding, this result is also substantively surprising because the skill in question is at making common value policy investments that benefit everyone. Indeed, this result is a striking demonstration of how “good policy” considerations cannot really be considered separately from “ideological” ones even if are theoretically distinct, because of how strategic ideological actors can exploit
them. Specifically, in our model an improvement in one developer’s ability would benefit everyone if it did not affect also his policy’s ideology. However, because a more skilled developer will exploit his skill to develop a more ideologically-extreme policy that remains competitive, in equilibrium that skill harms his opponent.

We conclude by examining how unilateral changes in ability effect the decisionmaker’s welfare.

**Proposition 9.** Unilateral changes in ability have the following effects on the decisionmaker.

- If the developers begin symmetric \( x_i = x_{-i}, \alpha_i = \alpha_{-i} \) and developer \( i \) becomes more skilled, then the decisionmaker’s utility locally increases.

- As a developer becomes increasingly skilled \( \alpha_i \to 1 \), the competitor’s probability of developing a policy approaches 0, but the decisionmaker’s utility approaches a strictly positive bound; this bound is strictly increasing in the inactive developer’s extremism and ability.

The overall pattern is thus that the decisionmaker benefits when a developer becomes unilaterally more skilled; even though this also brings more extreme policies from the expert and less overall participation. Specifically, if the developers begin symmetric and one becomes more skilled, the decisionmaker unambiguously benefits despite the resulting imbalance in participation. Balanced competition is thus always strictly worse from the decisionmaker than at least some asymmetry. Moreover, as one developer becomes increasingly skilled \( \alpha_i \to 1 \), the effect again resembles that of a developer becoming increasingly extreme \( x_i \to \infty \), but with some notable differences. As before, the “dominated” developer (here the amateur, formerly the moderate) is eventually driven out of the policy process, but the decisionmaker still concretely benefits from his potential participation (in the sense that her limiting utility does not approach her utility under monopoly). Rather than increasing unboundedly, however, the decisionmaker’s utility approaches a strictly positive value that depends on the traits of the amateur.\(^5\) Thus, even when the amateur is effectively driven out of the policy process, his traits exert a measurable effect on the expert’s behavior. Specifically, if an amateur who is vanishingly unlikely to participate becomes more extreme or skilled (but remains vanishingly unlikely to participate), the expert still reacts by crafting policies that are concretely better.

---

\(^5\)It is also true that if the developers begin symmetric and then developer \( i \) becomes arbitrarily skilled, the decisionmaker will be better off in the limit of complete asymmetry than she was with full symmetry as long as \( \alpha_{-i} \geq \alpha \approx 1.0435 \).
for the decisionmaker. The empirical implication is that the preferences and abilities of an “inactive” participant in the policy process may still exert outsized influence on political outcomes. This effect is natural in sequential models of competitive policy development (Hitt, Volden and Wiseman (2017); Lax and Cameron (2007)) because the equilibrium always involves one developer being “inactive.” However, it is quite unusual when the developers craft their policies simultaneously (so that both sometimes active in equilibrium), and once again stems from the fact that the developers care about policy outcomes even when they lose.

Conclusion

We have developed a model that explores the implications of strategic policy development by competing ideological actors with differing ideologies and abilities. We find that the process exhibits several intuitive patterns, including unequal participation, inefficiently unpredictable and extreme outcomes, wasted effort, and an apparent bias toward extreme policies. We explore the politics of asymmetric extremism and ability, and find that despite increasingly imbalanced policies and outcomes, and even in an apparent absence of competition against an ideologically extreme faction, a moderate decisionmaker may nevertheless strongly benefit (although at the expense of the extremist’s ideological competitor). The model thus provides rationale for how ideological extremism may come to dominate policymaking and even benefit moderate decisionmakers; one rooted in the nature of productive policy competition rather than dysfunction, bias, capture, or some other systemic failure. We further show that the politics of asymmetric ability are essentially observationally equivalent to those of asymmetric extremism.

We model costly policy development as an individual or group making an ideology-specific investment in quality that is valued by all participants in the policy process, in contrast to a large literature that models policy expertise as the acquisition of private knowledge about an “unknown state of the world.” Our model is intentionally sparse and lacking in institutional detail in order to be applicable to the wide variety of settings in which competitive policy development occurs, including but not limited to legislatures, bureaucracies, and courts. Broadening the interpretation of disagreement beyond “ideology” in a classical left-right sense, the model is applicable to any political environment where there are a mixture of competing and common interests, freedom among several individuals or groups.
to make proposals, and a decisionmaker who must make a single final choice.

Our analysis suggests several broad avenues for follow-on work. The first is to include additional elements that would make it better suited to studying specific settings. For example, what if there are not just two but many potential policy developers, as in a legislature? It is straightforward to show that when ability is common, there is always an equilibrium in which only the two most ideologically-extreme developers are active. However, there may be other equilibria with broader participation (see Baye, Kovenock and de Vries (1996)), and ability may be very unevenly distributed among potential policy developers (see Hitt, Volden and Wiseman (2017)). Or, what if the developers can also engage in unproductive or even destructive activities in conjunction with policy development? For example, they may try to bribe the decisionmaker with transfers, engage in unproductive advertising, lobbying, or grassroots mobilization; try to harm the reputation of their opponent or their opponent’s policy; or even actively sabotage its functioning. Finally, what if the competing policy developers are not individuals but teams (as in McCarty (2020)) – for example, aligned legislators and interest groups (Hall and Deardorff (2006)) – who have common ideological interests, but must figure out how to distribute the costs of developing high quality policies that achieve those interests among them?

The second avenue (following the classical literature on policy expertise) is to consider how political institutions can be designed in order to encourage the development of high quality policies. For example, what if the developers can actually be chosen by the decisionmaker, as in the literature on legislative committee composition (Krehbiel (1992))? What if there are existing policy developers – when would the decisionmaker want to subsidize their activities, and how? What if it is not the identities of the developers under consideration, but that of the decisionmaker, as in the Presidential appointment of an agency head to consider proposals from career staff and outside groups (Lewis (2008))? Finally, what if the decisionmaker is not a unitary actor but a collective choice body, as in a legislature? What sorts of collective choice rules will best encourage the development of high quality legislation? We hope to explore these and other avenues in future work.

