THE RADIAL GRADIENT IN THE SUN'S ROTATION

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Received 1988 September 12; accepted 1988 November 9

ABSTRACT

The solar oscillation data of Libbrecht are inverted, and it is found that there is a sharp radial gradient in the Sun's rotation at the base of the convection zone. The existence of a sharp radial gradient there may be used to suggest that it is the site of the dynamo which drives the sunspot cycle.
Subject headings: Sun: interior — Sun: oscillations — Sun: rotation

I. INTRODUCTION

The well-studied 22 yr sunspot cycle is generally expected to be a consequence of dynamo action arising from the interplay of internal rotation, magnetism, and convection. We do not know precisely where this dynamo is buried. In recent years, however, we have been able to determine relevant internal properties of the Sun.

Helioseismology is the new science developed to determine internal properties of the Sun, like rotation, by studying solar oscillations. Duvall, Harvey, and Pomerantz (1986) used oscillation data to demonstrate that the convection zone rotates with the surface differential rate—meaning there is no apparent radial gradient in rotation in that zone. Brown et al. (1988) used the oscillation data of Brown and Morrow (1987) to show that there apppears to be a gradual gradient in rotation from the convection zone to the radiative interior. The more certain result of Brown et al. (1988) is that $d\Omega/dr > 0$ at low latitudes and the less certain result is that $d\Omega/dr < 0$ at higher latitudes. Since the standard $a - \Omega$ dynamo theories require a radial gradient, it seems that the dynamo must be at least as deep as the base of the convection zone. Helioseismological data have also been used, by Christensen-Dalsgaard et al. (1985), to determine the location of the base of the convection zone. Unfortunately, efforts like that of Dziembowski and Goode (1986), to use oscillation data to determine the internal magnetic field have not enjoyed comparable success. The problem here is that even if the magnetic field and rotation energies were comparable, rotation would be easier to determine. The point is that advection is linear in the rotation rate, while the lowest order magnetic field perturbation is quadratic in the field.

It is our purpose to use the data of Libbrecht (1989) to improve the description of the gradient in solar rotation near the base of the convection zone. We also show a consistency between the point of inflection of this gradient and the location of the base of the convection zone. We further use this oscillation data to place a limit on the magnetic field in the region near the base of the convection zone.

II. DATA

Libbrecht (1989) presents his frequency splitting data in the form introduced by Duvall, Harvey, and Pomerantz (1986) as modified by Brown and Morrow (1987). That is,

$$v_{nlm} - v_{nlo} = l \sum_{i=1}^{5} a_i P_i\left(\frac{m}{l}\right),$$ (1)

where the (nl)-multiplets are labeled by n, the radial order, and l, the angular degree. Rotation and/or magnetism induce a fine structure in each (nl)-multiplet. The fine structure is labeled by m, the angular order. The odd-a coefficients reflect the fine structure due to rotation as given by, for instance,

$$\Omega(r, \theta) = \Omega_0(r) + \Omega_1(r)\mu^2 + \Omega_2(r)\mu^4,$$ (2)

where μ is the cosine of the polar angle. The odd-a coefficients of Libbrecht (1989) are shown in Figure 1 as a function of l. The data in the figure are weighted averages, in bins five l values wide, with the variance being shown. There is a relatively large dispersion in the averaged results presumably due to random errors and finite lifetime effects—nonetheless, the trends in the data are regarded as being meaningful. A mean rotation-law, from the data of Libbrecht (1989), with $\Omega_0 = 460.2 \pm 0.2$, $\Omega_1 = -58.3 \pm 1.8$, and $\Omega_2 = -73.1 \pm 2.6$ nHz is used to calculate the a-values represented by the solid lines in the figure. This mean rotation law exhibits surface-like differential rotation. For the a_1 and a_3 coefficients, the high-l data lie above the calculated values, while those for the low-l data lie below. Since $l > 40$ oscillations are largely confined to the convection zone and lower degree oscillations sample more deeply, we anticipate a radial gradient in the true rotation near the base of the convection zone. In that region, the decrease in the magnitude of the a_3 and a_5 coefficients with decreasing l leads us to expect a trend away from differential rotation to solid body rotation. Oscillations of degree 10–60 sample best the region between 0.4 and 0.9 of the solar radius.

