Competitive Policy Development

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June 4, 2014

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Abstract

We present a model of policy development in which factions have different ideologies, yet agree on certain common objectives. Policy developers can appeal to a decisionmaker by making productive investments to improve the quality of their proposals. These investments are specific to a given proposal, which means that policy developers can potentially obtain informal agenda power. Competition undermines this agenda power, forcing policy developers to craft policies that are better for the decisionmaker. This beneficial effect is strongest if policy developers have divergent ideological preferences, because their intense desire to affect policy motivates them to develop higher-quality proposals.

During the early years of the New Deal, President Franklin Delano Roosevelt faced an enormous challenge. He believed that dramatic policy innovation was urgently needed on a wide range of issues, including agriculture, trade, banking, employment, and social insurance. However, although Roosevelt was well-versed in policy, crafting workable proposals for such a far-reaching agenda was well beyond his capacity, or that of his immediate advisors. Instead, the President had to rely on a broader array of politicians and bureaucrats to develop new policies. Although these individuals shared some common goals–e.g., ending the Great Depression–they also had deep-seated ideological disagreements among themselves, and with the President, about what policies the federal government should pursue.

Classic and contemporary theories of bureaucratic politics would suggest that this situation was ripe for exploitation and inefficiency. Weber (1942) famously argued that the typical ruler of a modern state is reduced to being a "mere dilettante" when dealing with institutionalized bureaucratic actors. More recently, scholars building on the work of Crawford and Sobel (1982) have argued that policymaking is less effective when the experts who provide advice to a decisionmaker don't share his goals. However, the individuals and organizations that crafted New Deal policy initiatives were generally unable to subvert the President's policy goals and achieve their own particularistic objectives. Instead, what resulted was one of the most dramatic periods of policy innovation in American history.

Historians have argued that a major reason for the general success of policy development during the New Deal was competition within the Roosevelt administration (Schlesinger 1958). Bureaucrats and advisors who sought to influence policy in accordance with their own preferences had to contend with others who favored different approaches. In trade policy, there was intense competition between proponents of quid pro quo deals versus proponents of the most favored nation principle. In public works, there was intense competition between those who favored shovel-ready projects, those who favored development of infrastructure, and those who wanted to minimize costs. And in the area of soil conservation, the Agriculture and Interior Departments fought heatedly over control of the Forest Service, as well as over policies for mitigating the Dust Bowl.

These conflicts unsurprisingly resulted in a certain amount of pathological bureaucratic infighting. However, they also generated competition to develop better and more effective policies. The reason was simple: anyone who wanted to convince Roosevelt to adopt a particular approach rather than competing alternatives had to produce a well-crafted policy that would also achieve the President's policy goals. For example, Schlesinger (1958; 349, 535) notes that competition between Agriculture and Interior "spurred each Department to redouble its efforts in the conservation cause" and feuds over public works stimulated "more effective accomplishment" of public goals. Historians (Leuchtenburg 1963, 328-9) and political scientists (Bendor 1985) have followed Schlesinger in concluding that administrative competition played a crucial role in the development of New Deal policies.

Moreover, far from being concerned about the ideological biases of his advisors, Roosevelt found it useful to draw on a wide range of sources for proposals. As he said in 1944, "You sometimes find something pretty good in the lunatic fringe. In fact, we have got as part of our social and economic government today a whole lot of things which in my boyhood were considered lunatic fringe, and yet they are now part of everyday life."[1] Policy developers within the administration thus included Democrats, Republicans, bankers like Joseph P. Kennedy, and leftists like Harry Hopkins. In fact, the administration appeared to be "composed of human opposites put into their positions with the specific intent of generating conflict" (Gerlak and McGovern 1999).

In this paper, we analyze the role of competition and extremism in an all-pay contest model, in which policy developers exert costly effort to improve the quality of proposals that they make to a decisionmaker. Contest models have been used to study lobbying (e.g., Tullock 1980; Baye, Kovenock, and de Vries 1993; Che and Gale 1998), but most models focus on allocation of a prize, rather than adoption of a policy. Our model differs from this literature in several respects. First, the actors are policy-motivated and value winning only as a means to achieve their policy goals. Second, although actors have ideological disagreements, they also have common interests, in the sense of preferring high-quality policies over low-quality ones. Finally, actors choose both the quality and the ideology of their proposals from a continuum, which means that compromise can arise endogenously.

A key assumption of our model is that any quality developed to improve a policy proposal is specific to that proposal (see also Londregan 2000, Ting 2011, and Hirsch and Shotts 2012). This assumption contrasts with models of endogenous acquisition of general expertise, in which an expert worries that a decisionmaker will expropriate her investments to achieve different policy goals (e.g., Gilligan and Krehbiel 1987). As shown by Aghion and Tirole's (1999) and Callander's (2008) models of informal authority, expertise that is non- or partially-transferable has strategic properties that are very different from general expertise. In our model, a policy developer can exploit policy-specific quality to compel the decisionmaker to accept policies that promote her ideological interests. This setup is appropriate for analyzing effort that is strategically expended to craft a particular proposal–for example, as head of

[1] http://www.presidency.ucsb.edu/ws/?pid=16513, accessed 4/5/2014.

the Public Works Administration, Harold Ickes did not try to come up with ideas for shovel-ready projects that he opposed, and instead focused on developing detailed plans for long-run infrastructure projects that he hoped would be adopted (Schlesinger 1958, 281-9).

Our model generates several insights about competitive policy development. First, we show that competition always benefits the decisionmaker. Absent competition, a monopolistic policy developer would craft policies that promote her interests without providing much benefit to the decisionmaker. Competition serves to discipline policy developers, forcing them to craft policies that are closer to the decisionmaker's ideal point, and often higher-quality.

Second, we assess a natural intuition; that a decisionmaker would prefer policy developers whose ideologies are closely aligned with his own, to ensure that they develop policies in line with his own preferences. However, consistent with the experience of the Roosevelt administration, we show that it is actually better to have competing policy developers with divergent preferences, whose intense desire to affect policy motivates them to exert effort to develop high-quality proposals.

Finally, we show that although the decisionmaker, ceteris paribus, prefers moderate policies, the model generates endogenous extremism of policy choices and outcomes. That is, the decisionmaker benefits from extreme policy proposals, and chooses them over moderate ones. This happens because quality is endogenous, and a policy developer who makes an extreme proposal will exert sufficient effort improving its quality to overcompensate the decisionmaker for his ideological losses. Our model thus provides an account for why extremist policy developers may be present in the policymaking arena, successful in getting their policies enacted, and beneficial for moderates.

1 The Model

Two entrepreneurs (*L*eft and *R*ight) develop competing policies for consideration by a decisionmaker (D). A policy $b = (y, q)$ consists of an *ideology* $y \in \mathbb{R}$ and a level of *quality* $q \in [0, \infty) = \mathbb{R}^+$, which captures features valued by all players, such as cost savings, efficient administration, or economic growth. All players care about the ideology and quality of the policy that is ultimately chosen, an assumption that differs crucially from most previous contest models, in which participants compete for a fixed prize. Utility functions take the form $U_i(b) = q - (x_i - y)^2$, where x_i is player $i's$ ideological ideal point. The decisionmaker's ideal point is $x_D = 0$, with entrepreneurs on either side ($x_L < 0$ and $x_R > 0$).

The game has two stages. In the policy development stage, the entrepreneurs simultaneously craft policies $b_i = (y_i, q_i)$. Producing quality q_i on a policy with ideology y_i costs $c_i(q_i) = \alpha_i q_i$ up-front, reflecting the investment of time and resources required to develop a well-designed policy proposal. For simplicity, the marginal cost of generating quality is constant, independent of ideology, and greater than the marginal benefit ($\alpha_i > 1$).[2]

In the policy choice stage, the decisionmaker chooses one of the two new policies or a *reservation policy*. This is assumed to be $(0, 0)$, capturing the idea that the decisionmaker lacks policy-development capacity, and so will simply select a poorly-designed policy that reflects his ideological preferences. The decisionmaker also cannot transfer quality generated for one policy to another policy; in the terminology of Hirsch and Shotts (2012) it is *policy-specific.* For example, if an entrepreneur invests time and effort to develop an effective and equitable school voucher program, the decisionmaker cannot expropriate those investments to develop an alternative policy that improves the quality of public schools.

For most of the analysis, we consider a symmetric variant in which the entrepreneurs are equally-extreme ($|x_L| = x_R = x$) and equally-skilled at developing quality ($\alpha_L = \alpha_R = \alpha$). The parameter x represents the extremism of ideological interests in the policymaking arena, and α captures the efficiency of the technology for generating high-quality policies.

Preliminaries Absent competition, our model is similar to Snyder's (1991) model of vote-buying without price discrimination. A policy "monopolist" develops a policy that balances the marginal ideological benefit of moving policy in her direction against the marginal cost of producing just enough quality to obtain the decisionmaker's approval.[3] The decisionmaker is left no better off than with the reservation policy, because the monopolist extracts all the benefits of quality in the form of ideological rents.

With competition, the model is a variant of an all-pay contest (Siegel 2009), with some distinctive properties that we highlight later. As in Che and Gale (2003), bids (y_i, q_i) are two-dimensional, and there is a score function $s(y_i, q_i)$ determining the winner. In our model, a policy's score is the utility it provides to the decisionmaker $s(y, q) = U_D(y, q) = q - y^2$, because the decisionmaker cannot commit in advance to which policy he will choose.

For much of the analysis, we rewrite strategies and utilities in terms of the score. With this transformation, an entrepreneur effectively chooses a target level of utility s_i to offer to the decisionmaker, and a combination of ideology y_i and quality $q_i = s_i + y_i^2$ to achieve that target. Quality costs $\alpha_i(s_i + y_i^2)$ to produce, so more ideologically extreme policies–while

[2]While the quality return is deterministic, it could be uncertain provided that the true value is unknown to the decisionmaker, and the expected value is known and linear in effort.

[3]See also Hitt, Volden, and Wiseman's (2011) analysis of legislative effectiveness.

better for the entrepreneur in the event of adoption–are more costly up-front because they require greater investments in quality to achieve the same utility for the decisionmaker.

In Lemma 1 in the Online Appendix, we show that all equilibria satisfy intuitive conditions. Each entrepreneur i's strategy can be described by two components: a univariate probability distribution $F_i(s_i)$ over scores, and a unique ideology $y_i(s_i)$ for each score $s_i > 0$, capturing how she trades off quality investments and ideological demands. The entrepreneurs mix smoothly over a common nonempty interval of scores $[0, \bar{s}]$, so there are no atoms or ties. Finally, at least one entrepreneur j is always active, in the sense of developing a policy strictly better than the reservation policy ($F_j(0) = 0$). Using these properties we heuristically derive the equilibrium strategies and results. Technical details are in the Online Appendix.[4]

Optimal Ideologies An entrepreneur's utility from a policy with score s and ideology y is $V_i(s, y) = U_i(y, s + y^2) = -x_i^2 + s + 2x_i y$. This is linearly increasing in score holding ideology fixed, and it is also linear in ideological movements along the decisionmaker's indifference (score) curves. Entrepreneur i's utility for developing a policy (s_i, y_i) with score $s_i > 0$ is

$$\Pi_i(s_i, y_i; F_{-i}(\cdot), y_{-i}(\cdot)) = -\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int_{s_i}^{\infty} V_i(s_{-i}, y_{-i}(s_{-i}))\, dF_{-i}. \quad (1)$$

The first term is the up-front cost of producing quality, and the second is the probability of victory multiplied by the policy payoff. The third term is $i's$ policy utility when losing.

We note two distinctive properties of Equation 1 that arise from our assumptions that the entrepreneurs invest in quality, and are purely policy-motivated. First, unlike Che and Gale (2003) there is no pure all-pay component of the strategies. Quality has a marginal up-front cost of α_i, but with probability $F_{-i}(s_i)$ the policy is adopted and yields an intrinsic marginal benefit of 1. This means that the entrepreneurs are more willing to invest in improving the quality of policies that are more likely to be adopted, because they are also more likely to enjoy the intrinsic benefits. Second, because an entrepreneur cares about policy, rather than winning per se, calculating her utility when she loses requires integrating over all the endogenous policies $(s_{-i}, y_{-i}(s_{-i}))$ in her opponent's strategy that would defeat her own. Thus, there are *rank-order spillovers* (Baye, Kovenock, and de Vries 2012), in the sense that the winner's strategy has a direct effect on the loser's utility.

We now characterize an entrepreneur's optimal combination of ideology and quality at

[4]The equilibrium concept is Subgame Perfect Nash. We also require entrepreneurs to use strategies with a score c.d.f. $F_i(s_i)$ that can be written as the sum of an absolutely continuous and a discrete distribution.

every score. The key simplification is that holding the probability of winning (i.e. score) fixed, i's policy only affects her utility when she wins, and her opponent's policy only affect her utility when she loses. Thus, the only aspect of $-i$'s strategy that affects $i's$ optimal combination of ideology and quality at score s_i is the probability $F_{-i}(s_i)$ that $-i$ develops a lower-score policy, which is equal to the probability that i's policy wins. Taking the first order condition of Equation 1 with respect to y_i then yields the optimal policy at each score.