References


Online Appendix

This Appendix is divided into two parts. Appendix A is a linear and self-contained treatment of the model with its own lemmas and propositions. Appendix B describes where to locate each result stated in the main text propositions in the general treatment in Appendix A.
A General Treatment

We begin with a slightly more general formulation of the model than stated in the main text. Two proposers labelled −1 (left) and 1 (right) develop competing policies for consideration by a decision-maker (DM), labelled player 0. A policy (γ, q) consists of an ideology γ ∈ R and a level of quality q ∈ [0, ∞) = R+. Utility over proposals takes the form

\[ U_i (\gamma, q) = \lambda q - (\gamma - X_i)^2, \]

where \( X_i \) is player i’s ideological ideal point, and \( \lambda \) is the weight all players place on quality. The proposers’ ideal points are on either side of the decisionmaker (\( X_{-1} < X_D < X_1 \)).

The game is as follows. First, the proposers simultaneously choose proposals \((\gamma_i, q_i)\). Making a proposal with quality \( q_i \) costs \( c_i(q_i) = a_iq_i \) up-front, where \( a_i > \lambda \). Second, the DM chooses one of the two proposals or something else from an exogenous set of outside options \( \Omega \), where \( \Omega \) may contain the DM’s ideal point with no quality \((0,0)\) and/or proposals that are strictly worse (and can be empty).

A.1 Preliminary Analysis

The game is a multidimensional contest in which the scoring rule applied to “bids” \((\gamma, q)\) is just the DM’s utility \( U_D (\gamma, q) = \lambda q - (X_D - \gamma)^2 \). To facilitate the analysis we thus reparameterize proposals \((\gamma, q)\) to be expressed in terms of \((s, y)\), where \( y = \gamma - X_D \) is the (signed) distance of a proposal’s ideology from the DM’s ideal, and \( s = \lambda q - y^2 \) is the DM’s utility for a proposal, or its score. The implied quality of a proposal \((s, y)\) is then \( q = \frac{s+y^2}{\lambda} \). Using this we re-express the proposers’ utility and cost functions in terms of \((s, y)\). Note that the decisionmaker’s ideal point with 0-quality has exactly 0 score, and is the most competitive “free” proposal to make.

Definition A.1.

1. Player i’s utility for proposal \((s, y)\) is

\[ V_i (s, y) = U_i \left( y + X_D, \frac{s + y^2}{\lambda} \right) = -x_i^2 + s + 2x_iy \]
where \( x_i = X_i - X_D \) is the (signed) distance of i’s ideal from the DM.

2. Proposer i’s cost to make proposal \((s, y)\) is

\[
c_i \left( \frac{s + y^2}{\lambda} \right) = \frac{a_i}{\lambda} (s + y^2) = \alpha_i (s + y^2)
\]

where \( \alpha_i = \frac{a_i}{\lambda} \) is i’s weighted marginal cost of generating quality.

Definition 1 reparameterizes proposals into score and ideological distance (henceforth just ideology) \((s, y)\), and the five primitives \((X_i, a_i, \lambda)\) into four parameters \((x_i, \alpha_i)\) describing the proposers’ (signed) ideal ideological distance from the DM \(x_i = X_i - X_D\) (henceforth just ideal ideology) and weighted marginal costs of generating quality \(\alpha_i = a_i \lambda\) (henceforth just costs). Note that this notation differs *slightly* from the main text, where \(x_i\) and \(y\) denote the *unsigned* distance of a proposer’s ideal point and an ideological location from the DM.

### A.1.1 Necessary and Sufficient Equilibrium Conditions

In the reparameterized game, a proposer’s pure strategy \((s_i, y_i)\) is a two-dimensional element of

\[ \mathcal{B} \equiv \{(s, y) \in \mathbb{R}^2 | s + y^2 \geq 0\} \]

A mixed strategy \(\sigma_i\) is a probability measure over the Borel subsets of \(\mathcal{B}\), and let \(F_i(s)\) denote the CDF over scores induced by i’s mixed strategy \(\sigma_i\).\(^6\)

We now derive necessary and sufficient equilibrium conditions in a series of four lemmas. Let \(\Pi_i(s_i, y_i; \sigma_{-i})\) denote i’s expected utility for making proposal \((s_i, y_i)\) with \(s_i \geq 0\) if a tie would be broken in her favor. Clearly this is i’s expected utility from making a proposal at any \(s_i > 0\) where \(-i\) has no atom, and \(i\) can always achieve utility arbitrarily close to \(\Pi_i(s_i, y_i; \sigma_{-i})\) by making \(\varepsilon\)–higher score proposals. Now \(\Pi_i(s_i, y_i; \sigma_{-i}) =

\[ -\alpha_i (s_i + y_i^2) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_{-i} > s_i} V_i(s_{-i}, y_{-i}) \, d\sigma_{-i}. \]  

(A.1)

The first term is the up-front cost of generating the proposal’s quality. The second term is the probability i’s proposal is selected, times her utility for it. The third term is i’s utility should she

\(^6\)For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.
lose, which requires integrating over all her opponent’s proposals with higher score than $s_i$. Taking the derivative with respect to $y_i$ yields the first Lemma.

**Lemma A.1.** At any score $s_i > 0$ where $F_{-i}(\cdot)$ has no atom, the proposal $(s_i, y_i^*(s_i))$, where $y_i^*(s_i) = F_{-i}(s_i) \cdot \frac{\alpha_i}{\alpha_i}$, is the strictly best score-$s_i$ proposal.

**Proof:** Straightforward. QED

Lemma A.1 states that at almost every score $s_i > 0$, proposer $i$’s unique best combination of ideology and quality to generate that score is just a weighted average of the proposer’s and DM’s ideal ideologies $\frac{\alpha_i}{\alpha_i}$, multiplied by the probability $F_{-i}(s_i)$ that $i$’s opponent makes a lower-score proposal. Note that $i$’s optimal ideology does not depend directly on her opponent $-i$’s ideologies, since a proposal’s ideology (holding score fixed) only matters conditional on winning. The optimal ideology also depends on the exact score $s_i$ only indirectly through probability $F_{-i}(s_i)$ the proposal wins the contest, since $i$’s utility conditional on winning is additively separable in score and ideology.

The second lemma establishes that at least one of the proposers is always active, in the sense of making a proposal with strictly positive score (all positive-score proposals are positive-quality, but the reverse is not necessarily true).

**Lemma A.2.** In equilibrium $F_k(0) > 0$ for at most one $k \in \{L, R\}$.

**Proof:** Suppose not, so $F_i(0) > 0 \forall i$ in some equilibrium. Let $U_i^*$ denote proposer $i$’s equilibrium utility, which can be achieved by mixing according to her strategy conditional on making score-$s \leq 0$ proposal. Let $\bar{y}^0$ denote the expected ideological outcome and $\bar{s}^0$ the expected score outcome conditional on both sides making score $\leq 0$ proposals. Since $x_L < 0 < x_R$, we have $V_k(\bar{s}^0, \bar{y}^0) \leq V_k(0,0)$ for at least one $k$, which implies $k$ has a profitable deviation since $U_k^* \leq \bar{\Pi}_k(0,0; \sigma_{-k}) < \bar{\Pi}_k(0, y_k^*(0); \sigma_{-k})$ (since $F_{-k}(0) > 0$). QED

The third Lemma establishes that in equilibrium there is 0 probability of a tie at a positive score.

