Solar-cycle effects on solar oscillation frequencies

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Measurements of solar oscillations taken in 1986 and 1988 show systematic changes in the Sun's acoustic-mode frequencies of the order of 1 part in 10,000. These data reveal that the frequency shifts are the result of latitude-dependent changes in the structure of the Sun which are correlated with the Sun's magnetic-activity cycle.

JUST over a decade ago, precise observations of the Doppler shift of integrated sunlight (that is, not imaged) demonstrated that the Sun exhibited globally coherent five-minute oscillations[1,2]. Sharply peaked features were found in power spectral analyses of the integrated Doppler measurements, and these features were interpreted as global normal modes of the Sun. These modes, called p-modes because pressure is the dominant restoring force, are small amplitude ($<20\ \mathrm{cm\ s^{-1}}$ in surface velocity) acoustic oscillation modes, which are probably excited by turbulent convection in the Sun's outer layers[3,4].

Subsequent observations of Doppler and intensity images of the Sun showed that at least several hundred thousand globally coherent p-modes are excited to observable amplitudes. The perturbation of a scalar quantity, for example pressure, resulting from the excitation of a single mode can be characterized by three 'quantum' numbers, n, l and m (assuming a slowly rotating Sun): $\delta p_{nlm}(r, \theta, \phi, t) = \mathrm{Re}\,[\delta p_{nl}(r)\, Y_l^m(\theta, \phi)\, e^{i2\pi\nu_{nlm}t}]$, where $\delta p_{nl}(r)$ is a radial wavefunction with n nodes, $Y_l^m(\theta, \phi)$ is a spherical harmonic of degree l and order m, and ν_{nlm} is the frequency of the mode. Because different modes are described by different wavefunctions, observations of p-mode frequencies can be used as a probe of the structure of the solar interior, as has been discussed in several recent review articles[5-7].

Ever since the discovery of global p-modes in the Sun, it has been expected that solar-cycle changes in the Sun's interior structure would result in observable changes in the mode frequencies, and therefore provide a possible indirect probe of a deep-seated magnetic dynamo. Although there are indications in recent low-l data that mode frequencies near 3 mHz are perhaps $\sim$0.4 μHz greater at solar maximum than at solar minimum, the signal-to-noise ratio in the published observations is not very high, and there is some disagreement between different observations[8,9]. Measurements of intermediate-l mode frequencies ($5 \leq l \leq 100$) have also been used to search for