Corollary 1. *With probability* 1 *an entrepreneur's policies* (s_i, y_i) *are either* 0*-quality and never win, or satisfy* $y_i = y_i^*(s_i) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s_i)$.[5]

This equilibrium relationship yields a number of insights about the connection between ideology and quality when both are chosen by strategic actors. First, more ideologically-extreme entrepreneurs produce more ideologically-extreme policies ceteris paribus, because they are more willing to pay the sure costs of developing quality for the uncertain benefits of ideological gains. Second, more skilled entrepreneurs (i.e., lower α_i) also produce more extreme policies ceteris paribus, because they are better able to generate quality and exploit it to realize ideological gains. Finally, more ideologically-extreme policies in the support of an entrepreneur's strategy are not just higher quality–they are also strictly better for the decisionmaker, and more likely to be adopted. The entrepreneurs therefore *overcompensate* the decisionmaker for his ideological losses when they propose more extreme policies.

The equilibrium association between extremism, quality, and decisionmaker utility is counterintuitive, but it emerges naturally from the entrepreneurs' incentives. Policies (s_i, y_i) that are better for the decisionmaker have a higher probability $F_{-i}(s_i)$ of adoption, so an entrepreneur is more willing to pay the certain quality costs of proposing a more extreme ideology in exchange for the uncertain ideological gains in the event of victory.

Equilibrium Score Conditions To derive the equilibrium score CDFs (F_i, F_{-i}), note that every score $s_i \in [0, \bar{s}]$ in the common support of the entrepreneurs' score CDFs must maximize $i's$ utility when the optimal policies $y_i^*(s)$ are developed; i.e., with probability 1 i chooses a $s_i \in \arg\max_{s_i} \left\{\Pi_i \left(s_i, y_i^*(s_i); F_{-i}(\cdot), y_{-i}^*(\cdot)\right)\right\}$. Since i is indifferent over all scores in $[0, \bar{s}]$, differentiating $\Pi_i \left(s_i, y_i^*(s_i); F_{-i}(\cdot), y_{-i}^*(\cdot)\right)$ with respect to s_i and setting it equal to zero yields a pair of differential equations that must be jointly satisfied in equilibrium:

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i \left(\left(\frac{x_i}{\alpha_i}\right) F_{-i}(s) - \left(\frac{x_{-i}}{\alpha_{-i}}\right) F_i(s) \right) \quad \forall s \in [0, \bar{s}] \text{ and } i \in \{L, R\} \quad (2)$$

[5] The property is immediate at scores $s_i > 0$ where $-i$'s score CDF has no atom; however, the claim also relies on the absence of score atoms above 0 and the fact that one entrepreneur is always active (Lemma 1).

Equation 2 has a natural interpretation. The left side is $i's$ net marginal cost of increasing the score she offers: she pays $\alpha_i > 1$ to generate quality, but with probability $F_{-i}(s)$ she wins and enjoys the marginal benefit, 1, of that quality. The right side is $i's$ marginal ideological gain from increasing her score: with probability $f_{-i}(s)$ she goes from losing to winning the contest, which shifts the outcome from $y^*_{-i}(s) = \left(\frac{x_{-i}}{\alpha_{-i}}\right) F_i(s)$ to $y^*_i(s) = \left(\frac{x_i}{\alpha_i}\right) F_{-i}(s)$.

Notably, the entrepreneurs' policy motivation and the resulting rank-order spillovers induce a mutual dependence between the equilibrium score CDFs. The reason is that entrepreneur $i's$ score CDF directly affects the ideology of her opponent's policy at each score. In particular, if i starts to drop out of the contest (a higher $F_i(s)$), her opponent's equilibrium response is to be more ideologically-aggressive (a more extreme $y^*_{-i}(s)$). Because i is policy-motivated, this increases the harm to her of dropping out. The entrepreneurs' policy motivation thus magnifies the intensity of competition in the model.

Equilibrium of the Symmetric Model We now heuristically derive the unique equilibrium of the symmetric model; details are in the proof of Proposition 1. The equilibrium involves a common score CDF $F(s)$ satisfying $F(0) = 0$, i.e., both entrepreneurs are always active. Applying symmetry to Equation 2 yields the single differential equation $\alpha - F(s) = f(s) \cdot 4\frac{x^2}{\alpha} F(s)$. A simpler equation may be derived on the inverse $F^{-1}(F)$ by substituting in $F^{-1}(F)$ for s and observing that $f(F^{-1}(F)) = \frac{\partial}{\partial F}(F^{-1}(F))$, which produces

$$\frac{\partial}{\partial F}\left(F^{-1}(F)\right) = 4x^2 \left(\frac{F/\alpha}{\alpha - F}\right). \tag{3}$$

Solving and applying the boundary condition $F(0) = 0 \iff F^{-1}(0) = 0$ (i.e., both entrepreneurs are always active) then yields the unique equilibrium.

Proposition 1. *The inverse of the equilibrium score CDF in the symmetric model is* $F^{-1}(F) = 4x^2 \left(\ln\left(\frac{\alpha}{\alpha - F}\right) - \frac{F}{\alpha}\right)$*, and equilibrium proposal ideologies are* $y_i(s) = sign(x_i) \cdot \frac{x}{\alpha} F(s)$.

The equilibrium can be expressed more intuitively in terms of a common probability distribution $G(y)$ over the *ideological extremism* of entrepreneurs' proposals, and a function $s(y)$ that maps the ideological extremism of each policy to its score (recall that the quality $q(y)$ of a policy with ideological extremism y is $s(y) + y^2$). From Corollary 1, $y(s) = \frac{x}{\alpha} F(s) \iff s(y) = F^{-1}\left(\frac{y}{x/\alpha}\right)$. To derive $G(y)$, observe that the probability that an entrepreneur develops a policy less extreme than y is also the probability $F(s(y))$ that she develops a lower-score policy, $F\left(F^{-1}\left(\frac{y}{x/\alpha}\right)\right) = \frac{y}{x/\alpha}$. We thus obtain the following result.

Corollary 2. *In equilibrium, the ideological extremism of the entrepreneurs' policies is uniform on* $[0, \frac{x}{\alpha}]$*, and the score* $s(y)$ *of a policy with extremism* y *is* $4x\left(x \ln\left(\frac{x}{x-y}\right) - y\right)$.

2 Equilibrium Properties

The equilibrium strategies of the competitive symmetric model are illustrated in Figure 1 in ideology-quality space. For purposes of comparison, the figure also shows the monopoly policy that each entrepreneur would develop absent competition. In the competitive equilibrium, the entrepreneurs mix smoothly over developing policies located on two symmetric curves that extend from $(0,0)$ out to $\left(\pm\frac{x}{\alpha}, \bar{s} + \left(\frac{x}{\alpha}\right)^2\right)$. The distribution of each entrepreneur's policies projected onto the ideology (x) axis is uniform.

Benefits of Competition As a monopolist, each entrepreneur would produce a non-centrist policy with just enough quality to make the decisionmaker indifferent with the reservation policy. Competition forces the entrepreneurs to moderate the ideology of their proposals. Although they do not always produce policies that are higher-quality than the monopoly policy, their proposals are always strictly better for the decisionmaker. Thus, the decisionmaker always benefits from competition. This result contrasts with Rotemberg and Saloner's (1994) argument that firms benefit from narrowing their focus and eliminating internal competition. A key difference is that entrepreneurs in our model care about the policy outcome even if they lose, which intensifies productive competition.

Endogenous Extremism A striking property of equilibrium is that the decisionmaker always chooses the more ideologically-extreme proposal–in Figure 1, more-extreme proposals are located on higher decisionmaker indifference (i.e., score) curves. The decisionmaker's preference for policies on the "lunatic fringe" emerges endogenously from the entrepreneurs' strategic incentives to invest in quality. Because they are only interested in influencing outcomes, entrepreneurs never develop an extreme policy unless they also make the up-front investments necessary to give it a high chance of success (see Corollary 1).

While this prediction is no doubt unusual, it is important to recall that policy entrepreneurs in the real world may propose extreme policies for reasons other than a desire to influence outcomes. For example, they may engage in position-taking, i.e., making proposals that are unlikely to succeed, in order to please certain constituencies. Or, they may have intrinsic preferences over their policy proposals irrespective of the outcome. These factors would make it more difficult to observe our prediction in real-world policymaking. At a minimum, however, our model shows that empirical studies should take into account how strategic actors endogenously affect difficult-to-observe features of the policies that they propose (Triossi, Valdivieso, and Villena-Roldan 2013).

The decisionmaker's endogenous preference for extreme policies also generates equilibrium polarization in policy outcomes. As shown in Figure 2, more-extreme policies have higher density than less-extreme ones within the interval $\left[-\frac{x}{\alpha}, \frac{x}{\alpha}\right]$.[6] While polarization is often perceived as being a sign of political dysfunction, in our model it arises from ideologically-motivated actors' productive engagement in the policy process.

Comparative Statics The consequences of having more ideologically-extreme (higher x) and/or more skilled (lower α) entrepreneurs are simple to derive from the CDF over ideology $G(y) = \frac{y}{x/\alpha}$, the inverse CDF over score $F^{-1}(F)$ from Proposition 1, and the inverse CDF $H^{-1}(H)$ over quality, which is equal to $F^{-1}(H) + \left(H\frac{x}{\alpha}\right)^2$ because $q(y) = s(y) + y^2$.[7]

Proposition 2. *As the entrepreneurs become more extreme (higher x) or more skilled (lower α), their proposals become first-order stochastically more extreme, but also first-order stochastically higher quality and better for the decisionmaker.*

Polarized entrepreneurs naturally produce ideologically-extreme policies. However, as the entrepreneurs become more polarized their disagreement actually benefits the decisionmaker, because they invest more in quality. They have two reasons for doing this. First, they care more intensely about ideological gains. Second, each entrepreneur wants to prevent the other's increasingly-extreme policies from being adopted.

The effect of increasing the entrepreneurs' skill is actually similar to the effect of ideological polarization. Entrepreneurs who are better able to generate high-quality policies attempt to exploit that ability to achieve ideological gains, which results in extreme policy proposals, but also benefits the decisionmaker. Thus, an increase in observed policy extremism may occur as a by-product of greater skill at policy development, and does not necessarily indicate that actors with centrist preferences are worse off.

Entrepreneurs' Utility So far we have focused on how competitive policy development affects the decisionmaker, showing that he chooses more extreme (endogenous) policies over

[6] In this interval, the density over ideological outcomes is $\frac{|y|}{(x/\alpha)^2}$.

[7] The decisionmaker's equilibrium utility is $\int_0^{\bar{s}} \frac{\partial}{\partial s}\left([F(s)]^2\right) s \cdot ds = \int_0^1 \frac{\partial}{\partial F}\left(F^2\right) F^{-1}(F) \cdot dF$.

less extreme ones, benefits from competition, and prefers to have more skilled entrepreneurs despite the greater policy extremism that results. We now turn to the entrepreneurs, and show that these results do not, in general, extend to them.

First, for reasonable values of the cost parameter ($\alpha \geq 3$) each entrepreneur prefers the moderate policies within the support of her opponent's strategy, since the additional quality of extreme ones is insufficient to compensate for her ideological losses. The entrepreneurs are thus a "team of rivals," in the sense that disagreement endures even after they have made common-value investments in their respective policies.

Second, each entrepreneur is always harmed by the presence of a competitor.[8] In addition, for most values of α ($> \bar{\alpha} \approx 1.23$) the entrepreneurs would prefer to jointly give up policy development capacity and allow the decisionmaker to choose the reservation policy. The decisionmaker is thus the main beneficiary of competitive policy development, even though the entrepreneurs make common-value investments. This feature is shared with common agency models of influence (Dixit, Grossman, and Helpman, 1997), where equal and opposing interest groups are hurt by competition because influence is costly and policy is unchanged.

Finally, for $\alpha > \hat{\alpha} \approx 2.108$ entrepreneurs are harmed by shared improvements in their skill at policy-development, because of the greater ideological extremism of their policies as well as the more intense competition over quality. We summarize these observations below.

Proposition 3. *The entrepreneurs (i) prefer their opponents' more moderate equilibrium policies to their more extreme ones when $\alpha \geq 3$, (ii) prefer being a monopolist to facing competition, (iii) prefer the reservation policy to the competitive equilibrium when $\alpha > \bar{\alpha} > 1$, and (iv) prefer higher (common) costs of developing quality when $\alpha > \hat{\alpha} > \bar{\alpha}$.*

Only when $\alpha \in [1, \bar{\alpha}]$, i.e., when the entrepreneurs come sufficiently close to valuing quality for its own sake, do they benefit both from their joint ability to develop policies, and from further improvements in their capacity.