**Lemma A.3.** In equilibrium there is 0-probability of a tie at scores $s > 0$.

**Proof:** Suppose not, so each proposer’s strategy generates an atom of size $p_i^s > 0$ at some $s > 0$. Proposer $i$ achieves her equilibrium utility $U_i^*$ by mixing according to her strategy conditional on a
score-s proposal. Let $\bar{y}^s$ denote the expected ideological outcome conditional on both sides making score-s proposals; then $V_k(s, \bar{y}^s) \leq V_k(s, 0)$ for at least one $k$, who has a profitable deviation. If $k$’s proposal at score $s$ is $(s, 0)$, then $U_k^s \leq \Pi_k(s, 0; \sigma_{-k}) < \Pi_k(s, y_k^s(s); \sigma_{-k})$ (since $F_{-k}(s) > 0$). If $k$ sometimes proposes something else, then $U_k^s < \left( 1 - \frac{p_{-k}}{F_{-k}(s)} \right) \Pi_k(s, E[y_k|s]; \sigma_{-k}) + \left( \frac{p_{-k}}{F_{-k}(s)} \right) \Pi_k(s, 0; \sigma_{-k})$, which is $k$’s utility if she were to instead propose $(s, 0)$ with probability $\frac{p_{-k}}{F_{-k}(s)}$, and the expected ideology $E[y_k|s]$ of her strategy at score $s$ with the remaining probability (and always win ties). QED

Lemmas A.1 – A.3 jointly imply that in equilibrium, proposer $i$ can compute her expected utility as if her opponent only makes proposals of the form $(s_{-i}, y_{-i}^s(s_{-i}))$. The utility from making any proposal $(s_i, y_i)$ with $s_i > 0$ where $-i$ has no atom (or a tie would be broken in $i$’s favor) is therefore

$$\Pi_i^*(s_i, y_i; F) = -\alpha_i(s_i + y_i^2) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_i}^{\infty} V_i(s_{-i}, y_{-i}^*(s_{-i})) dF_{-i}.$$  \hspace{1cm} (A.2)

Proposer $i$’s utility from making the best proposal with score $s_i$ is $\Pi_i^*(s_i, y_i^*(s_i); F)$, which we henceforth denote $\Pi_i^*(s_i; F)$.

Fourth and finally, we establish that equilibrium score CDFs must satisfy the following natural properties arising from the all pay component of the contest.

**Lemma A.4.** Support of the equilibrium score CDFs over $\mathbb{R}^+$ is common, convex, and includes 0.

**Proof:** We first argue $\hat{s} > 0$ in support of $F_i \rightarrow \Pi_{-i}(s) < F_{-i}(\hat{s}) \forall s < \hat{s}$. Suppose not; so $\exists s < \hat{s}$ where $-i$ has no atom and $F_{-i}(s) = F_{-i}(\hat{s})$. Then $\Pi_i(s, y_i; F) - \Pi_i(s, y_i^*; F) = -(\alpha_i - F_{-i}(\hat{s})) \cdot (\hat{s} - s) < 0$, implying $i$’s best score-$s$ proposal is strictly better than her best score-$\hat{s}$ proposal, a contradiction. We now argue this yields the desired properties. First, an $\hat{s} > 0$ in $i$’s support but not $-i$ implies $\exists \delta > 0 \text{ s.t. } F_{-i}(s - \delta) = F_{-i}(s)$. Next, if the common support were not convex or did not include 0, then there would $\exists \hat{s} > 0$ in the common support s.t. neither proposer has support immediately below, so $F_i(s) < F_i(\hat{s}) \forall i, s < \hat{s}$ would imply both proposers have atoms at $\hat{s}$, a contradiction. QED

We conclude by combining the preceding lemmas to state a preliminary characterization of all equilibria in the form of necessary and sufficient conditions.
Proposition A.1. Necessary conditions for SPNE are as follows:

1. **(Ideological Optimality)** With probability 1, proposals are either
   
   (a) negative score $s_i \leq 0$ and 0-quality $(s_i + y_i^2 = 0)$
   
   (b) positive score $s_i > 0$ with ideology $y_i = y_i^*(s_i) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s_i)$.

2. **(Score Optimality)** The profile of score CDFs $(F_i, F_{-i})$ satisfy the following boundary conditions and differential equations.

   - **(Boundary Conditions)** $F_k(0) > 0$ for at most one proposer $k$, and there $\exists \bar{s} > 0$ such that $\lim_{s \to \bar{s}} \{F_i(s)\} = 1 \ \forall i$.
   
   - **(Differential Equations)** For all $i$ and $s \in [0, \bar{s}]$,
     
     $$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i (y_i^*(s) - y_{-i}^*(s))$$

The above and $F_i(s) = 0 \ \forall i, s < 0$ are sufficient for equilibrium.

**Proof:** **(Score Optimality)** A score $\bar{s} > 0$ in the common support implies $[0, \bar{s}]$ in the common support (by Lemma A.4) implying $\lim_{s \to \bar{s}-} \{\Pi_i(s; F)\} \geq U_i^*$. Equilibrium also requires $\Pi_i(s; F) \leq U_i^*$ $\forall s$ so $\Pi_i(s; F) = U_i^* \forall s \in [0, \bar{s}]$, further implying the $F$’s are absolutely continuous over $(0, \infty)$ (given our initial assumptions), and therefore $\frac{\partial}{\partial s}(\Pi_i^*(s; F)) = 0$ for almost all $s \in [0, \bar{s}]$. This straightforwardly yields the differential equations for score optimality, with the boundary conditions implied by Lemma A.4. **(Ideological Optimality)** At most one proposer $(k)$ makes $\leq 0$-score proposals with positive probability, so $F_{-k}(0) = 0$. Such proposals lose for sure and never influence a tie, and therefore must be 0-quality with probability 1, yielding property (a). Atomless score CDFs $\forall s > 0$ implies $(s, y_i^*(s))$ is the strictly best score-$s$ proposal (by Lemma A.1), yielding property (b). **(Sufficiency)** Necessary conditions imply all $(s, y_i^*(s))$ with $s \in (0, \bar{s}]$ yield a constant $U_i^*$. $F_{-k}(0) = 0$ implies $k$’s strictly best score–0 proposal is $(0, y_k^*(0)) = (0, 0)$ and yields $\Pi_k(0; F)$, and $F_k(s) = 0$ for $s < 0$ implies $k$ has a size $F_k(0)$ atom here. Thus both proposers’ mixed strategies yield $U_i^*$, and neither can profitably deviate to $s \in (0, \bar{s}]$. To see neither can profitably deviate to $s > \bar{s}$, observe $\Pi_i^*(s; F) - \Pi_i^*(\bar{s}; F) = -(\alpha_i - 1)(s - \bar{s}) < 0$. To see $k$ cannot profitably deviate to $s_k \leq 0$, $F_{-k}(0) = 0$
implies such proposals lose and never influence a tie, and so yield utility \( \leq U^*_k \). To see \(-k\) cannot profitably deviate to \(s_{-k} \leq 0\), observe all such proposals result in either \((0, y_{-k})\) or \((0, 0)\) when \(s_k \leq 0\) (since the DM’s other choices are \((0, 0)\) and \(\varnothing\)), and thus yield utility \( \leq \max \{ \Pi_{-k} (0, 0; F), \Pi_{-k} (0, y_{-k}; F) \} \) which is \( \leq U^*_{-k} \). QED