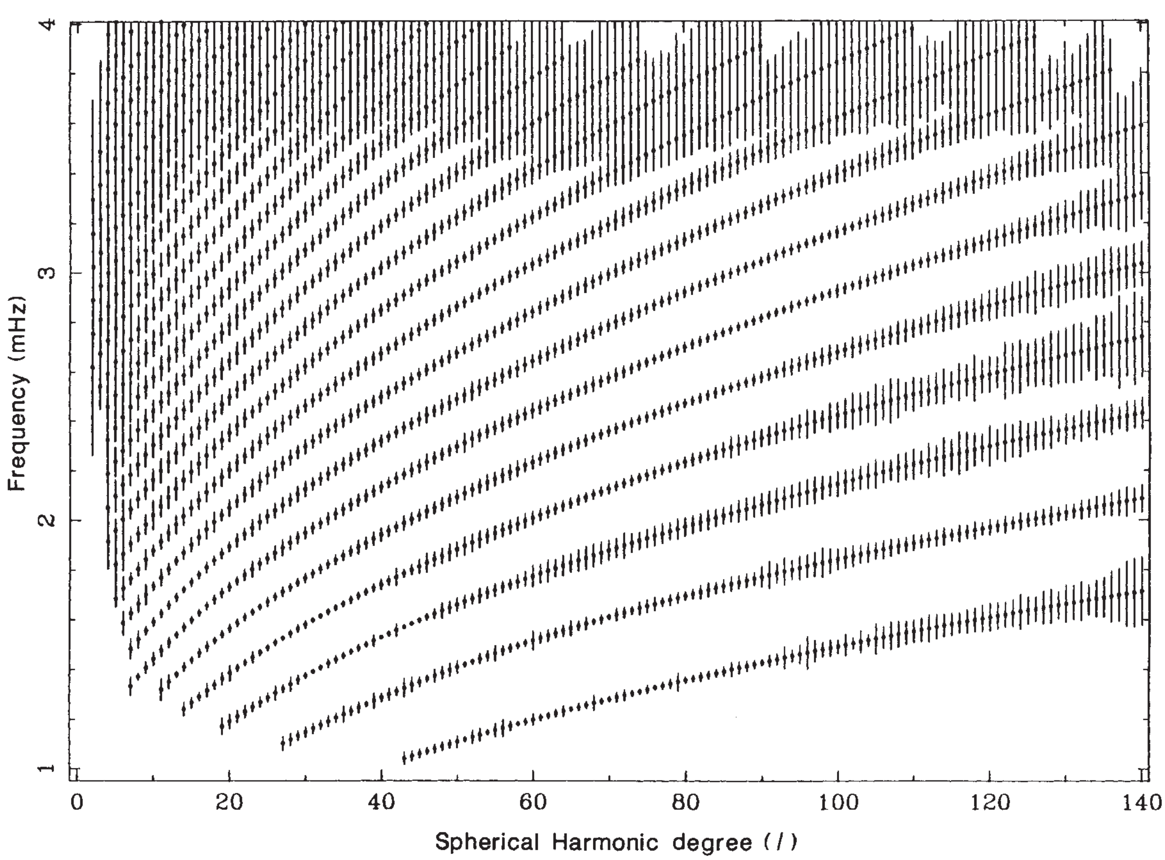

FIG. 1 Schematic plot showing the observed solar p-mode frequency measurements, where the usual 1σ error bars have been magnified by a factor of 1,000. Each ridge contains modes with a fixed number of radial nodes n, with the lowest frequency ridge having $n=1$.

TABLE 1 Splitting coefficients (nHz) at 3 mHz

	1986	1988	1988 − 1986
$\Delta\nu$	—	—	186.0 ± 1.7
α_2	−21.4 ± 3.4	62.8 ± 3.5	84.0 ± 4.8
α_4	42.0 ± 4.9	−62.0 ± 4.5	−104.1 ± 6.6
α_6	−19.8 ± 5.8	−181.7 ± 6.4	−161.9 ± 8.6

frequency variations with time, but again the signal-to-noise ratio has been fairly low and the results derived from different observations are inconsistent[10-12].

One area where many different observers have been in relatively good agreement is the time variation of the even-index p-mode splitting coefficients. If we expand the p-mode frequencies of a single (nl) multiplet as a sum of Legendre polynomials

$$\nu_{nlm} = \nu_{nl} + \sum_{i=1} \alpha_i(n, l) P_i(m/L) \qquad (1)$$

with $L = \sqrt{l(l+1)}$, then the α_i with even i measure a latitude-dependent variation in the effective propagation speed for acoustic waves (owing to, for example, real variations in the speed of sound inside the Sun and changes in the structure of the surface boundary), and the α_i with odd i measure a corresponding east-west propagation asymmetry (the solar rotation). Kuhn[13] first suggested that the observed $\bar{\alpha}_2$ and $\bar{\alpha}_4$ (here averaging $\alpha_i(n, l)$ over observed n and l) varied systematically with solar cycle, and all the measurements to date, made by several different observers, support this[10,14,15].

The data presented here at Big Bear Solar Observatory using our dedicated helioseismology telescope[16,17], over a period of approximately four months in the summer of 1986 and again in 1988. The very long observing runs resulted in very accurate measurements of p-mode frequencies, as shown in Fig. 1. The 1986 data set has already been used to measure p-mode amplitudes and linewidths[4,17], splittings[15,18] and accurate mode frequencies[19]. Because 1986 was near solar minimum whereas by 1988 the new magnetic cycle had reached considerable amplitude, these data are well suited to search for solar-cycle frequency changes.