3 Additional Questions

Overall, our model paints a picture of vigorous engagement by ideological extremists in a competitive policy environment, a pattern that is consistent with prior contest-theoretic models of social conflict (e.g., Esteban and Ray 1999). Unlike previous work, however, the key vehicle for

[8]Entrepreneurs' equilibrium utility is $\Pi_i^*(\bar{s}; F) = -\alpha\left(\bar{s} + (x/\alpha)^2\right) + V_i(\bar{s}, x_i/\alpha_i) = -(1 - 1/\alpha)x^2 - (\alpha - 1)\bar{s}$, i.e., their utility from producing score $\bar{s} = F^{-1}(1)$ with ideology $\pm\frac{x}{\alpha}$ and winning for sure.

competition is productive investments in quality, which are strategically useful because they are valued by a decisionmaker who freely chooses which policy to implement. Consequently, competition benefits the decisionmaker despite observably-extreme ideological outcomes. In addition, the decisionmaker benefits from increasingly-polarized policy developers.

We now discuss variants of our model, to better understand these results and assess their robustness. We first comment on what happens if extremists play a direct role in the decision-making process. We then consider alternative assumptions about the set of entrepreneurs, to see whether centrists are likely to be active in addition to, or instead of, the two extremists in our model. Finally, we consider alternative assumptions about entrepreneurs' utilities and strategies, analyzing what happens if they disagree about "quality," if they can sabotage each other's policies, or if they are dogmatically unwilling to compromise.

Decisionmaking Authority In our model, the only way extremists can achieve influence is by crafting policies that appeal to a centrist decisionmaker. However, in many political institutions, decisionmaking authority is divided among several actors–e.g., passing a law in the United States requires approval of the President, the House, and enough Senators to overcome the threat of a filibuster. Particularly given the recent increase in elite-level political polarization, it is useful to consider what happens when extremists not only can develop policies, but also play a formal role in the policy choice process. In related work (Hirsch and Shotts 2014), we analyze the effect of noncentrist veto players. We show that their presence sometimes benefits a centrist decisionmaker, because they demand greater quality investments in exchange for ideological concessions. However, extremist veto players may also harm the decisionmaker when they are so resistant to change that they deter potential policy developers from attempting to craft new high-quality policies.

An Entrepreneur Aligned with the Decisionmaker In our model, the decisionmaker is assumed to lack policy-development capacity. However, some political leaders have direct subordinates who share their goals, as well as close allies that they could invite to develop policy proposals. A natural intuition is that the decisionmaker in our model would benefit if one of the extremist entrepreneurs were replaced by a centrist. In fact, the reverse is true: competition vanishes, the remaining extremist behaves as a monopolist, and the decisionmaker is no better off than with the reservation policy.

Proposition 4. *Fix $x_L < 0$ and $\alpha_L > 1$. If R is centrist ($x_R = x_D = 0$) and has any level of skill ($\alpha_R > 1$), the unique equilibrium is as if she were absent. The left entrepreneur develops* *her monopoly policy, the right entrepreneur does nothing, and the decisionmaker is no better off than with the reservation policy.*

The reason for this result is simple: in our model, productive investments are inspired by the prospect of ideological gains. A centrist entrepreneur's ideological interests are already protected by the decisionmaker, so to her, the left entrepreneur's policies are equivalent to centrist policies with quality s_L. Her cost of developing higher-quality centrist policies $\alpha_R s_R$ outweighs the benefit $s_R - s_L$, so she remains inactive.

Additional Entrepreneurs In many policymaking environments, multiple actors could choose to enter the fray and develop new policies. We now consider what happens when additional moderate entrepreneurs are introduced between the two extremists in our model.

Proposition 5. *The equilibrium with two symmetric entrepreneurs* $(-x_E, x_E)$ *and common costs* α *remains an equilibrium when* N *more-moderate* ($|x_i| \leq x_E$) *and less-skilled* ($\alpha_i \geq \alpha$) *entrepreneurs are present. In this equilibrium, the additional entrepreneurs are inactive.*

With additional entrepreneurs, there always exists an equilibrium in which (weakly) more-moderate and less-skilled entrepreneurs allow two extremists to shoulder the burden of policy development.[9] Compared to them, a moderate has less to gain from engaging in the policy contest–she values ideological gains less, and already benefits from an aligned extremist's participation. This suggests a tendency for ideologues to dominate policy contests.

Entrepreneurs Who Don't Value Quality on Each Other's Policies In some political environments, there are non-policy costs to the losing faction that increase with the quality of the winning policy–for example, the winning faction's strength may increase in future electoral, legislative, or bureaucratic contests. Moreover, "quality" may have very different meanings for factions on opposite sides of the political spectrum.[10] For example, a teachers' union may prefer that a voucher system be inefficient, cumbersome, and unreliable, because this would deter parents from pulling their children out of the public schools.

As a reduced form for analyzing such considerations, we check whether our results are robust to assuming that an entrepreneur places a lower, zero, or even negative value on the quality of her opponent's policies. Suppose each entrepreneur i values the quality of

[9] We do not rule out other equilibria. However, the equilibrium with two active extremists is unique if the additional entrepreneurs are at $x_D = 0$.

[10] However, Kendall, Nannicini, and Trebbi's (2013) study of Italian voters suggests that some voters don't discount the quality of ideologically-distant policies or candidates.

her opponent's policies at $(1-\beta)\,q_{-i}$ where $\beta \geq 0$, so that i's payoff from $-i$'s policy is $V_i\left(s_{-i}, y_{-i}\right) - \beta\left(s_{-i} + y_{-i}^2\right)$. With this setup, each entrepreneur effectively faces a higher cost of losing the contest. Solving this variant of our model yields the following results.

Proposition 6. *For all $\beta \geq 0$ there is a unique symmetric equilibrium in which: (i) The ideology of equilibrium policies is the same as in the baseline model. (ii) Proposition 2 holds. (iii) The quality and score of entrepreneurs' policies and the final policy outcome are first-order stochastically increasing in β. (iv) The marginal effect of extremism x on the decisionmaker's utility is increasing in β.*[11]

This proposition shows that our main results are actually strengthened when entrepreneurs discount each other's quality. Policy outcomes are no more ideologically extreme, but are higher-quality. Moreover, the benefits of extremism are stronger, in the sense that the marginal benefit to the decisionmaker of greater polarization is higher. These effects arise for the same reasons as our main results: the entrepreneurs' dislike of each other's policies, whether for ideological or "quality" reasons, is funneled into productive investments.

Sabotage An additional potential concern with our model is that real-world competition isn't always productive; political factions sometimes interfere with policy implementation or engage in other forms of sabotage to shift policy decisions in their favor. Although a complete analysis of sabotage is beyond the scope of our paper,[12] we briefly comment on why we believe it is reasonable to expect costly political activity to focus on productive investments.

Consider a variant of our model in which each entrepreneur i can also pay an up-front cost $\alpha_i^s q_i^s$ to engage in sabotage and thereby reduce the quality of her opponent's policy by q_i^s. If each entrepreneur uses a combination of productive investment and sabotage, then the decisionmaker prefers $i's$ proposal over $-i's$ i.f.f. $\left(q_i - q_{-i}^s\right) - y_i^2 \geq \left(q_{-i} - q_i^s\right) - y_{-i}^2$. Rearranging, this is equivalent to the decisionmaker's utility from $i's$ policy $q_i - y_i^2$, plus $i's$ level of sabotage q_i^s, exceeding that of her opponent.

What combination of investment q_i and sabotage q_i^s will entrepreneur i use? Clearly, if i engages in sabotage and fails to win, then she is worse off because her sabotage reduces the quality of the actual policy outcome. Less obvious, however, is the fact that using sabotage also harms i when she wins, because it redirects her effort away from productive investment. Thus, it can be shown that sabotage will only disrupt the equilibrium in Proposition 1 if its

[11] Note that we do not assert for this variant that all equilibria are symmetric.

[12] In particular, it is an open question whether the decisionmaker benefits from extremism of entrepreneurs in equilibria that involve a combination of sabotage and productive investment.

marginal cost is sufficiently lower than the marginal cost of investing in quality, i.e., $\alpha_i^s < \alpha_i - 1$. Only when this condition holds is sabotage sufficiently cheap to compensate the entrepreneur for both the harm she does to her opponent's policy, and the foregone value of investing in her own policy's quality.[13]

Note that this analysis uses our baseline assumption that entrepreneurs fully value quality even when they lose. However, because sabotage is more appealing for entrepreneurs who don't value each other's quality, we generalize the analysis to cover that case as well.

Proposition 7. *If entrepreneur i values $-i$'s quality at $(1-\beta)\, q_{-i}$ there exists an equilibrium without sabotage if and only if $\alpha^s \geq \alpha - (1-\beta)$.*

Discounting an opponent's quality thus increases the range of α^s where sabotage must occur. Nevertheless, as long as the entrepreneurs place some positive weight on each other's quality, sabotage can be absent even when it is strictly cheaper than investing in quality.[14]

Dogmatic Entrepreneurs A final issue we consider has to with the fact that the entrepreneurs in the model are risk-averse over ideology. This implies that, compared to moderates, extremists place a higher value on small shifts away from the reservation policy, and thus are more willing to invest in quality to achieve ideological gains. Although risk aversion is a standard assumption in the literature, it cannot capture the preferences of one type of political extremist: someone who only values policy done her way, and therefore places little or no weight on gains achieved via compromise.

We now consider how dogmatic, risk-loving entrepreneurs would influence our results. Suppose for simplicity that each entrepreneur's utility function takes the form $U_i(b) = q + 1_{y=x_i} \cdot B\,|x_i|$ where $B \geq 0$, so that she receives a net benefit of $B\,|x_i|$ whenever her ideal policy is chosen, but is indifferent over all other ideological outcomes. As in our baseline model, each entrepreneur's benefit to getting her ideal is increasing in her extremism. But unlike our baseline model, she only values her ideal ideological outcome and thus is strongly risk-loving. The preferences of the decisionmaker remain unchanged.

In contrast to our main model, a dogmatic entrepreneur will only develop positive-score policies at her own ideal point, because she doesn't value compromise victories. Thus each entrepreneur's $F_i(0)$ must be sufficiently high to make her opponent $-i$ willing to develop

[13] Konrad (2000) shows that sabotage often is not an optimal strategy in multiplayer rent-seeking contests; our analysis differs because we consider only two players, and quality investments are directly productive.

[14] Similarly, if sabotage only makes the policy less appealing to the decisionmaker (e.g., if it is negative campaigning) then an equilibrium with no sabotage exists if and only if $\alpha^s \geq \alpha$.

a proposal at x_{-i} with quality $q_{-i} \geq x_{-i}^2$. Conditional on developing a proposal, each entrepreneur mixes over a range of positive scores. The following proposition analyzes equilibria, including the one that is decisionmaker-optimal.

Proposition 8. *With dogmatic entrepreneurs, there exists an equilibrium in which the decisionmaker's utility is strictly positive if and only if* $x \in \left(0, \frac{B}{\alpha-1}\right)$. *The Online Appendix characterizes a symmetric equilibrium in which decisionmaker utility is (i) weakly greater than in any other equilibrium (ii) continuous and (iii) strictly quasiconcave over* $x \in \left[0, \frac{B}{\alpha-1}\right]$.

Thus, when the entrepreneurs have uncompromising preferences, it is no longer true that more polarization always benefits the decisionmaker. Instead, extremism is only good up to a point. It can spur investments in quality, but eventually the entrepreneurs' cost of compensating the decisionmaker for their ideal policies becomes so high that they drop out.

Also note that proposing a 0-quality extreme policy in lieu of the reservation policy is consistent with equilibrium–such a policy is free, will never be adopted, and so is effectively 0-score. "Inactivity" by dogmatic extremists in the real world may therefore be manifested as a form of position-taking: proposing low-quality policies that are hopelessly-extreme.

4 Related Literature

Our model provides a new approach to studying competition for intra- and inter-organizational influence. Several models of influence via information (Dewatripont and Tirole 1999; Landier, Sraer, and Thesmar 2010; Gul and Pesendorfer 2012) focus on a binary set of alternatives. More similar to our approach are models of competitive signaling in which a decisionmaker chooses from a continuum of policy options (Gilligan and Krehbiel 1989, Battaglini 2003). In contrast to our model, these models feature informational invertibility, an assumption that has been criticized by Callander (2008) for being unrealistic. Another odd feature of competitive signaling models is that each expert benefits from the other expert's presence, due to reduced variance of policy outcomes. In our policy-development model, in contrast, each entrepreneur would prefer be a monopolist rather than having to compete.