### A.1.2 Preliminary Observations about Equilibria

Proposition A.1 implies that all equilibria have a simple form. At least one proposer (henceforth labelled \(-k\)) is always active – thus, competition not only strictly benefits the DM in expectation, but with probability 1. The other proposer (henceforth labelled \(k\)) may also always be active \((F_k (0) = 0)\), or be inactive with strictly positive probability \((F_k (0) > 0)\). Inactivity may manifest as proposing the DM’s ideal point with no quality \((0, 0)\), or as “position-taking” with more distant 0-quality proposals that always lose \((s_k < 0 \text{ and } s_k + y_k^2 = 0)\). However, any equilibrium exhibiting the latter is payoff-equivalent to one exhibiting the former; we thus focus on the former for comparative statics.\(^7\)

When either proposer \(i\) is active, she mixes smoothly over the ideologically-optimal proposals \((s, y_i^*(s)) = \left( s, \frac{x_i}{\alpha_i} F_{-i} (s) \right) \) with scores in a common mixing interval \([0, \bar{s}]\) according to the CDF \(F_i (s)\).\(^8\)

The differential equations characterizing the equilibrium score CDFs arise intuitively from the proposers’ indifference condition over \([0, \bar{s}]\). The left hand side is \(i\)’s net marginal cost of making a higher-score proposal, given a fixed probability \(F_{-i} (s)\) of winning the contest; the proposer pays marginal cost \(\alpha_i > 1\) for sure, but with probability \(F_{-i} (s)\) her proposal is chosen and she enjoys a marginal benefit of 1 (because she values quality). The right hand side represents \(i\)’s marginal ideological benefit of increasing her score. Doing so increases by \(f_{-i} (s)\) the probability that her proposal wins, which changes the ideological outcome from her opponent’s optimal ideology \(y_{-i}^* (s)\) at score \(s\) to her own optimal ideology \(y_i^* (s)\).

\(^7\)Profiles with “position-taking” are equilibria if the position-taking does not invite a deviation by \(-k\) to negative scores; whether this is the case depends on \(k\)’s score-CDF below 0 and the DM’s outside options \(\varnothing\). When \((0, 0) \in \varnothing\) the necessary conditions are also sufficient.

\(^8\)Technically, the proposition does not state that the support interval is also bounded \((\bar{s} < \infty)\), but this is later shown indirectly through the analytical equilibrium derivation.
A.2 Activity, Strength, Dominance, and Ideology

We first derive properties of equilibrium that do not require a complete characterization of the equilibrium score CDFs. To do so we use the following simple result that describes the equilibrium relationship between the proposers’ score CDFs.

**Lemma A.5.** In any SPNE, \( \epsilon_i (F_{-i} (s)) = \epsilon_{-i} (F_i (s)) \) \( \forall s \geq 0 \), where

\[
\epsilon_i (p) = \int_p^1 \frac{|x_i|}{\alpha_i - q} \, dq = |x_i| \log \left( \frac{\alpha_i - p}{\alpha_i - 1} \right)
\]

**Proof:** Rearranging the differential equation in score optimality yields \( \frac{f_{-i}(s) \cdot |x_i|}{\alpha_i - F_{-i}(s)} = \frac{f_i(s) \cdot |x_i|}{\alpha_i - F_i(s)} \) \( \forall s \in [0, \bar{s}] \rightarrow \int_0^{\bar{s}} \frac{f_{-i}(s) \cdot |x_i|}{\alpha_i - F_{-i}(s)} \, ds = \int_0^{\bar{s}} \frac{f_i(s) \cdot |x_i|}{\alpha_i - F_i(s)} \, ds \forall s \in [0, \bar{s}]; \) a change of variables and the boundary condition \( F_i (\bar{s}) = 1 \) yields \( \int_0^{\bar{s}} \frac{f_{-i}(s) \cdot |x_i|}{\alpha_i - F_{-i}(s)} \, ds = \int_0^1 \frac{|x_i|}{\alpha_i - q} \, dq = \epsilon_i (F_{-i} (s)). \) The relationship holds trivially for \( s > \bar{s}. \) QED

We refer to the property in Lemma A.5 as the *engagement equality*. To see why, observe that the decreasing function \( \epsilon_i (p) \) captures \( i \)'s relative willingness to deviate from a proposal winning with probability \( p \) to one that wins for sure (since the marginal ideological benefit of moving policy in her direction is \( |x_i| \), and the net marginal cost of increasing score on a proposal winning the contest with probability \( q \) is \( \alpha_i - q \)). We call this function \( i \)'s *engagement at probability \( p \). The engagement equality \( \epsilon_i (F_{-i} (s)) = \epsilon_{-i} (F_i (s)) \) states that at every score \( s \geq 0 \) both proposers must be equally engaged given the resulting probabilities of winning the contest, and therefore equally willing to deviate to the maximum score \( \bar{s} \). It is easily verified that \( \epsilon_i (1) = 0 \) \( \forall i \) and \( \epsilon_i (p) \) is strictly increasing in \( |x_i| \) and decreasing in \( \alpha_i \) \( \forall p \in [0, 1] \).

Usefully, the engagement equality implies a simple functional relationship between the players' score CDFs that must hold in equilibrium regardless of their exact values. Letting

\[
p_i (e) = \alpha_i - (\alpha_i - 1) e^{\frac{e}{|x_i|}}
\]

denote the inverse of \( \epsilon_i (p) \) (which is decreasing in \( p \), increasing in \( |x_i| \), and decreasing in \( \alpha_i \)) equilibrium requires that \( F_i (s) = p_{-i} (\epsilon_i (F_{-i} (s))) \forall s \in [0, \bar{s}] \).
A.2.1 Activity and Strength

We first use the engagement equality to derive the identity of the sometimes-inactive proposer $k$ and
the probability $F_k(0)$ that she is sometimes inactive, as well as perform comparative statics on $F_k(0)$.