Frequency shifts

The change in the mode frequencies, $\Delta\nu_{nl} = \nu_{nl}(1988) - \nu_{nl}(1986)$ depends strongly on mode frequency ν_{nl} and only weakly on l (Fig. 2). the data are fairly well described by $\Delta\nu(\nu, l) \propto M^{-1}(\nu, l)$, where M is the so-called mode 'mass', defined as the ratio of the mode energy to the square of the surface velocity, the latter evaluated at a 5,000-Å optical depth of $\tau_{5,000} = 0.05$.

We suggest that the most significant solar-cycle changes in the Sun's structure, inasmuch as they affect the p-mode frequencies, occur near the surface. Such a hypothesis is qualitatively supported by the observed $\Delta\nu(\nu, l)$, and can be understood by considering the details of the radial structure of the mode eigenfunctions. For a given l, the upper reflection point for lower-ν modes is deeper in the Sun than for higher-ν modes. At fixed frequency, the eigenfunctions are roughly l-independent near the surface, but lower-l modes penetrate more deeply into the solar interior than higher-l modes. Consequently higher-ν, and to a lesser extent higher-l modes, are more sensitive to surface perturbations, which is consistent with the observed $\Delta\nu(\nu, l)$.

If instead the perturbation were to extend over a significant fraction of the solar radius, then from asymptotic theory the (fractional) mode frequency shift would depend mainly on the ratio ν_{nl}/l, which labels different acoustic rays. Because the observed $\Delta\nu(\nu, l)$ is not well described as a function of ν_{nl}/l we conclude that the relevant structure changes occur mainly in a thin layer. The effect of perturbing a thin layer in the propagating regions of the modes was studied by Thompson[20], who found an oscillatory frequency dependence in $\Delta\nu$, which we do not observe. Thus the dominant effect on the frequencies is not the direct result of, say, changes in the magnetic field at the base of the convection zone, although the effect of such a hypothetical perturbed layer could conceivably show up in a more careful analysis. A perturbation confined mainly to the evanescent regions of the modes near the centre of the Sun can also be ruled out because the frequency shift does not increase with decreasing l.

Without resorting to a detailed model, it is our view that the p-mode frequencies are responding to changes in the strength of solar magnetic activity near the Sun's surface. In addition to

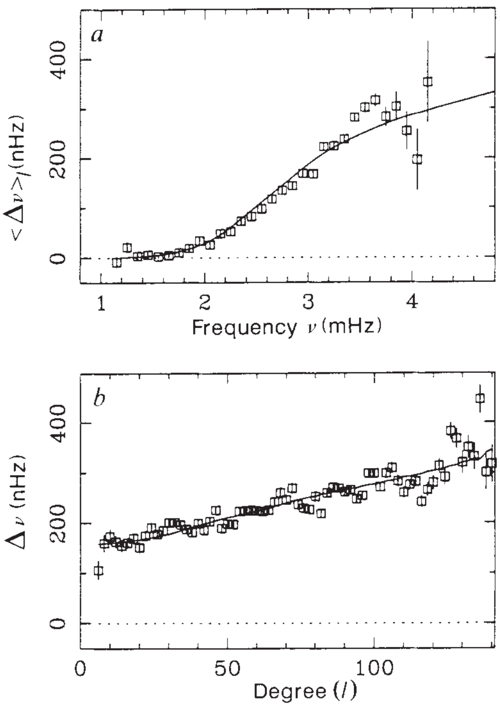

FIG. 2 *a*, p-mode frequency differences between 1988 and 1986, $\Delta\nu = \nu_{88} - \nu_{86}$, as a function of frequency, after averaging over measured modes with degree $5 \leq l \leq 60$. The solid line is the inverse mode mass at $l = 20$, $M^{-1}(\nu, l = 20)$, scaled to fit the data, using mode masses at $\tau_{5,000} = 0.05$ (P. Kumar, personal communication) based on a solar model by Christensen-Dalsgaard[26]. *b*, p-mode frequency differences at 3 mHz, as a function of spherical harmonic degree l. The points were obtained by fitting the observed frequency shifts in each l range to $M^{-1}(\nu, l = 20)$ and evaluating the fit at 3 mHz. Also plotted is the inverse mass function at 3 mHz, $M^{-1}(\nu = 3\text{ mHz}, l)$, scaled to fit the data.