III. THE REGULARIZED INVERSION

To determine $\Omega(r, \theta)$, we solve the inverse problem posed by

$$v_{nlm} - v_{nlo} = m \sum_{s=0}^{2} \int_0^R K_{nlms}(r)\Omega_s(r)dr,$$ (3)

where the left-hand side of the equation is given by the frequency splitting data as represented in equation (1). The quantity K is the splitting kernel which is calculated using a
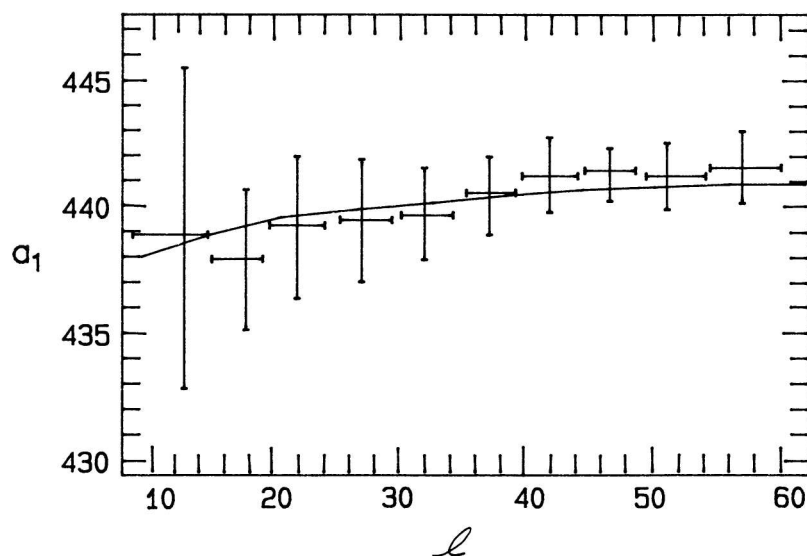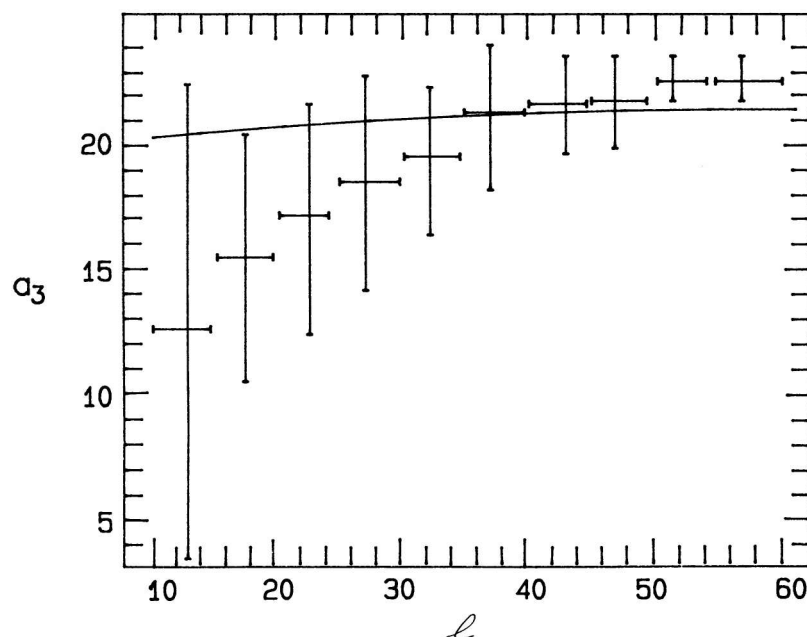
Fig. 1a

Fig. 1b

FIG. 1.—Odd-a splitting coefficients (nHz) vs. l. The solid line represents the determination of the coefficients from the mean rotation law.

standard solar model. For a detailed formulation and discussion of equation (3) see, for instance, Durney, Goode, and Hill (1987) or Brown et al. (1988). We connote Ω as a frequency rather than a rate—our only nonstandard notation—to avoid making that conversion explicit with (2π)-factors. Usually $\Omega_s(r)$ is obtained from equations (1) and (3) by minimizing χ_s^2. For all but the coarsest grids, the resulting solutions for $\Omega_s(r)$ will exhibit short-wavelength variations which reflect the ill-conditioned nature of the problem rather than the internal rotation of the Sun. We employ the method of regularization, as reviewed by Goncharski, Cherepashczuk, and Yagola (1978) and Craig and Brown (1986) to better condition the problem.