Proposition A.2. In equilibrium $k \in \arg \min_i \{\epsilon_i(0)\}$ and

$$F_k(0) = p_{-k}(\epsilon_k(0)) = \alpha_{-k} - (\alpha_{-k} - 1) \left( \frac{\alpha_k}{\alpha_k - 1} \right)^{\frac{x_k}{x_{-k}}}.$$  

The probability $k$ is inactive $F_k(0)$ is decreasing in her distance from the DM $|x_k|$ and her opponent’s quality costs $\alpha_{-k}$, and increasing in her opponent’s distance from the DM $|x_{-k}|$ and her own quality costs $\alpha_k$. In addition, $\lim_{|x_k| \to 0} \{F_k(0)\} = \lim_{|x_{-k}| \to \infty} \{F_k(0)\} = \lim_{\alpha_{-k} \to 1} \{F_k(0)\} = 1.$

Proof: Suppose $\epsilon_k(0) < \epsilon_{-k}(0)$; then $F_k(0) = 0$ and the engagement equality would imply
$F_{-k}(0) < 0$, a contradiction. Since $F_i(0) = 0$ for some $i$ we must have $F_{-k}(0) = 0$ and $F_k(0) =
 p_{-k}(\epsilon_k(0)) > 0$. Comparative statics and limit statements follow from previous observations on $\epsilon_i(\cdot)$
and $p_i(\cdot)$. QED

The sometimes-inactive proposer is thus the one with the lowest engagement at probability 0 – that
is, who is least willing to participate in the contest entirely.

We next use the engagement equality to derive the players’ probabilities of victory.

Proposition A.3. In equilibrium the probability proposer $k$ loses the contest is

$$\int_0^1 p_{-k}(\epsilon_k(p)) \, dp = \int_0^1 \left( \alpha_{-k} - (\alpha_{-k} - 1) \left( \frac{\alpha_k}{\alpha_k - 1} \right)^{\frac{x_k}{x_{-k}}} \right) dp$$

which is decreasing in her distance from the DM $|x_k|$ and her opponent’s quality costs $\alpha_{-k}$, and
increasing in her opponent’s distance from the DM $|x_{-k}|$ and her own quality costs $\alpha_k$.

Proof: The probability $k$ loses the contest is $\int_0^2 f_{-k}(s) \, F_k(s) \, ds$; applying the engagement equality
this is $\int_0^2 p_{-k}(\epsilon_k(F_{-k}(s))) \, f_{-k}(s) \, ds$, and applying a change of variables of $F_{-k}(s)$ for $p$ (recalling
$F_{-k}(0) = 0$) yields the result. QED
The probability \( k \) loses thus obeys the same comparative statics as her probability of inactivity. Somewhat paradoxically, she becomes less likely to win when her preferences are closer to the DM or her opponent’s are more distant. More intuitively, she becomes more likely to win if she is more able or her opponent less able.

A.2.2 Dominance

In the standard asymmetric two-player all-pay contest there is always an unambiguously weaker player, who makes bids that are first-order stochastically worse for the DM. In the present contest, in contrast, there may be no unambiguously weaker player in this sense.

**Proposition A.4.** Proposer \( i \) is dominated \((F_-(s) < F_i(s) \forall s \in (0, \bar{s}))\ i.f.f.\ she is less engaged at every probability \( p \) \((\epsilon_i(p) < \epsilon_-(p) \forall p \in (0, 1))\). Equivalently, she is dominated i.f.f. both \( \int_0^1 \frac{|x_{-i}|}{\alpha_{i-q}} dq \leq \int_0^1 \frac{|x_i|}{\alpha_{i-q}} dq \) and \( \frac{|x_i|}{\alpha_{i-1}} \leq \frac{|x_{-i}|}{\alpha_{i-1}} \), where the latter condition is stronger than the former i.f.f. \( i \) has a cost advantage.

**Proof:** Lemma A.5 and the engagement function \( \epsilon_i(p) \) strictly decreasing when \( p \in [0, 1) \) immediately implies \( \text{sign} (\epsilon_{-k} (F_{-k} (s)) - \epsilon_k (F_{-k} (s))) = \text{sign} (F_k (s) - F_{-k} (s)) \forall s \in [0, \bar{s}) \), which straightforwardly yields the first statement. Now let \( \delta(p) = \epsilon_{-k} (p) - \epsilon_k (p) \), so \( \delta(0) \geq 0 = \delta(1) \). We argue \( \delta'(1) \leq 0 \) is necessary and sufficient. For necessity, \( \delta'(1) > 0 = \delta(1) \rightarrow \delta(p) < 0 \) in a neighborhood below 1. For sufficiency, it is easily verified that \( \delta'(p) = \frac{|x_k|}{\alpha_k - p} - \frac{|x_{-k}|}{\alpha_{-k} - p} \) crosses 0 at most once when the proposers are asymmetric; thus \( \delta(0) \geq 0 = \delta(1) \geq \delta'(0) \) implies \( \delta(p) \) strictly quasi-concave over \([0, 1]\) and \( \delta(p) > \min \{ \delta(0), \delta(1) \} \geq 0 \) for \( p \in (0, 1) \).

We last argue \( \delta(0) \geq 0 \) and \( \alpha_k > \alpha_{-k} \rightarrow \delta'(1) < 0 \). Observe that \( \alpha_k < \alpha_k \) and \( \delta'(0) = \frac{z_k}{\alpha_k} - \frac{x_{-k}}{\alpha_{-k}} \leq 0 \rightarrow \delta'(1) = \frac{|x_k|}{\alpha_k} \left( \frac{1}{1-1/\alpha_k} \right) - \frac{|x_{-k}|}{\alpha_{-k}} \left( \frac{1}{1-1/\alpha_{-k}} \right) < 0 \). If \( \delta'(0) \leq 0 \) we are done; if \( \delta'(0) > 0 \) then \( \delta'(1) \geq 0 \rightarrow \delta'(p) > 0 \forall p \in [0, 1] \rightarrow \delta(1) > 0 \), a contradiction. QED

Clearly, a proposer \( k \) who is both less extreme (\( |x_k| \leq |x_{-k}| \)) and less able (\( \alpha_k \geq \alpha_{-k} \)) (with one strict) satisfies both conditions and is therefore dominated. However, when one proposer is more extreme while the other is more able, then lower engagement at probability 0 is necessary but not sufficient for the more able proposer to be dominated.
A.2.3 Ideology

Lastly, the engagement equality directly yields simple expressions for the probability distribution over the ideology of each player’s proposals.