the direct effect of magnetic fields on the dynamics of oscillatory motion, the fields can modify the thermal structure of the Sun, particularly near the upper boundary, on which the mode frequencies depend.

The surface evanescent layer of a typical observed mode extends to a depth $\leq 1\%$ of the solar radius, so it is likely that the perturbation is not strong much deeper than this. On the other hand, a simple calculation of the effect of a perturbation confined strictly to the photosphere yields an even stronger frequency dependence for $\Delta\nu(\nu, l)$ than is observed. The calculation, based on a Lagrangian formalism[14,21], assumed that the vertical extent of the perturbed layer does not exceed one pressure scale height and used mode eigenfunctions appropriate to an isothermal photosphere. The effects of a surface perturbation were calculated, in an approximate way, within this formalism, and the treatment was simplified by considering $l = 0$ modes only. For modes with frequencies below the acoustic cutoff, the rough frequency dependence of $\Delta\nu$ was found to be

$$\Delta\nu \propto \frac{\nu^3}{M(\nu)} \tag{2}$$

where $M(\nu)$ is the mass of the $l = 0$ modes evaluated at the location of the perturbation.

As the observations are reasonably well fit by a simple $M(\nu)^{-1}$ dependence at low l, we conclude that the observed $\Delta\nu$ cannot originate solely in the photosphere, and therefore the sub-photospheric layers must contribute significantly to the observed shift. It should be noted that the choice, $\tau_{5,000} = 0.05$, at which M was evaluated for comparison with the data is not crucial to this conclusion, because the function $M(\nu, l)$ at this optical depth adequately represents the frequency dependence the mode mass for photospheric values of τ. P. Goldreich *et al.* (personnal communication) have found from detailed modelling that the observed $\Delta\nu(\nu, l)$ can follow from quite reasonable assumptions about how the top of the convection zone is perturbed as the result of solar magnetic activity.

Frequency splittings

The above $\Delta\nu(\nu, l)$ suggest that between 1986 and 1988 there was a change in the structure of near-surface layers of the Sun, a change that affected the propagation of acoustic waves in that region. If the responsible time-dependent perturbation also depends on latitude, as one might expect for a solar-cycle effect, then it should also be observable in the even-index splitting coefficients $\alpha_{2j}(n, l)$ in both the 1986 and 1988 data sets. In particular, if the perturbation can be separated into the product of a time-independent radial function and a time-dependent function of latitude, then the α_{2j} should show the same frequency dependence as the $\Delta\nu$. To investigate this hypothesis, we measured the frequency splittings $\alpha_i(n, l)$ in equation (1) up to $i = 6$. Averaging over l for $5 \leq l \leq 60$ we obtain the results shown in Fig. 3 for the 1986 and 1988 data. Clearly all the α_{2j} show a frequency dependence that is consistent with the inverse mass function. We also found that the α_{2j} depend only weakly on l for $5 \leq l \leq 60$, consistent with the measured $\Delta\nu(\nu, l)$.