Other models analyze political influence via transfers (Groseclose and Snyder 1996; Dixit, Grossman, and Helpman 1997) and intra-firm competition is analyzed by several authors, including Milgrom and Roberts (1988) and Lazear and Rosen (1981). Many of these models assume contractibility. Within political institutions, however, nominal principals typically have sharply-limited means to control their subordinates (Moe 1984) and formal commitment is often impossible due to the lack of external enforcement. In some environments lacking formal contracts, repetition makes it possible to create informal relational ones (Baker, Gibbons, and Murphy 2002). However, this is less feasible in political environments, where many decisionmakers are short-lived (Heclo 1977).[15] Moreover, as was the case in the early years of the New Deal, political leaders typically face a high degree of urgency to enact policy on a given issue–as noted in Kingdon's (1995) classic book, "policy windows" open only briefly. This urgency further undermines leaders' ability to pressure subordinates to develop better policy proposals. In our model, competition serves as such a disciplining device.

Finally, we note that variants of our model could be applied to other environments where actors propose policies and compete to have them enacted by exerting costly effort, e.g., lobbying (Epstein and Nitzan 2004, Meirowitz and Jordan 2012) or valence competition in elections (Ashworth and Bueno de Mesquita 2009). In many such models, it would be natural to analyze the simultaneous choice of ideology and effort. However, to the best of our knowledge, previous research on lobbying and elections has focused on models in which actors choose ideological and nonideological offers sequentially, rather than analyzing them as two components simultaneously chosen in an all-pay contest.

5 Conclusion

We have presented a model of policy development in which factions have different ideologies or preferences, yet also agree on certain common objectives. Competing policy developers can appeal to a decisionmaker by making productive, policy-specific investments to improve the quality of their proposals. Rather than being tailored narrowly to any specific institution, our model is designed to capture features of many different political organizations, including legislatures, parties, democratic polities, NGOs, militaries, and government agencies.

The key features of our model's empirical domain are that different actors can develop policies for consideration by a decisionmaker, and that policy consists of both a common values component and a component over which actors disagree. For example, the commissions in charge of some U.S. government agencies have multiple members with different preferences, and even if a single decisionmaker like the board median is ultimately decisive, the other board members have the opportunity to develop policy proposals. Similarly, a wide range of interest groups are allowed to make policy suggestions in notice and comment procedures for

[15]Most high-level political appointees in the U.S. Federal Government stay for less than 18 months, and their subordinates are keenly aware of this fact.

regulatory rulemaking. Our model suggests that these institutions promote beneficial policy-development competition. It also suggests an explanation for the importance of programmatic parties in promoting effective governance in developing countries.[16] Parties with strong policy preferences have an incentive to invest in quality in order to increase their chances of gaining control of the government. Because a crucial form of quality in developing countries is choosing non-corrupt candidates for office, our model suggests that countries with programmatic parties will exhibit lower levels of corruption when compared to countries where parties don't have firm policy objectives.

Our analysis suggests several avenues for future work. One possibility is to analyze policy entrepreneurs' choice about whether to use targeted benefits like pork or collective benefits like policy quality to acquire support for their proposals. Another possibility is to consider aspects of institutional design, including subsidies for policy development, endogenous selection of entrepreneurs, addition of veto players, or delegation of decisionmaking authority.

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[16] We thank Georgy Egorov and Frederico Finan for suggesting this application of our model.

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Figure 1: Symmetric Model Equilibrium

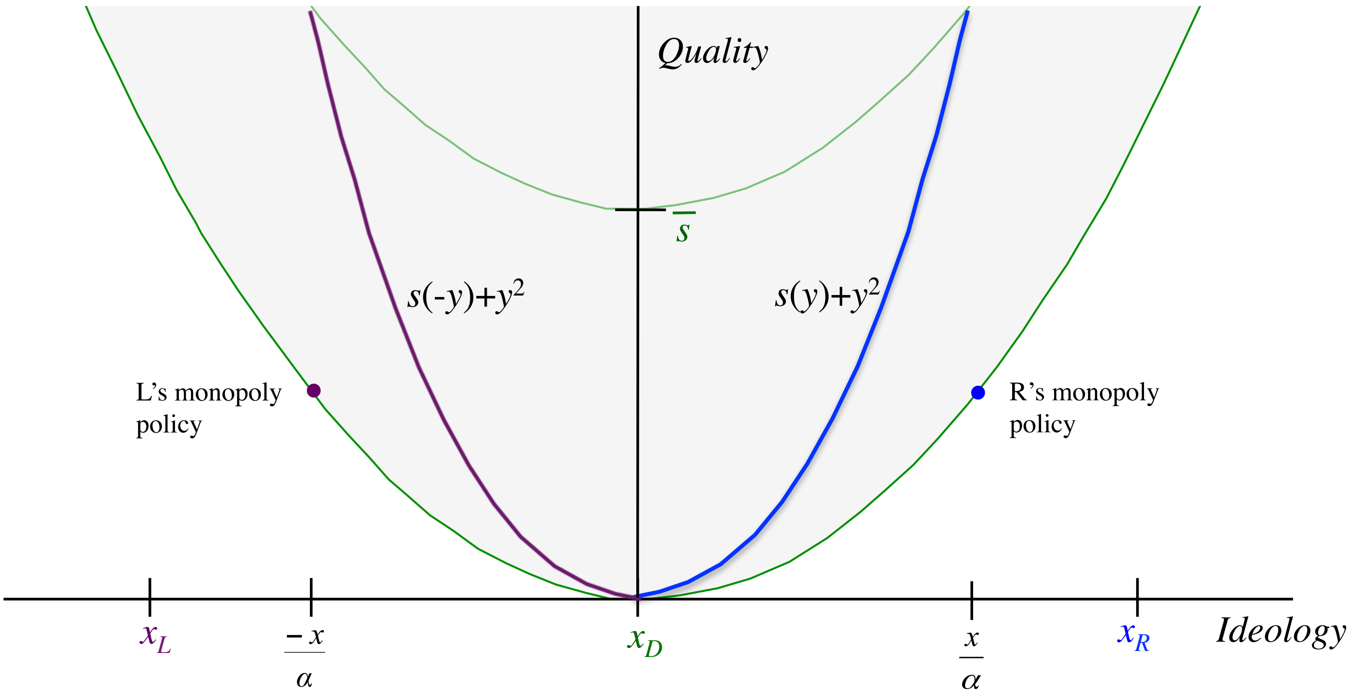
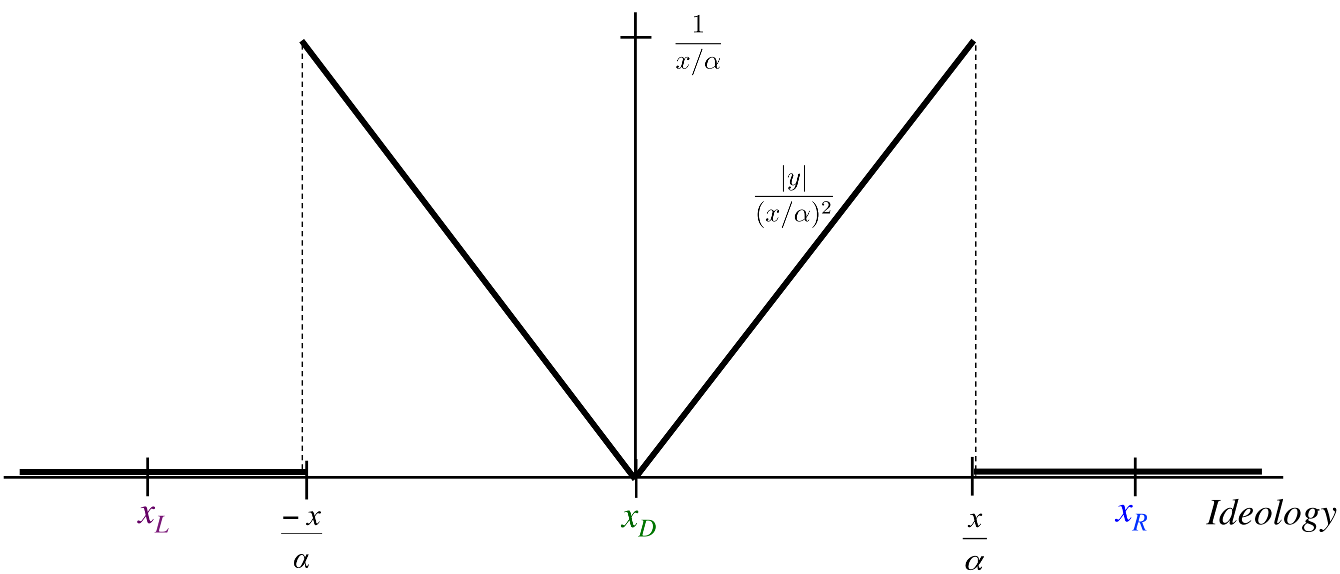


Figure 2: Density of Ideology of Final Policy



7 Online Appendix

This appendix is divided into three parts. Appendix A proves results in the main text. Appendix B is a complete treatment of the general model (which allows each entrepreneur to value the quality of her opponent's policies at $(1-\beta)\,q_{-i}$) and proves Lemma 1, which contains necessary conditions for equilibrium. Appendix C is a complete treatment of the variant with dogmatic entrepreneurs, and proves Lemma 2 which contains necessary conditions for equilibrium in that variant.

A Main Proofs

We first transform strategies (y, q) to be expressed in terms of score and ideology. An entrepreneur's pure strategy $b_i = (s_i, y_i)$ is a two-dimensional element of $\mathbb{B} \equiv \{(s, y) \in R^2 \mid s + y^2 \geq 0\}$, or the set of scores and ideologies that imply positive-quality policies. A mixed strategy σ_i is a probability measure over the Borel subsets of $\mathbb{B}$, and let $F_i\,(s)$ denote the CDF over scores induced by i's mixed strategy σ_i. For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution. The decisionmaker is the last mover, so equilibrium requires that he choose a policy (s, y) with the maximum score. While a complete description also requires specifying his tie-breaking rules, equilibria are invariant to this decision so we omit the additional notation.

In Appendix B we prove the following properties of equilibrium for both the baseline model, and the variant in which each entrepreneur values the quality of her opponent's policies at $(1-\beta)\,q_{-i}$.

Lemma 1. *The following properties hold for* $\beta \geq 0$.

1. *At any score* $s_i > 0$ *where* $-i$ *has no atom, developing* $(s_i, y_i^*(s_i))$ *with* $y_i^*(s_i) = F_{-i}(s_i)\frac{x_i}{a_i}$ *is strictly better than developing any other policy.*

2. *In any equilibrium,* $F_k\,(0) = 0$ *for some* $k \in \{L, R\}$ *and the support of the score CDFs* (F_i, F_{-i}) *over* $s \geq 0$ *is common, convex, atomless, and includes* 0.

We now prove the remaining results in the main text using this result.

Proof of Proposition 1 Since the CDFs are atomless over $(0, \infty)$ and at such scores developing $(s_i, y_i^*\,(s_i))$ is strictly better than any other policy (by Lemma 1), in equilibrium the probability a policy (s_i, y_i) with $s_i > 0$ satisfies $y_i = y_i^*\,(s_i)$ is 1. In addition, the score conditions in Lemma 1 (combined with our restriction on the strategy space) immediately imply that CDFs are absolutely continuous over $[0, \bar{s}]$ with $\bar{s} > 0$ and satisfy $F_k\,(0) = 0$ for some k and $\lim\limits_{s \to \bar{s}} \{F_i\,(s)\} = 1\ \forall i$.

Now the proof proceeds in three steps. First we derive a pair of differential equation on the score CDF (F_i, F_{-i}) that must be satisfied. Next we prove that with symmetric entrepreneurs the CDFs must be identical $F_i = F_{-i} = F$ and derive a unique solution to the system. Finally we prove that the resulting strategies yield an equilibrium.

Step 1

Define the function $\Pi_i^*(s_i; F)$ over all scores $s_i \geq 0$ to be equal to:

$$\Pi_i^*(s_i; F) = -\alpha_i \left(s_i + [y_i^*(s_i)]^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i^*(s_i)) + \int_{s_i}^{\infty} V_i\left(s_{-i}, y_{-i}^*(s_{-i})\right) dF_{-i}. \quad \text{(A.1)}$$

This is an entrepreneur's expected utility were she always to win at a score-tie (with her opponent or the reservation policy) with the policies $y_i^*(s_i)$ substituted in.

At an equilibrium strategy profile σ^* entrepreneur i's utility from developing the optimal policy at any score $s_i > 0$ is exactly equal to $\Pi_i^*(s_i; F)$. The statement is not necessarily true at $s_i = 0$, but i can achieve utility arbitrarily close to $\Pi_i^*(0; F)$ by developing a score ε above. Consequently $U_i^* \geq \Pi_i^*(s_i; F)$ for all $s_i \geq 0$. In addition, $\Pi_i^*(s_i; F) \geq U_i^*$ and thus $= U_i^*\ \forall s_i \in [0, \bar{s}]$; if instead for some $s_i \in [0, \bar{s}]$ we had $U_i^* > \Pi_i^*(s_i; F)$ then by continuity of (F_i, F_{-i}) over $s_i > 0$ (and right continuity at 0) i would be developing scores with positive probability that yield strictly lower utility than her equilibrium utility, a contradiction.