**Proposition A.5.** Let \( G_i (y) = \Pr (|y_i| \leq |y|) \) denote the probability that \( i \)’s proposal is closer to the DM than \( y \). Then

\[
G_i (y) = p_{-i} \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right) = \alpha_{-i} - (\alpha_{-i} - 1) \left( \frac{x_i - y}{x_i - x_i/\alpha_i} \right) \left| \frac{x_i}{x_i - \alpha_i} \right|
\]

which is first-order stochastically increasing in \( i \)'s extremity \(|x_i|\), decreasing in her costs \( \alpha_i \), decreasing in her opponent’s extremity \(|x_{-i}|\), and increasing in her opponent’s costs \( \alpha_i \).

**Proof:** Proposer \( i \)'s ideology at score \( s \) is \( y_i^* (s) = \frac{x_i}{\alpha_i} F_{-i} (s) \) (from ideological optimality), so \( F_{-i} (s_i^* (y)) = \frac{y}{x_i/\alpha_i} \) where \( s_i^* (y) \) is the inverse of \( y_i^* (s) \). That is, the probability \(-i\) makes a proposal with score \( \leq s_i^* (y) \) is \( \frac{y}{x_i/\alpha_i} \). Now the probability \( G(y) \) that \( i \) makes a proposal closer to the DM than \( y \) is \( F_i (s_i^* (y)) \), which is \( p_{-i} (\epsilon_i (F_{-i} (s_i^* (y)))) = p_{-i} \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right) \) from the engagement equality. Comparative statics are straightforward. QED

When a proposer \( i \) becomes more ideologically extreme (higher \(|x_i|\)) or able (lower \( \alpha_i \)), she makes first-order stochastically more extreme proposals. Proposer \( i \)'s opponent \(-i\) reacts to \( i \) becoming more extreme or more able by moderating the ideological extremism of her own proposals.

A.3 Payoffs

We complete the analysis by calculating payoffs. This requires first characterizing score CDFs \( F_i (s) \) satisfying Proposition A.1, which are shown constructively to be unique.

**Proposition A.6.** The unique score CDFs over \( s \geq 0 \) satisfying Proposition A.1 are \( F_i (s) = p_{-i} (\epsilon (s)) \) \( \forall i \), where \( \epsilon (s) \) is the inverse of

\[
s(\epsilon) = 2 \int_{\epsilon}^{\epsilon_k (0)} \sum_j \frac{|x_j|}{\alpha_j} p_j (\hat{e}) d\hat{e}.
\]
The inverse score CDFs are \( s_i(F_i) = s(\epsilon_{-i}(F_i)) \) \( \forall i \), and the score targeted at each ideology is \( s_i^*(y) = s\left(\epsilon_i\left(\frac{y}{\alpha_i}\right)\right) \). The function \( s(\cdot) \) is strictly increasing in \( x_i \) and strictly decreasing in \( \alpha_i \) \( \forall \epsilon \in [0, \epsilon_k(0)) \), and the maximum score is \( \bar{s} = s(0) \).

**Proof:** From the engagement equality \( \epsilon_i(F_{-i}(s)) = \int_{F_{-i}(s)}^{1} \frac{|x_i|}{\alpha_i - q} dq = s(\epsilon) \forall i, s \) for some \( s(\cdot) \). We characterize the unique \( s(\cdot) \) implying score CDFs \( F_i(s) = p_{-i}(\epsilon(s)) \) and optimal ideologies \( y_i(s) = \frac{x_i}{\alpha_i} p_i(\epsilon(s)) \) that satisfy score optimality. First observe that \( \epsilon'(s) = f_i(s) \epsilon'_{-i}(F_i(s)) = -\frac{f_{-i}(s)|x_i|}{\alpha_i - F_{-i}(s)} \).

Next the differential equations may be rewritten as \( \frac{\alpha_i - F_{-i}(s)}{F_{-i}(s) - |x_i|} = 2 \sum_j y_j(s) \). Substituting the preceding observations into both sides yields \( \frac{1}{\epsilon'(s)} = -2 \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon(s)) \), and rewriting in terms of the inverse \( s(\cdot) \) yields \( s'(\cdot) = -2 \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \). Lastly \( \epsilon_k(F_{-k}(s)) = s(\epsilon) \) and \( F_{-k}(0) = 0 \) imply the boundary condition \( s(\epsilon_k(0)) = 0 \) so \( s(\epsilon) = \int_{\epsilon_k(0)}^{\epsilon(0)} s'(\epsilon) d\epsilon = 2 \int_{\epsilon_k(0)}^{\epsilon(0)} \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) d\epsilon \). Now \( s(\epsilon) \) is straightforwardly increasing in \( |x_i| \) and decreasing in \( \alpha_i \) given previous observations about \( p_j(\epsilon) \). QED

The maximum score \( \bar{s} \) thus changes continuously with the parameters of both proposers, even when one is dominant. This contrasts with the standard 2-player all pay contest, where the mixing interval is unaffected by the parameters of the stronger player.

The preceding characterization transparently yields the following comparative statics.

**Corollary A.1.** Increasing a proposer’s extremism \( |x_i| \) or decreasing her costs \( \alpha_i \) first-order stochastically increases her own score CDF, but has ambiguous effects on her opponent’s score CDF.

To see that the effect of a proposer’s parameters on her opponent’s score CDF is necessarily ambiguous, suppose that the always-active proposer \(-k\) becomes even more extreme or able. Then her opponent \( k \) becomes less likely to be active, but also the range of scores \([0, \bar{s}]\) over which she mixes when she is active increases. She thus has a higher probability of making very high-score proposals, even while she is simultaneously less likely to enter the contest.

**A.3.1 Proposer Payoffs**

Using Proposition A.6, the proposers’ equilibrium payoffs are as follows.

**Proposition A.7.** Proposer \( i \)’s equilibrium utility is \( \Pi^*_i(\bar{s}; F^*) = -\left(1 - \frac{1}{\alpha_j}\right)x_i^2 - (\alpha_i - 1)\bar{s}, \) which is decreasing in her own costs \( \alpha_i \) as well as either players’ extremism \( |x_j| \forall j, \) and increasing in her opponent’s costs \( \alpha_{-i} \).
Proof: A proposer’s equilibrium utility is straightforward since \((\bar{s}, y^*_i(\bar{s}))\) is in the support of their strategy and wins for sure. Comparative statics of a proposer’s \(i\)'s parameters on her opponent \(-i\)'s utility, as well as of \(x_i\) on her own utility, follow immediately from previously-shown statics on \(\bar{s} = s(\epsilon)\).