We have not corrected the data in Fig. 3 for the second-order effects of solar rotation, which are expected to produce a measurable α_2. The calculated rotational α_2 is small (P. Goode, personnel communication), however, smaller even than the observed

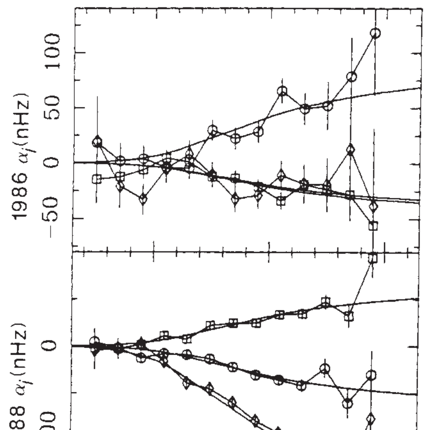

FIG. 3 Plot of the even-index splitting coefficients, α_i, $i = 2, 4, 6$, as a function of frequency for 1986 and 1988, averaging over measured multiplets with $5 \leq l \leq 60$. α_2, α_4 and α_6 are represented by boxes, circles and diamonds, respectively. Lines connect the data points, and fits to $M^{-1}(\nu, l = 20)$ are also drawn for each of the α_i. As for the $\Delta\nu$ data in Fig. 2*a*, the frequency dependence of the α_i is well represented by $M^{-1}(\nu, l = 20)$, particularly in 1988.

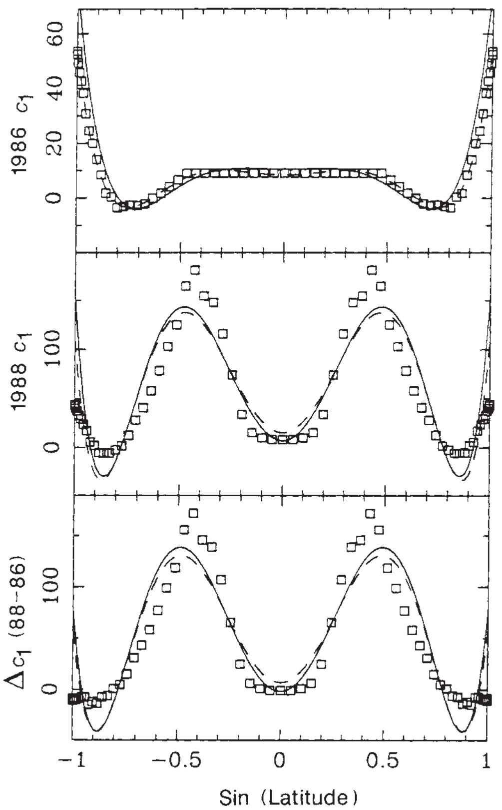

FIG. 4 Latitude inversions of the splitting data, compared with solar-limb brightness measurements. The solid lines are the inversions, the points are the limb measurements, and the dotted lines are fits to the limb measurements using equation (3), keeping only terms up to β_6. All the plots contain the same arbitrary scale factor.

α_2 in 1986, so little is lost by neglecting this effect. Dziembowski and Goode performed an inversion of the 1986 splittings, and found evidence for a strong magnetic field at the base of the convection zone[22]. This seems to contradict our conclusion above that the dominant perturbation is near the solar surface, but further work may clarify this.

Assuming that the dominant cause of the measured non-zero α_{2j} is a near-surface perturbation, the data support our hypothesis that the radial structure of solar magnetic activity near the surface is independent of both time and latitude. The support is not irrefutable, however, because in 1986 the signal-to-noise ratio in the α_{2j} measurements is just high enough to discern the $M^{-1}(\nu)$ functional form, and in 1988 the signal is dominated, as we will see below, by solar activity that is concentrated in a fairly narrow latitude band.

Averaging the $\Delta\nu$ and α_{2j} over $5 \leq l \leq 60$, fitting each to $M^{-1}(\nu, l=20)$, and evaluating the fit at $\nu = 3$ mHz, we obtain the fit coefficients given in Table 1.