Finally, since (F_i, F_{-i}) are also absolutely continuous and strictly increasing $\forall s \in [0, \bar{s}]$, $U_i^* = \Pi_i^*(s_i; F)\ \forall s_i \in [0, \bar{s}] \iff \frac{\partial}{\partial s_i}(\Pi_i^*(s_i; F)) = 0$ for almost all $s \in [0, \bar{s}]$, which yields

$$\alpha_i - F_{-i}(s) = f_{-i}(s) \cdot 2x_i \left(\left(\frac{x_i}{\alpha_i}\right) F_{-i}(s) - \left(\frac{x_{-i}}{\alpha_{-i}}\right) F_i(s)\right). \quad \text{(A.2)}$$

Step 2

Substituting symmetric parameters into (A.2) yields

$$\alpha - F_{-i}(s) = f_{-i}(s) \cdot 2\frac{x^2}{\alpha}(F_L(s) + F_R(s)) \text{ for almost all } s \in [0, \bar{s}] \text{ and } i \in \{L, R\}.$$

We first prove that a solution must be symmetric. The above implies $\frac{f_L(s)}{\alpha - F_L(s)} = \frac{f_R(s)}{\alpha - F_R(s)}$ a.e., which then $\rightarrow \exists C$ s.t. $\log C - \log(\alpha - F_L(s)) = -\log(\alpha - F_R(s))$ a.e. and hence everywhere (since the F's are continuous), which then $\rightarrow \frac{\alpha - F_L(s)}{\alpha - F_R(s)} = C$. We now show that $F_L(0) = F_R(0)$, which $\rightarrow C = 1 \iff F_L(s) = F_R(s) = F(s)\ \forall s \in [0, \bar{s}]$. Suppose not, so there $\exists k$ s.t. $F_k(0) > 0$. Lemma 1 requires that $F_{-k}(0) = 0$, so $C > 1$, and rearranging the equality yields $1 - F_k(s) = (C-1)(\alpha - 1) + C(1 - F_{-k}(s))$. Now continuity and no atoms above 0 requires that $\lim_{s \to \bar{s}}\{1 - F_i(s)\} = 0\ \forall i$, but if $\lim_{s \to \bar{s}}\{1 - F_{-k}(s)\} = 0$ then $\lim_{s \to \bar{s}}\{1 - F_k(s)\} = (C-1)(\alpha - 1) > 0$, a contradiction; hence the entrepreneurs must use identical score CDFs $F(s)$ with $F(0) = 0$. The ideologies $y_i(s) = \frac{x_i}{\alpha} F(s)$ then follow from Lemma 1. The rest of the derivation is contained in the main text; also observe the differential equation on $F^{-1}(F)$ satisfies a Lipschitz condition and so has a unique solution.

Step 3

The strategies derived (a unique common score CDF and policies equal to $y_i^*(s) = \frac{x_i}{\alpha} F(s)$ with probability 1 since $F(0) = 0$) have been shown to be necessary for equilibrium. We now show they are an equilibrium and hence the unique one. First, observe that all policies $(s_i, y_i^*(s_i))$ s.t. $s_i \in (0, \bar{s}]$ yield utility equal to a $\hat{U}_i^*$ that is strictly higher than the utility from any other policy (s_i, y_i) with $s_i > 0$, since by construction $\Pi_i^*(s_i; F)$ is constant over $[0, \bar{s}]$ and strictly decreasing above (for $s_i > \bar{s}$, $\Pi_i^*(s_i; F) - \Pi_i^*(\bar{s}; F) = -(\alpha_i - 1)(s_i - \bar{s}) < 0$). This is also i's utility from playing her strategy. Second, at negative scores $s_i \leq 0$, developing a 0-quality policy (i.e. ideology $y_i = \pm\sqrt{-s_i}$) is strictly better than developing any other policy; $F(0) = 0$ implies neither entrepreneur ever ties with the reservation policy, so a weakly negative score policy with positive quality both always loses and never influences a tie. Finally, again since $F(0) = 0$ and is right continuous, negative-score 0-quality policies (including the reservation policy) yield $\Pi_i^*(0; F) = \hat{U}_i^*$. All policies thus yield the same or strictly less utility than $\hat{U}_i^*$ and we have an equilibrium. ■

Proof of Proposition 2 The CDF over ideological extremism $G(y) = \frac{y}{x/\alpha}$ is decreasing in x and increasing in α, showing the desired effects of ideology and costs. For the remaining CDFs we must work with their inverses. Observe that for a parameter p, $F^{-1}(F(s;p);p) = s \to \frac{\partial F}{\partial p} = -\frac{\partial F^{-1}/\partial p}{\partial F^{-1}/\partial F}$; now, since the score CDF and its inverse $\partial F^{-1}/\partial F$ are increasing functions, a CDF $F(s;p)$ is first order stochastically increasing in p i.f.f. its inverse is increasing in p. Now the inverse score CDF is $F^{-1}(F) = 4x^2\left(\ln\left(\frac{\alpha}{\alpha - F}\right) - \frac{F}{\alpha}\right)$ which is clearly increasing in x and decreasing in α, showing the desired comparative statics. Finally, the inverse quality CDF is $H^{-1}(H) = F^{-1}(H) + \left(H\frac{x}{\alpha}\right)^2$; both components are increasing in x and decreasing in α. ■

Proof of Proposition 3 To see that an entrepreneur's utility for her opponent's equilibrium policies is decreasing in their extremism when $\alpha \leq 3$, observe that i's utility when her opponent develops a policy of extremism y is $V_i(s(y), -sign(x_i)y) = -x^2 + s(y) - 2xy$; differentiating yields $s'(y) - 2x = \frac{4xy}{x-y} - 2x \leq 0$ i.f.f. $y \leq \frac{x}{3}$. Since ideological extremism is uniformly distributed over $\left[0, \frac{x}{\alpha}\right]$, the derivative is negative for all ideologies in the support i.f.f. $\alpha \leq 3$.

To show statements about the entrepreneurs' equilibrium utility, observe it is equal to

$$-\alpha\left(\bar{s} + (x/\alpha)^2\right) + V_i(\bar{s}, x_i/\alpha_i) = -(1 - 1/\alpha)x^2 - (\alpha - 1)\bar{s},$$

i.e., their utility from producing score $\bar{s}$ with ideology $\pm\frac{x}{\alpha}$ and winning for sure. From this it immediately follows that each entrepreneur is strictly worse off with competition than as a monopolist, because her utility as a monopolist is $-\alpha\left(x/\alpha\right)^2 + V_i\left(0, x_i/\alpha_i\right) = -\left(1 - 1/\alpha\right)x^2$ and $\bar{s} > 0$ in equilibrium.

Next, substituting in $\bar{s} = F^{-1}(1)$ and rearranging yields utility

$$-x^2\left(\alpha - 1\right)\left(4\ln\left(\frac{\alpha}{\alpha - 1}\right) - \frac{3}{\alpha}\right)$$

To show the remaining desired properties we show that this function is strictly single-troughed with minimum at $\hat{\alpha} > 1$, approaches 0 as α approaches 1, is strictly less than $-x^2$ (their utility from the reservation policy) at $\hat{\alpha}$, and approaches $-x^2$ as $\alpha \to \infty$. Thus, cost increases benefit the entrepreneurs for $\alpha \geq \hat{\alpha}$ (and over this set they are strictly worse off with competition than just getting the reservation policy). In addition, there $\exists \bar{\alpha} \in (1, \hat{\alpha})$ s.t. their utility is $> -x^2$ for $\alpha \in (1, \bar{\alpha})$.

Writing utility as $-x^2 f(\alpha)$, where $f(\alpha) = (\alpha - 1)\left(4\ln\left(\frac{\alpha}{\alpha-1}\right) - \frac{3}{\alpha}\right)$, we see that $\lim_{\alpha\to 1^+} f(\alpha) = 0$ and $\lim_{\alpha\to\infty} f(\alpha) = 1$. Next, we show $\exists \alpha^*$ s.t. a) $f(\alpha)$ is strictly concave below α^*, b) $f(\alpha^*) > 1$, and c) $f'(\alpha) < 0$ for $\alpha \geq \alpha^*$. These properties imply that $f(\alpha)$ has a unique maximum $\hat{\alpha} \in (1, \alpha^*)$ and $f(\hat{\alpha}) > 1$. Finally, the preceding observations imply that $f(\alpha) = 1$ at some $\bar{\alpha} < \hat{\alpha}$, and that $f(\alpha) < 1$ for $\alpha < \bar{\alpha}$ and > 1 for $\alpha > \bar{\alpha}$.

Property a) can be shown by taking the second derivative $f''(\alpha)$ and setting equal to 0; the solution is $\alpha^* = 3$. For property b) just evaluate at α^*. Property c) can be shown by rearranging the first derivative to be

$$\begin{aligned} & \frac{1}{\alpha^2}\left(4\alpha^2\log\left(\frac{\alpha}{\alpha - 1}\right) - (3 + 4\alpha)\right) = \frac{1}{\alpha^2}\left(\int_0^1 \frac{4\alpha^2}{\alpha - q}dq - 2\int_0^1 (3 + 4\alpha)\,q\,dq\right) \\ = & \frac{1}{\alpha^2}\left(\int_0^1 \frac{-4\alpha^2 - 6\alpha + q\left(6 + 8\alpha\right)}{\alpha - q}dq\right). \end{aligned}$$

The numerator is clearly < 0 $\forall q \in [0, 1]$ when $\alpha \geq \alpha^* = 3$. ■

Proof of Proposition 4 First note that our arguments about the form of equilibria do not apply here since they are based on the assumption that $x_i, x_{-i} \neq 0$. Now let $\Pi_i(s_i, y_i)$ denote i's expected utility from developing policy (s_i, y_i) (suppressing the dependence on the other players' strategies). Suppose $x_k = 0$ and $x_{-k} \neq 0$; then $V_k(s, y) = s$. Since k has the same utility as the DM she no longer cares about how ties are broken, so it is simple to verify that her utility from developing any policy (s_k, y_k) is equal to,

$$\Pi_k(0, 0) - \alpha_k\left(s_k + y_k^2\right) + F_{-k}\left(\max\{s_k, 0\}\right) \cdot \left(s_k - E\left[\max\{s_{-k}, 0\} \mid s_{-k} \leq s_k\right]\right).$$

The above is $= \Pi_i(0, 0)$ for $s_i + y_i^2 = 0$ and $< \Pi_i(0, 0)$ otherwise; so regardless of the other strategies k's $0-$quality policies are all equivalent, and strictly dominate all other policies.

Now consider $-k$; if her opponent is developing only 0-quality (and thus ≤ 0 score policies) then she can win for sure with score ε and achieve utility arbitrarily close to her monopoly utility from developing $\left(0, \frac{x_{-k}}{\alpha_{-k}}\right)$. It is also simple to verify that this utility is strictly higher than that from developing any other policy. Thus, if the DM picks $\left(0, \frac{x_{-k}}{\alpha_{-k}}\right)$ with probability < 1 when developed, $-k$'s best-response correspondence is empty. Conversely, any strategy profile in which k mixes over 0-quality policies, $-k$ develops $\left(0, \frac{x_{-k}}{\alpha_{-k}}\right)$, and the DM chooses it with probability 1 is an equilibrium. ■

Proof of Proposition 5 In the symmetric equilibrium there are no atoms, and both entrepreneurs develop strictly positive-score policies with probability 1. Developing any 0-quality policy is therefore exactly equivalent, and strictly better than developing a weakly negative score policy with positive quality. We thus henceforth refer to developing a 0-quality policy as being "inactive" and a strictly positive-score policy as being "active."

The net gain to the active entrepreneurs from producing $(s_i \geq 0, y_i)$ above inactivity is

$$-\alpha\left(s_i + y_i^2\right) + F\left(s_i\right) \cdot \left(\left(s_i - E\left[s \mid s \leq s_i\right]\right) + 2x_E\left(y_i + E\left[y\left(s\right) \mid s \leq s_i\right]\right)\right) \tag{A.3}$$

At the equilibrium strategies the maximum over all $(s_i \geq 0, y_i)$ is 0 since they achieve their equilibrium utility with the reservation policy.

Now the expected ideological outcome conditional on both active entrepreneurs producing scores $\leq s$ is 0 by symmetry. In addition let $s_{\max}$ denote the maximum score, which is distributed according to $\left[F\left(s_{\max}\right)\right]^2$, and of course $E\left[s \mid s \leq s_i\right] < E\left[s_{\max} \mid s_{\max} \leq s_i\right]$. So the net gain to an inactive additional entrepreneur from entering with policy $(s_i \geq 0, y_i)$ is

$$-\alpha_i\left(s_i + y_i^2\right) + \left[F\left(s_i\right)\right]^2 \cdot \left(\left(s_i - E\left[s_{\max} \mid s_{\max} \leq s_i\right]\right) + 2x_i y_i\right) \tag{A.4}$$

Equation A.3 is $\geq$ than Equation A.4 $\forall\left(s_i \geq 0, y_i\right)$ when $\alpha_i \geq \alpha$ and $|x_i| \leq |x_E|$; so the maximum gain from activity for such an entrepreneur is ≤ 0 and inactivity is optimal. ■

Proof of Proposition 6 Before beginning the main proof we prove an accessory lemma.