Taking the derivative with respect to \(\alpha_i\), substituting in \(\frac{\partial}{\partial \alpha_i} \left( \frac{p_i(\epsilon)}{\alpha_i} \right) = \frac{x_i x_i(\epsilon) p_i(\epsilon)}{(\alpha_i - 1) \alpha_i^2}, \quad \frac{\partial}{\partial \alpha_k} = -\frac{|x_k|}{\alpha_k (\alpha_k - 1)},\)

\[-\frac{p'_i(\epsilon) x_i}{\alpha_i - p_i(\epsilon)} = 1, \quad \text{performing a change of variables, and rearranging the expression yields} \quad -1_{i=k} \cdot 2 \int_{0}^{x_k(0)} \frac{|x_k|}{\alpha_k} \left( p_{-k}(\epsilon) - \left( \alpha_k \log \left( \frac{\alpha_k}{\alpha_k - 1} \right) \right)^{-1} p_{-k}(\epsilon_k(0)) \right) \, d\epsilon - \left( \frac{|x_i|}{\alpha_i} \right)^2 \left( 1 + 2 \int_{p_i(\epsilon_k(0))}^{1} \left( \frac{\alpha p}{\alpha_i - p} - 1 \right) \, dp \right). \]

The first term is negative since \(p_{-k}(\epsilon) > p_{-k}(\epsilon_k(0))\) for \(\epsilon < \epsilon_k(0)\) and \(\frac{1}{\alpha_k} < \int_{0}^{1} \frac{1}{\alpha_k - p} \, dp = \log \left( \frac{\alpha_k}{\alpha_k - 1} \right)\). The second term is also negative since \(1 + 2 \int_{p_i(\epsilon_k(0))}^{1} \left( \frac{\alpha p}{\alpha_i - p} - 1 \right) \, dp = \int_{p_i(\epsilon_k(0))}^{1} (2p - 1) \, dp \geq 0\). QED

A proposer’s equilibrium utility has two components. The first \(- \left( 1 - \frac{1}{\alpha_i} \right) x_i^2\) is her utility if she could make proposals as a “monopolist” (and the DM’s outside option included \((0, 0)\)). The second \(- (\alpha_i - 1) \bar{s}\) is the cost generated by competition, which forces her to make proposals that leave the DM strictly better off than the best “free” proposal \((0, 0)\) in order to maintain influence. This competition cost is increasing in \(i\)’s marginal cost \(\alpha_i\) of generating quality (holding \(\bar{s}\) fixed) as well as the maximum score \(\bar{s}\), which in turn is increasing in both proposers’ ideological extremism and decreasing in their costs everywhere in the parameter space. A proposer is thus strictly harmed when her competitor becomes more extreme or able. This is distinct from all pay contests without spillovers (Siegel (2009)), where the equilibrium utility of the “sometimes inactive” player is pinned at her fixed value for losing.

A proposer also worse off when her own preference become more distant from the decisionmaker. Finally, a proposer is worse off when her costs of producing quality increase – even though there is a countervailing effect of reducing the intensity of competition (and indeed, the competition cost \((\alpha_i - 1) \bar{s}\) alone is not generically monotonic in \(\alpha_i\)).

A.3.2 DM Payoffs

Lastly, again using Proposition A.6 the DM’s equilibrium utility and the proposers’ average scores (which bound the DM’s utility from below) are as follows.
**Proposition A.8.** The DM’s equilibrium utility is
\[ U_{DM}^* = \int_{c_t(0)}^{\epsilon_t(0)} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon} \left( \prod_j p_j(\epsilon) \right) d\epsilon = \]
\[ 2 \int_{0}^{{\epsilon_t(0)}} \left( 1 - \prod_j p_j(\epsilon) \right) \cdot \left( \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \right) d\epsilon \]

Proposer i’s average score is
\[ E[s_i] = \int_{c_t(0)}^{\epsilon_t(0)} s(\epsilon) \cdot \frac{\partial}{\partial \epsilon} (p_{-i}(\epsilon)) d\epsilon = \]
\[ 2 \int_{0}^{{\epsilon_t(0)}} (1 - p_{-i}(\epsilon)) \cdot \left( \sum_j \frac{|x_j|}{\alpha_j} p_j(\epsilon) \right) d\epsilon \]

**Proof:** \( F_i(s) F_{-i}(s) \) is the CDF of \( \max\{s, s_{-i}\} \) so the DM’s utility is
\[ \int_{0}^{\epsilon_t(0)} s \cdot \frac{\partial}{\partial s} \left( \prod_j F_j(s) \right) ds = \]
\[ \int_{0}^{\epsilon_t(0)} s \cdot \frac{\partial}{\partial s} \left( \prod_j p_j(s) \right) ds \] A change of variables from \( s \) to \( \epsilon \) yields the first expression and integration by parts and rearranging yields the second. A nearly identical series of steps yields i’s average score.
QED

Direct comparative statics on the DM’s utility \( U_{DM}^* \) are difficult because changing a proposer’s parameters has mixed effects on her opponent’s score CDF. We thus consider two special cases; breaking symmetry, and the limiting cases of extreme imbalance. The effect of breaking symmetry is as follows.

**Proposition A.9.** When the proposers are symmetric \( |x_i| = |x_{-i}| \) and \( \alpha_i = \alpha_{-i} \), the DM’s utility is locally increasing either’s extremism or ability.

**Proof:** First differentiating the DM’s utility \( U_{DM}^* \) with respect to \( |x_{-k}| \) and applying symmetry yields
\[ \frac{2}{a} \int_{0}^{\epsilon_t(0)} \left( \frac{1 - (p(\epsilon))^2}{\alpha} + (1 - (p(\epsilon))^2) p(\epsilon) \right) d\epsilon \] which is transparently \( \geq \frac{2}{a} \int_{0}^{\epsilon_t(0)} \left( 1 - 3(p(\epsilon))^2 \right) \frac{\partial p(\epsilon)}{\partial \epsilon} d\epsilon. \]
Now substituting \( \frac{\partial p(\epsilon)}{\partial \epsilon} = -\log \left( \frac{\alpha - p(\epsilon)}{\alpha - 1} \right) p'(\epsilon) \) and a change of variables yields
\[ \frac{2}{a} \int_{0}^{\epsilon_t(0)} \left( 1 - 3p(\epsilon)^2 \right) \log \left( \frac{\alpha - p(\epsilon)}{\alpha - 1} \right) dp = \]
\[ 2 \frac{2}{a} \int_{0}^{1} \left( \frac{p - p^2}{\alpha - p} \right) dp > 0. \]
Next differentiating \( U_{DM}^* \) w.r.t. \( \alpha_{-k} \) and applying symmetry yields
\[ \frac{2}{a} \int_{0}^{\epsilon_t(0)} \left( 1 - (p(\epsilon))^2 \right) p(\epsilon) \frac{\partial p(\epsilon)}{\partial \epsilon} \frac{\partial \epsilon}{\partial \alpha} \] Substituting \( \frac{\partial p(\epsilon)}{\partial \alpha} = \frac{x}{(\alpha - 1)\alpha^2} p'(\epsilon), \frac{\partial p(\epsilon)}{\partial \alpha} = -\left( \frac{1 - p(\epsilon)}{\alpha - 1} \right), -\frac{p'(\epsilon)x}{\alpha - p(\epsilon)} = 1, \) rearranging, and a change of variables yields
\[ \frac{2}{a} \int_{0}^{\epsilon_t(0)} \left( 2p^2 \left( \frac{\alpha - \alpha \epsilon}{\alpha - p} \right) - (1 - p^2) \right) dp < 0. \]
QED