Latitude inversions

It is straightforward and instructive to invert the α_{2j} and $\Delta\nu$ to obtain a measure of the strength of the perturbation as a function of solar latitude, assuming as above that its radial structure is independent of both time and latitude. Expressing the perturbation as an effective sound-speed perturbation $c_1(r, \theta, t)$, we can then write[13,23,24]

$$c_1(r, \theta, t) = \sum_j \beta_{2j}(t) f(r) P_{2j}(\cos\theta) \quad (3)$$

where θ is the colatitude, and the observable splitting coefficients are

$$\alpha_{2j}(\nu, l, t) = C(\nu, l)(-1)^j \frac{(2j-1)!!}{(2j)!!} \beta_{2j}(t) \quad (4)$$

where $C(\nu, l)$ is a function determined by the depth dependence $f(r)$ of the perturbation. This result is a good approximation when $j \ll l$, and therefore applies to all but the lowest-l modes studied here. Equation (4) also holds for $j=0$ if we identify $\Delta\alpha_0 = \Delta\nu$, giving $\Delta\nu(\nu, l) = C(\nu, l)\Delta\beta_0$. If we take $C(\nu, l) \propto M^{-1}(\nu, l)$, then we see from Figs 2 and 3 that all the data are consistent with our assumption of a time- and latitude-independent $f(r)$. It remains to be seen whether this consistency persists over the solar cycle, as better data are acquired.

Using the fit coefficients in Table 1, the depth dependence of the perturbation can be absorbed into a single scale factor, allowing an inversion to determine the latitude dependence of the effective perturbation. We expect that such an inversion will show that the perturbation is primarily confined to the active latitudes on the Sun. To investigate this, we have used the limb photometer measurements of Kuhn *et al.*[13,25]. (Because there are no 1986 limb data, we interpolated the data from 1985 and 1987 to estimate the 1986 limb brightness.) These data measure the relative effective temperature of the solar limb, and thus serve to outline the active latitudes, although the detailed physical changes near the solar surface which are responsible for the limb observations have yet to be identified.

The results of the inversions are shown in Fig. 4, for 1986 and 1988, as well as for the difference between the 1988 and 1986 data. Because β_0 cannot be determined absolutely, the 1986 and 1988 inversions were arbitrarily displaced to match the limb data. Comparing 1988 and 1986 data, however, we can determine $\Delta\beta_0$ along with the other $\Delta\beta_{2j}$, so the (1988−1986) inversion contains no arbitrary displacement. All the inversions were scaled using a single scale factor.

The fit of the limb temperature data to the splitting inversions is remarkably good. The splittings clearly give the expected result in that the effective sound-speed perturbation is confined mostly to the active latitudes when the activity is strong. The 1986 splittings show a substantial perturbation coming from the polar region, however, which again matches the limb brightness measurements. The presence of the polar perturbation results from the relatively large α_4 seen in 1986. Therefore the source of the perturbation cannot be the very strong atmosphere perturbation in active regions alone, because there are no active regions at the solar poles. Weaker magnetic fields, or large-scale temperature variations, must contribute significantly to the perturbation.

Discussion

These new measurements clearly show that p-mode frequencies change with time, presumably owing to changes in the structure of the Sun during the solar cycle. The measured frequency shifts $\Delta\nu(\nu, l)$ depend strongly on frequency and weakly on l for $5 \leq l \leq 140$, with a functional form that is well described by $\Delta\nu(\nu, l) \propto M^{-1}(\nu, l)$, the latter evaluated at optical depth, $\tau_{5,000} = 0.05$. This frequency dependence indicates that the dominant structural changes during the solar cycle, inasmuch as they affect p-mode frequencies, occur near the solar surface, although we do not yet have a detailed physical model of the perturbation. Further, the even-coefficient p-mode splittings show the same frequency dependence as the $\Delta\nu$, which suggests that the radial structure of the near-surface perturbation is independent of both time and latitude. Inversions of the splittings show that the hypothesized near-surface perturbation is strongest in the active latitudes, but includes a weak polar component at the solar-cycle minimum. These data open up the possibility of inverting $\Delta\nu(\nu, l)$ and $\alpha_{2j}(\nu, l)$ to determine the detailed depth dependence of the perturbation, and hopefully of constructing a model that describes the structure of solar activity as a function of depth. □

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