Lemma A.1. *Consider a continuous function $h\left(x\right)$ that is almost-everywhere differentiable, and let $h^i\left(x\right)$ denote the i'th derivative of h (with $h^0\left(x\right) = h\left(x\right)$). Then the following two conditions imply that $h\left(x\right)$ is increasing in $x \geq 0$.*

1. *$h^k\left(0\right) = 0 \;\forall$ integer $k \in [0, i]$, and $h^{i+1}\left(0\right) > 0$*
2. *$h\left(x\right) > 0 \rightarrow h'\left(x\right) > 0$ wherever h is differentiable*

Proof. We first argue that property (1) implies $h(x) > 0$ in a neighborhood above 0. First observe that $sign\{h(x)\} = sign\left\{\frac{h(x)}{x^{i+1}}\right\}$ for $x > 0$. Next, repeated applications of L'Hopital's rule imply that $\lim_{x\to 0}\left\{\frac{h(x)}{x^{i+1}}\right\} = \frac{h^{i+1}(0)}{(i+1)!} > 0$, which then implies $h(x) > 0$ in a neighborhood above 0. Next, when the preceding holds then by property 2 $h(x)$ is also strictly increasing in that neighborhood almost everywhere; it therefore must remain positive and strictly increasing thereafter. ■

The proof now proceeds in two steps. In Step 1 we derive the unique symmetric equilibrium, and in Step 2 we show it has the desired properties.

Step 1

Beginning with the necessary conditions in Lemma 1, the proof proceeds identically as the proof of Proposition 1 through the end of Step 1, yielding a modified differential equation

$$\alpha_i - F_{-i}(s) = f_{-i}(s)\cdot 2x_i\left(\left(\frac{x_i}{\alpha_i}\right)F_{-i}(s) - \left(\frac{x_{-i}}{\alpha_{-i}}\right)F_i(s) + \beta\left(s + \left(\frac{x_{-i}}{\alpha_{-i}}F_i(s)\right)^2\right)\right). \quad \text{(A.5)}$$

that must be satisfied a.e. in the support $[0,\bar{s}]$ and generates a constant $\Pi_i^*(s_i;F)$ for $s\in[0,\bar{s}]$. Relative to Equation 2, losing now entails an additional cost of $\beta\left(s + \left(\frac{x_{-i}}{\alpha_{-i}}F_i(s)\right)^2\right)$, which is the share of an opponent's quality that isn't valued.

In the baseline model we proved that the equilibrium with symmetric parameters is unique and symmetric; for this variant we make the weaker assertion that there is a unique symmetric equilibrium, but do not rule out other asymmetric equilibria. Substituting the symmetric parameters and common score CDF $F(s)$ yields the differential equation $\alpha - F(s) = f(s)\left(4\frac{x^2}{\alpha}F(s) + \beta\left(s + \left(\frac{x}{\alpha}F(s)\right)^2\right)\right)$. For notational simplicity we will henceforth use $s(F)$ to denote the inverse $F^{-1}(F)$, and again as in the argument for Proposition 1 we may derive a simpler differential equation for the inverse $s(F)$,

$$s'(F) = \frac{4\frac{x^2}{\alpha}F + \beta\left(s(F) + \left(\frac{x}{\alpha}F\right)^2\right)}{\alpha - F}. \quad \text{(A.6)}$$

The differential equation satisfies a Lipschitz condition and thus has a unique solution with the boundary condition $s(0)=0$ which is necessary. It can be verified that this solution is:

$$s(F;x,\alpha,\beta) = x^2\left(\frac{(8+6\beta)\cdot\left(\left(\frac{\alpha}{\alpha-F}\right)^{\beta} - 1 - \frac{\beta}{\alpha}F\right) - (1+\beta)\left(\frac{\beta F}{\alpha}\right)^2}{\beta(1+\beta)(2+\beta)}\right)$$

and yields a well defined $\bar{s} = s(1) < \infty\ \forall\beta > 0$. Finally, the argument that these strategies are indeed an equilibrium is the same as in step 3 in the proof of Proposition 1. The fact that the distribution of ideologies is identical to the baseline model follows directly from Lemma 1, using the fact that in a symmetric equilibrium $y_i^*(s_i) = F_{-i}(s_i)\frac{x_i}{a_i}$ and $F(0) = 0$.

Step 2

We show that $s(F; x, \alpha, \beta)$ is increasing in F, and $\forall F \in (0, 1]$ is increasing in x and β, decreasing in α, and satisfies increasing differences in (x, β). By the arguments in the proof of Proposition 2 this suffices to show the desired first-order stochastic changes in both the score and quality CDFs from (x, β, α). It also suffices to show that the cross partial in (x, β) of the DM's equilibrium utility is positive, since (from footnote 7) the DM's equilibrium utility is $\int_0^1 2F \cdot s(F; x, \alpha, \beta)\, dF$, and the cross partial in (x, β) is $\int_0^1 2F \cdot \frac{\partial^2(s(F;x,\alpha,\beta))}{\partial x \partial \beta} dF > 0$.

Despite a closed form solution, it is nevertheless easier to derive comparative statics directly from the differential equation in $(A.6)$; it will also be helpful to have the first, second, and third derivatives of $s(F)$ evaluated 0; repeated differentiation of $(A.6)$ and employing the boundary condition $s(0) = 0$ yields that $s'(0) = 0$, $s''(0) = 4\frac{x^2}{\alpha^2}$, and $s'''(0) = \left(\frac{2x^2}{\alpha^3}\right) \cdot (3\beta + 4)$.

We now show the desired properties of $s(F; x, \alpha, \beta)$ by repeatedly employing Lemma A.1.

- To see $s(F)$ increasing in F, observe that property 1 follows immediately from the derivatives evaluated at 0, and property 2 is easily verified from Equation $A.6$.

- To see $s(F; x)$ increasing in x for $F > 0$, we show that $s(F; \hat{x}) - s(F; x)$ is > 0 $\forall \hat{x} > x$ and $F > 0$ (it is equal to 0 for $F = 0$ and any values of $x, \hat{x}$). To do this, we show the stronger property that $s(F; \hat{x}) - s(F; x)$ is increasing in F by showing it satisfies properties 1 and 2 of Lemma A.1. Differentiating using Equation A.6 yields $s'(F; \hat{x}) - s'(F; x) = \frac{1}{\alpha - F} \cdot \left(\left(\frac{4F}{\alpha} + \beta\frac{F^2}{\alpha^2}\right)(\hat{x}^2 - x^2) + \beta\left(s(F; \hat{x}) - s(F; x)\right)\right)$, which is positive when $s(F; \hat{x}) - s(F; x) > 0$ and therefore satisfies property 2. Property 1 follows from observing (using the previously derived derivatives) that the first derivative of the difference at $F = 0$ is 0 and the second derivative is $\frac{4}{\alpha^2}(\hat{x}^2 - x^2) > 0$.

- For identical reasons we show $s(F; \hat{\beta}) - s(F; \beta)$ satisfies properties 1 and 2 for $\hat{\beta} > \beta \geq 0$. Differentiating using Equation A.6 yields $s'(F; \hat{\beta}) - s'(F; \beta) = \frac{\left(\hat{\beta} - \beta\right)\left(\frac{x}{\alpha}F\right)^2 + \left(\hat{\beta}s\left(F; \hat{\beta}\right) - \beta s(F; \beta)\right)}{\alpha - F}$. To see the function satisfies property 2 observe that $\hat{\beta}s(F; \hat{\beta}) - \beta s(F; \beta) > \hat{\beta}\left(s\left(F; \hat{\beta}\right) - s(F; \beta)\right)$. To see it satisfies property 1 observe that the first and second derivatives at 0 are 0, and the third derivative is $\left(\frac{6x^2}{\alpha^3}\right) \cdot \left(\hat{\beta} - \beta\right) > 0$.

- We show $s(F; \alpha) - s(F; \hat{\alpha})$ satisfies properties 1 and 2 for $1 \leq \alpha < \hat{\alpha}$. The function satisfies property 2 since $s'(F; \alpha)$ is increasing in $s(F; \alpha)$ and decreasing in α (holding $s(F; \alpha)$ fixed), so $s(F; x, \alpha) > s(F; x, \hat{\alpha}) \rightarrow s'(F; x, \alpha) > s'(F; x, \hat{\alpha})$. It also satisfies property 1 since $s''(0; \alpha) - s''(0; \hat{\alpha}) = 4x^2\left(\frac{\hat{\alpha}^2 - \alpha^2}{\alpha^2\hat{\alpha}^2}\right) > 0$.

- We show that $(s(F;\hat{x},\hat{\beta}) - s(F;x,\hat{\beta})) - (s(F;\hat{x},\beta) - s(F;x,\beta))$ satisfies properties 1 and 2 for $\hat{x} > x$ and $\hat{\beta} > \beta \geq 0$. Differentiating using Equation A.6 yields,

$$\frac{1}{\alpha - F}\left(\hat{\beta}\left(s\left(F;\hat{x},\hat{\beta}\right) - s\left(F;x,\hat{\beta}\right)\right) - \beta\left(s\left(F;\hat{x},\beta\right) - s\left(F;x,\beta\right)\right) + (\hat{\beta}-\beta)\frac{F^2\left(\hat{x}^2 - x^2\right)}{\alpha^2}\right).$$

  This is $> \frac{\hat{\beta}}{\alpha - F}\left(\left(s\left(F;\hat{x},\hat{\beta}\right) - s\left(F;x,\hat{\beta}\right)\right) - \left(s\left(F;\hat{x},\beta\right) - s\left(F;x,\beta\right)\right)\right)$ so the function satisfies property 2. It also satisfies property 1 since first and second derivatives at 0 are 0, but the third derivative is $\left(\frac{6}{\alpha^3}\right)\cdot\left(\hat{x}^2 - x^2\right)\cdot\left(\hat{\beta}-\beta\right) > 0$. ■

Proof of Proposition 7 Suppose we are at an equilibrium strategy profile satisfying Lemma 1 and nobody engages in sabotage. Consider a deviation by i to a profile (s_i, y_i, q_i^s) with sabotage $q_i^s > 0$. Then the probability that $-i$'s policy yields utility $\leq s$ is $F_{-i}(s + q_i^s)$. Since by Lemma 1 neither entrepreneur has an atom above 0 in equilibrium, with some sabotage an opponent's policy never ties with the reservation policy. So all the negative score 0 quality policies $s_i < 0$, $y_i = \pm\sqrt{s_i}$ are strictly better than the positive quality ones, and yield the same utility as the reservation policy with sabotage $(0, 0, q_i^s)$. In addition, winning outright when $s_{-i} \leq s_i + q_i^s$ and $s_i = 0$ is at least as good as tying with the reservation policy (since a tie generates a mixture between i's policy and the reservation policy). We therefore restrict attention to deviations with $q_i^s > 0$ and $s_i > 0$.

Now i's utility if she develops such a policy is,

$$-\alpha_i\left(s_i + y_i^2\right) - \alpha_i^s q_i^s + F_{-i}\left(s_i + q_i^s\right)V_i\left(s_i, y_i\right) + \int\limits_{s_{-i} > s_i + q_i^s}\left(V_i\left(s_{-i}, y_{-i}\right) - \beta\left(s_{-i} + y_{-i}^2\right) - (1-\beta)\,q_i^s\right)d\sigma_{-i}.$$

If instead she reallocated her sabotage to productive effort $(s_i + q_i^s, y_i, 0)$ then it is easy to verify that her net gain is weakly positive i.f.f.

$$F_{-i}\left(s_i + q_i^s\right)\cdot q_i^s + \left(1 - F_{-i}\left(s_i + q_i^s\right)\right)\cdot(1-\beta)\,q_i^s \geq \left(\alpha_i - \alpha_i^s\right)q_i^s.$$

The inequality is most difficult to satisfy if $F_{-i}\left(s_i + q_i^s\right) = 0$ (since $1 - \beta \leq 1$), and holds in this case $\iff$ $\alpha_i^s \geq \alpha_i - (1-\beta)$. Thus, if this condition holds then some strategy without sabotage is at least as good as any strategy with sabotage, and the no sabotage equilibrium holds. Conversely, if this condition fails then the inequality fails for $F_{-i}\left(s_i + q_i^s\right)$ sufficiently small. Since by Lemma 1 $F_{-i}(0) = 0$ for some i in any equilibrium, there $\exists q_i^s$ and s_i sufficiently small s.t. every policy $(s_i + q_i^s, y_i, 0)$ with only productive effort (including the one in i's support) is strictly worse than the policy (s_i, y_i, q_i^s) with sabotage; hence she has a profitable deviation and no sabotage is not an equilibrium. ■

Proof of Proposition 8 In this variant, the definition of a score and the DM's best responses remain unchanged. With the entrepreneurs' modified utility function, $i's$ utility for a policy (s, y) being implemented is $V_i(s, y) = (s + y^2) + 1_{y=x_i} \cdot Bx_i$. Observe that the entrepreneurs have dogmatic preferences over ideology, but nevertheless still value quality on their opponent's policy; the model therefore still has rank-order spillovers. We make this assumption to preserve comparability to the main model (although it stands to reason that a dogmatic entrepreneur would also discount her opponent's quality).