The DM thus strictly benefits locally if symmetry between the players is broken by one becoming more
extreme or able – even though the other also becomes less active. The effect of extreme asymmetries is as follows.

**Proposition A.10.** The DM’s utility exhibits the following limiting behavior

\[ 0 = \lim_{\alpha_i \to \infty} U_{DM}^* = \lim_{x_i \to 0} U_{DM}^* < \lim_{x_i \to \infty} U_{DM}^* = \infty \]

and \( \lim_{\alpha_i \to 1} U_{DM}^* = 2x_k \int_0^1 \left( \frac{1-p}{\alpha_k-p} \right) \left( \frac{x_k}{\alpha_k} p + x_{-k} \right) dp \), which is strictly increasing in \( x_k \) and strictly decreasing in \( \alpha_k \).

**Proof:** Observe that \( E [s_{-k}] \leq U_{DM}^* \leq \bar{s} \). For the first two limiting statements it is easily verified that \( \bar{s} \to 0 \) as \( \alpha_k \to \infty \) or \( x_k \to 0 \). For the third limiting statement observe that \( E [s_{-k}] \geq \frac{|x_{-k}|}{\alpha_{-k}} p_{-k} (\epsilon_{k}(0)) \cdot 2 \int_0^{\epsilon_{k}(0)} (1 - p_k (\epsilon)) \, d\epsilon \) which \( \to \infty \) as \( |x_{-k}| \to \infty \) since the first term \( \to \infty \) and the remaining terms are non-decreasing. For the fourth limiting statement, using the definition in Proposition A.8 and that \( \lim_{\alpha_k \to 1} \{ p_{-k} (\epsilon) \} = 1 \ \forall \epsilon \in [0, \epsilon_{k}(0)] \) yields a limit of \( 2 \int_0^{\epsilon_{k}(0)} (1 - p_k (\epsilon)) \cdot \left( \frac{x_k}{\alpha_k} p_k (\epsilon) + x_{-k} \right) \, d\epsilon \). Observing that \( -\frac{p_k^e (\epsilon)x_k}{\alpha_k - p_k (\epsilon)} = 1 \), substituting into the expression, and applying a change of variables yields the expression, which straightforwardly obeys the stated comparative statics. QED

If an extreme imbalance is the result of one proposer’s incompetence or ideological moderation, the DM’s utility approaches 0, her utility if \( -i \) were a “monopolist” (and the DM’s outside options included \((0, 0)\)). (Proposer \( -i \)’s utility also approaches her utility if she were a monopolist). However, if extreme imbalance is the result of one proposer’s greater ability to produce quality (specifically, if her marginal cost of producing quality approaches its intrinsic value), then the DM’s utility is bounded away from 0. In this case the DM strictly benefits from the potential for competition, even though actual competition is almost never observed (since \( F_{-i}(0) = F_k(0) \) approaches 1). Finally, unilateral ideological extremism benefits the decisionmaker in a strong sense; the DM can achieve arbitrarily high utility with a proposer whose preferences are sufficiently distant from her own.
B Guide to Main Text Propositions

In this Appendix we “prove” the main text propositions by describing where to locate each stated result in the preceding general treatment.

Proof of Proposition 1
The distribution over ideologies is provided in Proposition A.5. The score \( s_i^Y (y) = s \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right) \) at each ideology is provided in Proposition A.6 (which also defines the function \( s (\cdot) \)), so the implied quality is \( q_i^Y = y^2 + s \left( \epsilon_i \left( \frac{y}{x_i/\alpha_i} \right) \right) \) as stated in the main text.

Proof of Proposition 2
The first statement follows from Proposition A.2 on activity.

The second statement on ideology is an implication of the ideology comparative statics stated in Proposition A.5, which state that as a proposer becomes unilaterally more extreme her ideologies become more extreme and her opponent’s ideologies simultaneously become more moderate. To see this compare strategies when the proposers have the same extremism as the moderate (and therefore develop symmetrically extreme proposals) to when one proposer becomes more extreme. The second statement on score follows from the necessary and sufficient conditions for score-dominance in Proposition A.4 – a proposer being more extreme and able with at least one strict is a sufficient condition for score dominance. Finally, the second statement on quality is an immediately implication of the extremist having more ideologically extreme but also higher score proposals (first order stochastically).

The third statement is an immediate implication of the second statement.

Proof of Proposition 3
The first bullet point under both “own strategy” and “opponent” strategy follow from Proposition A.2 on activity. The second bullet point under both “own strategy” and “opponent” strategy follow from Proposition A.5 on ideology. The third bullet point under “own strategy” is a joint implication of the ideology comparative statics in Proposition A.5 and the comparative statics on “own score” in Corollary A.1. The third bullet point under “opponent strategy” also follows from Corollary A.1 and the subsequent discussion.

Proof of Proposition 4
Follows immediately from Proposition A.7.

**Proof of Proposition 5**
The first bullet point is from Proposition A.9. The second bullet point is a joint implication of Propositions A.2 (on activity) and A.10 (on decisionmaker’s welfare).

**Proof of Proposition 6**
Following from Propositions A.2, A.4, and A.5 according to an identical argument as in the proof of Proposition 2.

**Proof of Proposition 7**
Following from Proposition A.2, A.5, and Corollary A.1 by an identical argument as in the proof of Proposition 3.

**Proof of Proposition 8**
Follows immediately from Proposition A.7.

**Proof of Proposition 9**
The first bullet point is from Proposition A.9. The second bullet point is a joint implication of Propositions A.2 (on activity) and A.10 (on decisionmaker’s welfare).