In Appendix C we prove the following lemma.

Lemma 2. *Developing a policy* (s_i, y_i) *with* $s_i > 0$ *and either (i)* $y_i \neq x_i$*, or (ii)* $y_i = x_i$ *and* $F_{-i}(s_i) \leq \frac{\alpha_i x_i}{x_i + B}$*, is strictly worse than developing (0,0). In addition, the support of the equilibrium score CDFs over* $s \geq 0$ *is common, convex, atomless above 0, and includes* 0*.*

Since the CDFs are atomless over $(0, \infty)$ and at such scores developing (s_i, x_i) is strictly better than any other policy, in equilibrium (i) the probability a policy (s_i, y_i) with $s_i > 0$ satisfies $y_i = x_i$ is 1, (ii) the CDFs are absolutely continuous over $[0, \bar{s}]$ with $\bar{s} > 0$ and satisfy $\lim_{s \to \bar{s}} \{F_i(s)\} = 1 \; \forall i$. The proof now proceeds in two steps. First, we derive an equilibrium and show it must yield utility at least as high as any other equilibrium. Second, we prove that the decisionmaker's utility in the equilibrium described satisfies the conditions of Proposition 8.

Step 1

First we argue that in any equilibrium the entrepreneurs use a common score CDF $F_{-i}(s) = F_i(s) = F(s)$ for $s \geq 0$. Since the CDFs have common convex support, either $F_i(0) = 1 \; \forall i$ (in which case the properties hold trivially), or $F_i(0) < 1 \; \forall i$. As in the proof of Proposition 1 define $\Pi_i^*(s_i; F)$ over all scores $s_i \geq 0$ as:

$$\Pi_i^*(s_i; F) = -\alpha_i \left(s_i + x_i^2\right) + F_{-i}(s_i) \cdot \left(s_i + x_i^2 + Bx_i\right) + \int_{s_i}^{\infty} \left(s_{-i} + x_{-i}^2\right) dF_{-i}, \qquad \text{(A.7)}$$

and by identical arguments $\Pi_i^*(s_i; F) = U_i^* \; \forall s_i \in [0, \bar{s}]$. Substituting in the symmetric parameters this condition holds i.f.f. for almost all $s \in [0, \bar{s}]$,

$$\alpha - F_{-i}(s) = f_{-i}(s) \cdot Bx \quad \forall i. \qquad \text{(A.8)}$$

The above implies that $c_i - \log(\alpha - F_i(s)) = \frac{s}{Bx} \; \forall s \in [0, \bar{s}]$ with $\bar{s} < \infty$; since $F_i(\bar{s}) = F_{-i}(\bar{s}) = 1$ we have $c_i = c_{-i}$ and $F_i(s) = F(s) \; \forall s \in [0, \bar{s}]$, i.e., a common score CDF over this range. Inserting the starting value $F(0)$ yields $\log\left(\frac{\alpha - F(0)}{\alpha - F(s)}\right) = \frac{s}{Bx}$ in any equilibrium. In addition, $F(0) \geq \min\left\{\frac{\alpha x}{x+B}, 1\right\}$; if instead $F(0) < \frac{\alpha x}{x+B}$ then $\bar{s} > 0$ and by Lemma 2 both entrepreneurs are developing policies with scores close to 0 that yield strictly less utility than developing the reservation policy, a contradiction.

Second we argue that for distinct equilibria yielding score CDFs $\left(F, \hat{F}\right)$, the DM's utility is strictly greater in the former i.f.f. $F(0) < \hat{F}(0)$. In any equilibrium $\log\left(\frac{\alpha - F(0)}{\alpha - F(s)}\right) = \frac{s}{Bx}$ $\forall s \in [0, \bar{s}]$; so if there are two distinct equilibrium score CDFs $\left(F, \hat{F}\right)$ with $F(0) < \hat{F}(0)$ then it is easily verified that $\hat{F}(s) \geq F(s)$ $\forall s > 0$ with positive measure strict; so the DM's utility $\int_{s \geq 0} \frac{\partial}{\partial s}\left([F(s)]^2\right) ds$ is strictly higher with F than $\hat{F}$ (and equal if $F(0) = \hat{F}(0)$).

Finally we assert that the following symmetric strategies are an equilibrium (i) each entrepreneur develops either $(0,0)$ or (s, x_i) with $s > 0$, (ii) each entrepreneur uses the CDF $F(s) = 0$ for $s < 0$, $F(s)$ solving $\log\left(\frac{\alpha - \frac{\alpha x}{x+B}}{\alpha - F(s)}\right) = \frac{s}{Bx}$ for $s \in \left[0, Bx \log\left(\frac{\alpha - \frac{\alpha x}{x+B}}{\alpha - 1}\right)\right]$, and $F(s) = 1$ above. Using similar arguments to those in step 3 of the proof of Proposition 1, it is easily established that (i) all policies (s_i, x_i) s.t. $s_i \in (0, \bar{s}]$ yield utility equal to some value $\hat{U}_i^*$ that is strictly higher than the utility from any other policy (s_i, y_i) with $s_i > 0$, and (ii) at strictly negative scores $s_i < 0$ developing a 0-quality policy (i.e. ideology $y_i = \pm\sqrt{-s_i}$) is strictly better than developing any other policy, and yields the same utility as developing the reservation policy.[17] It is also easy to verify that developing the reservation policy yields $\hat{U}_i^*$; thus both entrepreneurs get $\hat{U}_i^*$ by playing their strategies.

It remains only to show that developing $(0, y_i)$ with $y_i \neq 0$ is not a profitable deviation. Since $-i$ develops (0,0) at 0, the utility from actually developing $(0, y_i)$ is weakly worse than the utility from developing it and always winning when $s_{-i} \leq 0$, which in turn is $\leq \Pi_i^*(0; F) = U_i^*$. All policies thus yield the same or strictly less utility than $\hat{U}_i^*$ and we have an equilibrium; since $F(0) = \frac{\alpha x}{\alpha + B}$, its lowest possible value, the DM's utility is at least as high as in any other equilibrium.

Step 2

We now show the decisionmaker's utility in the equilibrium characterized satisfies Prop. 8. If $\frac{\alpha x}{x+B} \geq 1$ then $F(0) = 1$ and the decisionmaker's equilibrium utility is 0. If $\frac{\alpha x}{x+B} < 1$ then since the DM's utility $U(x)$ is $\int_{F(0)}^{1} \frac{\partial}{\partial F}\left(F^2\right) F^{-1}(F) \cdot dF$ we have

$$U(x) = 2Bx \cdot \int_{\frac{\alpha x}{x+B}}^{1} F \log\left(\frac{\alpha - \frac{\alpha x}{x+B}}{\alpha - F}\right) dF.$$

Clearly this is equal to 0 at $x = 0$ and $\frac{B}{\alpha - 1}$, and positive in between. We now show that this expression is strictly quasi-concave (single peaked) in between, by showing that (i) $U'(0) > 0$, (ii) $U'\left(\frac{B}{\alpha-1}\right) = 0$, and (iii) $\exists x^* \in \left(0, \frac{B}{\alpha-1}\right)$ s.t. $U''(x) < (=)(>) 0 \iff x < (=)(>) x^*$. Then (iii) implies that U' is strictly single troughed and minimized at $x^* \in \left(0, \frac{B}{\alpha-1}\right)$, (ii) implies that $U'(x^*) < 0$, and (i) implies that $\exists \hat{x} \in (0, x^*)$ with $U' > 0$ for $x \in (0, \hat{x})$ and < 0 for

[17] The proof of Proposition 1 relies on $F_i(0) = 0$ $\forall i$; here we have "ties" at the 0-score but they are inconsequential because both entrepreneurs develop the reservation policy at this score.

$x \in \left(\hat{x}, \frac{B}{\alpha-1}\right)$, yielding strict single peakedness. The first derivative is

$$2B\left(\int_{\frac{\alpha x}{x+B}}^{1} F \log\left(\frac{\alpha - \frac{\alpha x}{x+B}}{\alpha - F}\right) dF - \left(1 - \frac{\alpha x}{x+B}\right) \cdot \left(\frac{x}{x+B}\right)\right)$$

and it is easily verified that this satisfies (i) and (ii). The second derivative is,

$$2B\left(x\frac{(B-(\alpha-1)\,x)+\alpha B}{(x+B)^3} - 2\left(1-\frac{\alpha x}{x+B}\right)\cdot\left(\frac{1}{x+B}\right)\right),$$

which is $>$ $(=)$ $(<)$ 0 i.f.f. $\frac{\alpha B x}{x+2B} > (=)\,(<)\,B-(\alpha-1)\,x$. The l.h.s. is strictly increasing in x, the r.h.s. is strictly decreasing in x, and it is easily verified that l.h.s.<r.h.s. when $x = 0$ and l.h.s.>r.h.s. when $x = \frac{B}{\alpha-1}$. ■

B Complete Treatment of Baseline Model

We prove Lemma 1 as a sequence of four lemmas. From the main text, an entrepreneur's utility for her own policy is $V_i(s_i, y_i)$, while for her opponent's policy is $V_i(s_{-i}, y_{-i}) - \beta\left(s_{-i} + y_{-i}^2\right)$ (since she discounts its quality if $\beta \geq 0$). Let $\Pi_i(s_i, y_i; \sigma_{-i})$ denote i's expected utility from developing policy (s_i, y_i) (suppressing the dependence on the DM's tie-breaking rules). At any score $s_i > 0$ where $-i$ has no atom, this expected utility is equal to

$$-\alpha_i\left(s_i + y_i^2\right) + F_{-i}(s_i) \cdot V_i(s_i, y_i) + \int\limits_{s_{-i} > s_i} \left(V_i(s_{-i}, y_{-i}) - \beta\left(s_{-i} + y_{-i}^2\right)\right) d\sigma_{-i}. \tag{B.1}$$

Taking the first order condition with respect to y_i yields the following.

Lemma B.1. *At any score $s_i > 0$ where $-i$ has no atom, developing $(s_i, y_i^*(s_i))$ with $y_i^*(s_i) = F_{-i}(s_i) \cdot \frac{x_i}{a_i}$ is strictly better than developing any other policy.*

Next we show that in equilibrium there is 0 probability of a tie at a strictly positive score.

Lemma B.2. *In equilibrium there is 0-probability of a tie at scores $s > 0$.*

Proof: Suppose not, i.e., each player's strategy generates an atom of size p_i at some common $s > 0$. Let $\bar{y}_t$ denote the expected ideological outcome conditional on the tie at score s (which depends on both players' strategies and the decisionmaker's tiebreaking rule). Let $F_{-i}^-(s_i) = \lim\limits_{s \to s_i^-} \{F_{-i}(s)\}$, and let $w_i(y_i, y_{-i}; s)$ denote the probability the DM chooses i's policy when the entrepreneurs develop (s, y_i) and (s, y_{-i}).

Entrepreneur k's utility from playing according to her strategy conditional on a tie (which may involve mixing over ideologies) is

$$\begin{aligned}
&-\alpha_k Var\left[y_k \mid s\right] - \alpha_k\left(s + \left(E\left[y_k \mid s\right]\right)^2\right) + F_{-k}^-(s) \cdot V_k\left(s, E\left[y_k \mid s\right]\right) \\
&+ p_{-k} \cdot \left(V_k(s, \bar{y}_t) - \beta \int\limits_{s_k = s} \int\limits_{s_{-k} = s} w_{-k}(y_k, y_{-k}; s) \cdot \left(s_{-k} + y_{-k}^2\right) \frac{d\sigma_k}{p_k} \frac{d\sigma_{-k}}{p_{-k}}\right) \\
&+ \int\limits_{s_{-k} > s} \left(V_i(s_{-k}, y_{-k}) - \beta\left(s_{-k} + y_{-k}^2\right)\right) d\sigma_{-k}
\end{aligned}$$

Since the entrepreneurs want to move ideology in strictly opposite directions conditional on a score, $V_k(s, 0) \geq V_k(s, \bar{y}_t)$ for at least one k. In addition, tying also involves a potential quality discount cost when $-k$'s policy wins. Rearranging, k's utility from playing according to her strategy conditional on a tie (which may involve mixing over ideologies), is $\leq$

$$-\alpha_k \left(s + E\left[y_k^2 \mid s\right]\right) + F_{-k}^{-}(s) \cdot V_k(s, E[y_k \mid s])$$
$$+p_{-k} \cdot V_k(s, 0) + \int_{s_{-k} > s} \left(V_i(s_{-k}, y_{-k}) - \beta\left(s_{-k} + y_{-k}^2\right)\right) d\sigma_{-k}.$$

Now let $\underline{y} = \lim_{s_k \to s^-} \{y_k^*(s_k)\}$; by definition $E[y_k \mid s] = \underline{y}$ with zero variance maximizes the first line. Moreover since $\underline{y}$ is weakly better than 0 for k, the above is $\leq \lim_{s_k \to s^+} \left\{\Pi_k\left(s_k, \underline{y}; \sigma_{-k}\right)\right\}$; but this in turn is strictly $< \lim_{s_k \to s^+} \{\Pi_k(s_k, y_k^*(s); \sigma_{-k})\}$ (because $p_{-k} > 0$ implies $\underline{y} \neq \lim_{s_k \to s^+} \{y_k^*(s_k)\}$, so tying must be strictly worse than just winning with $(s, y_k^*(s))$. ■

Having ruled out ties at strictly positive scores, we now show one of the entrepreneurs k must always be *active*, in the sense of developing a policy strictly better for the decisionmaker than the reservation policy ($F_k(0) = 0$ for some k). An immediate implication is that the decisionmaker is strictly better off with competition with probability 1.

Lemma B.3. *In equilibrium $F_k(0) = 0$ for some $k \in \{L, R\}$.*

Proof: Denote entrepreneur i's equilibrium utility as U_i^*, and suppose not, i.e., $F_i(0) > 0$ $\forall i$; this could be due to atoms at 0, developing scores lower than 0, or both. Let $\bar{y}_t$ denote the expected ideological outcome conditional on both players developing scores ≤ 0, which could be a complicated function of the players' strategies and the decisionmaker's tie-breaking rule. We will show equilibrium implies $\bar{y}_t = 0$, implying both entrepreneurs have a strict incentive to produce score ε and win with strictly positive probability bounded away from 0.

Entrepreneur i can achieve U_i^* by mixing according to her strategy conditional on generating score ≤ 0, and utility arbitrarily close to $\lim_{s_i \to 0^+} \{\Pi_i(s_i, 0; \sigma_{-i})\}$ by developing a policy with ε score and ideology at 0. Equilibrium thus implies that

$$U_i^* - \lim_{s_i \to 0^+} \{\Pi_i(s_i, 0; \sigma_{-i})\} \geq -\alpha_i \int_{s_i \leq 0} \left(s_i + y_i^2\right) \frac{d\sigma_i}{F_i(0)} + F_{-i}(0) \cdot \left(V_i(0, \bar{y}_t) - V_i(0, 0)\right)$$

is ≥ 0 (the l.h.s. = r.h.s. when $\beta = 0$). Since the above is true for both entrepreneurs $\{i, -i\}$ we have $\bar{y}_t = 0$ which furthermore implies that $U_i^* = \lim_{s_i \to 0^+} \{\Pi_i(s_i, 0; \sigma_{-i})\}$. But then by Lemma B.1 $\lim_{s_i \to 0^+} \{\Pi_i(s_i, y_i^*(0); \sigma_{-i})\}$ is strictly higher for both entrepreneurs since $y_i^*(0) = F_{-i}(0) \cdot \frac{x_i}{\alpha_i} \neq 0$, and they each have a strict incentive to deviate to an ε-higher score and produce their optimal ideology. ■

Note that the decisionmaker's access to the reservation policy is irrelevant for the proof; in the competitive model what matters about the reservation policy is that it is "free" to develop. This contrasts with the monopoly model, where the policy that the DM can unilaterally implement matters crucially, because the monopolist behaves as an agenda setter.

Last, we show additional natural properties of the equilibrium score CDFs.

Lemma B.4. *The support of the equilibrium score CDFs over* $s \geq 0$ *is common, convex, and includes* 0. *In addition, both CDFs are atomless* $\forall s > 0$.

Proof: We first show that if $\hat{s} > 0$ is in the support of F_i then $F_{-i}(\hat{s}) - F_{-i}(\hat{s} - \varepsilon) > 0$ $\forall \varepsilon > 0$. Suppose not; then $\exists \varepsilon > 0$ such that $F_{-i}(s)$ is constant over $[\hat{s} - \varepsilon, \hat{s}]$ and $-i$ has no atom at $\hat{s} - \varepsilon$ or $\hat{s}$. Intuitively, this can't happen because i would be playing scores above $\hat{s} - \varepsilon$ without getting a higher probability of victory. Formally $\Pi_i(\hat{s}, y_i; \sigma_{-i}) - \Pi_i(\hat{s} - \varepsilon, y_i; \sigma_{-i}) = -(\alpha_i - F_{-i}(\hat{s})) \cdot \varepsilon < 0 \; \forall y_i$, implying by an envelope argument that i's utility from developing $(\hat{s} - \varepsilon, y^*(\hat{s} - \varepsilon))$ is strictly higher than developing $(\hat{s}, y^*(\hat{s}))$; if $\hat{s}$ were in the support she could do strictly better by deviating to $(\hat{s} - \varepsilon, y^*(\hat{s} - \varepsilon))$, a contradiction.

Now the preceding argument implies several of the desired properties. If the players' score CDFs did not have common support over $s > 0$, then one player would have support at a score where the other player's CDF is constant below, violating the condition. If the common support did not include 0 or were not convex, then there would exist a score $s'' > 0$ in the common support and a strictly lower score $s' \geq 0$ such that $F_i(s)$ was constant $\forall i$ over $[s', s'')$, at least one k had $F_k(s'') = F_k(s')$ (since both cannot have atoms at s'' by Lemma B.2), and the condition would again be violated.

Finally we show that no entrepreneur has an atom above 0 by contradiction. Suppose $-i$ has an atom at $\hat{s} > 0$ of size p_{-i}; then i does not (by Lemma B.2) which implies $E[y_{-i} \mid \hat{s}] = y^*_{-i}(\hat{s})$ (by Lemma B.1). By the argument in the preceding paragraph, i's support includes $[0, \hat{s}]$, which implies $F_i(\hat{s}) > 0$ and $y^*_{-i}(\hat{s}) \neq 0$. In addition, $\lim_{s_i \to \hat{s}^-} \{\Pi_i(s_i, y^*_i(s_i); \sigma_{-i})\} \geq U^*_i$ (since otherwise i would be putting positive probability on scores yielding strictly less utility than her equilibrium utility). Now let $\hat{y}_i = \lim_{s_i \to \hat{s}^-} \{y^*_i(s_i)\} \neq 0$, i.e., i's optimal ideology if she developed score $\hat{s}$ and expected to always lose a tie. It is easily verified that

$$\begin{aligned} & \lim_{s_i \to \hat{s}^+} \{\Pi_i(s_i, \hat{y}_i; \sigma_{-i})\} - \lim_{s_i \to \hat{s}^-} \{\Pi_i(s_i, y^*_i(s_i); \sigma_{-i})\} \\ = \;& p_{-i}\left(V_i(\hat{s}, \hat{y}_i) - V_i\left(\hat{s}, y^*_{-i}(\hat{s})\right) + \beta\left(\hat{s} + \left(y^*_{-i}(\hat{s})\right)^2\right)\right) > 0, \end{aligned}$$

meaning it would yield utility strictly higher than i's equilibrium utility to develop ideology $\hat{y}_i$ and score just above $\hat{s}$ to win for sure, which is a contradiction. ■

C Complete Treatment of Variant with Dogmatists

Using the previous notation, $\Pi_i(s_i, y_i; \sigma_{-i})$ for any $s_i > 0$ is

$$-\alpha_i\left(s_i + y_i^2\right) + F_{-i}^{-}(s_i)\cdot V_i(s_i, y_i) + \int\limits_{s_{-i}>s_i} V_i(s_{-i}, y_{-i})\, d\sigma_{-i}.$$

$$+\int\limits_{s_{-i}=s_i}\left(w_i(y_i, y_{-i}; s_i)\cdot V_i(s_i, y_i) + w_{-i}(y_i, y_{-i}; s_i)\cdot V_i(s_i, y_{-i})\right) d\sigma_{-i} \qquad \text{(C.1)}$$

With some manipulation this may be rewritten as,

$$\begin{aligned}&\Pi_i(0,0;\sigma_{-i}) - (\alpha_i - F_{-i}(s_i))\, s_i - (\alpha_i - F_{-i}(s_i))\, y_i^2 + F_{-i}(s_i)\, Bx_i\cdot 1_{y_i=x_i}\\ &-\int\limits_{s_{-i}=s_i}\left(w_{-i}(y_i, y_{-i}; s_i)\cdot V_i(s_i, y_i) + w_i(y_i, y_{-i}; s_i)\cdot V_i(s_i, y_{-i})\right) d\sigma_{-i}\\ &-\int\limits_{s_{-i}=0} w_{-i}(0, y_{-i}; 0)\cdot V_i(0, y_{-i})\, d\sigma_{-i} - \int\limits_{s_{-i}\in(0,s_i)} V_i(s_{-i}, y_{-i})\, d\sigma_{-i}. \qquad \text{(C.2)}\end{aligned}$$

This is i's utility $\Pi_i(0,0;\sigma_{-i})$ from developing the reservation policy, plus a sequence of terms. The remaining terms in the first line are i's policy utility (net of costs) if she were to always win when $s_{-i} \le s$. The negative terms in the second line arise from the fact that $-i$ may have an atom at s; if she goes from $(0,0)$ to tying at (s_i, y_i) she sometimes defeats her opponent's policies at s_i, and if she goes from tying to winning she always defeats her opponent's policies at s_i. The third line nets off the foregone policy utility from defeating $-i$'s policies with score $s_{-i} \in [0, s)$; since $V_i(s, y) > 0\ \forall s > 0$ or $s = 0, y \neq 0$, the second and third lines are weakly negative. It is then straightforward to show the following.

Lemma C.1. *Developing a policy (s_i, y_i) with $s_i > 0$ and either (i) $y_i \neq x_i$, or (ii) $y_i = x_i$ and $F_{-i}(s_i) \le \frac{\alpha_i x_i}{x_i + B}$, is strictly worse than developing the reservation policy.*

Proof: For $s_i > 0$, it is easy to verify from Equation C.2 that $-(\alpha_i - F_{-i}(s_i))\cdot y_i^2 + F_{-i}(s_i)\, Bx_i \cdot 1_{y_i=x_i} \le 0 \to \Pi_i(s_i, y_i; \sigma_{-i}) < \Pi_i(0,0;\sigma_{-i})$; this is exactly the case when $y_i \neq x_i$ or $y_i = x_i$ and $F_{-i}(s_i) \le \frac{\alpha_i x_i}{x_i+B}$. ■

Using this, we prove the remaining parts of Lemma 2.

Lemma C.2. *The support of the equilibrium score CDFs over $s \ge 0$ is common, convex, and includes 0. In addition, both CDFs are atomless $\forall s > 0$.*

Proof: We first rule out ties at scores $s > 0$. Suppose not so that both entrepreneurs have an atom at $s > 0$ in equilibrium; by Lemma C.1 they must be developing (x_i, s) and (x_{-i}, s), so $\lim_{s_i \to s+}\{\Pi_i(s_i, x_i; \sigma_{-i})\} - \Pi_i(s_i, x_i; \sigma_{-i}) = p_{-i}(s)\cdot Bx_i \cdot w_{-i}(x_i, x_{-i}; s)$ (where

$p_i(s)$ denotes the size of i's atom at s). Since $w_i + w_{-i} = 1$ this is > 0 for at least one entrepreneur k, who has a strict incentive to deviate and produce ε-higher score.

Next, it is simple to verify using identical arguments as in the proof of Lemma B.4 that $\hat{s} > 0$ in support of $F_i \to F_{-i}(\hat{s}) - F_{-i}(\hat{s} - \varepsilon) > 0\ \forall \varepsilon > 0$, which then implies support over $s \geq 0$ is common, convex, and includes 0. Last we rule out atoms above 0 by contradiction. Suppose $-i$ has an atom at $\hat{s} > 0$ of size $p_{-i}(\hat{s})$; by Lemma C.1 she develops $(\hat{s}, x_{-i})$. By preceding arguments i's support includes $[0, \hat{s}]$ so $\lim_{s_i \to \hat{s}^-} \{\Pi_i(s_i, x_i; \sigma_{-i})\} \geq U_i^*$, but

$$\lim_{s_i \to \hat{s}^+} \{\Pi_i(s_i, x_i; \sigma_{-i})\} - \lim_{s_i \to \hat{s}^-} \{\Pi_i(s_i, x_i; \sigma_{-i})\} = p_{-i}(\hat{s}) \cdot Bx_i > 0,$$

so i can do strictly better than her equilibrium utility by developing $(s_i + \varepsilon, x_i)$ for sufficiently small ε, a contradiction. ■