We invoke the *a priori* constraint that $\Omega_s(r)$ is a slowly

varying function. Formally, we impose this by minimizing

$$\rho_s^2 = \chi_s^2 + \lambda_s MP_s, \qquad (4)$$

rather than χ_s^2, where M is the number of (nl) multiplets in the data and

$$P_s = \int \left| \frac{d\Omega_s}{dr} \right|^2 dr ; \qquad (5)$$

and we vary λ_s until $\chi_s^2 = 1$. When χ_s^2 (per degree of freedom) is unity, the calculated $\Omega_s(r)$ is precisely consistent with the 1 σ observational errors. To examine the robustness of features in
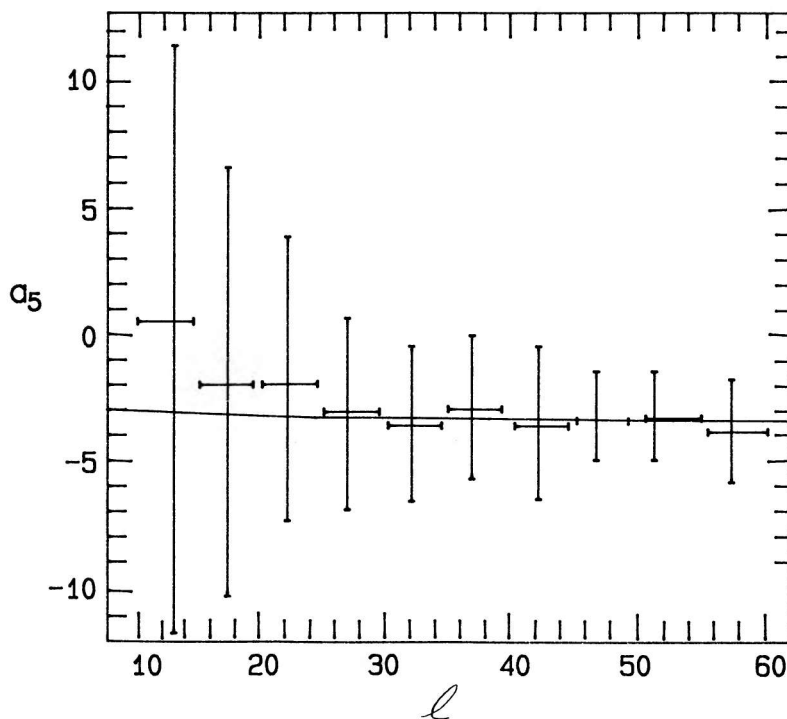
FIG. 1c

$\Omega_s(r)$, we reduce λ_s in our calculations until short-wavelength variations begin to appear.

IV. THE RESULTS OF THE INVERSION

The rotation frequency $\Omega_0(r)$ is better determined than $\Omega_1(r)$ or $\Omega_2(r)$ because it alone depends on the a_1 coefficient which is the most accurately determined splitting coefficient. For $\Omega_0(r)$, the monotonic function, ρ_0^2, varies by more than an order of magnitude when λ_0 changes from zero to infinity. For λ_s much

less than 10^{-7}, short-wavelength features begin to appear in $\Omega_0(r)$, $\Omega_1(r)$ and $\Omega_2(r)$. Figure 2 shows roughly consistent $\Omega_0(r)$ values for a wide range of λ_0 values. The inversions were performed using a grid with steps of 0.03 of the solar radius from 0.4 to 1.0. The error bars in the figure are the least-squares errors from the inversion with $\lambda_0 = 10^{-7}$. Each of the three continuous lines in the figure—included to aid in viewing the results—is fit to smoothly connect the 21 calculated frequencies between 0.4 and 1.0. It is clear that the most robust
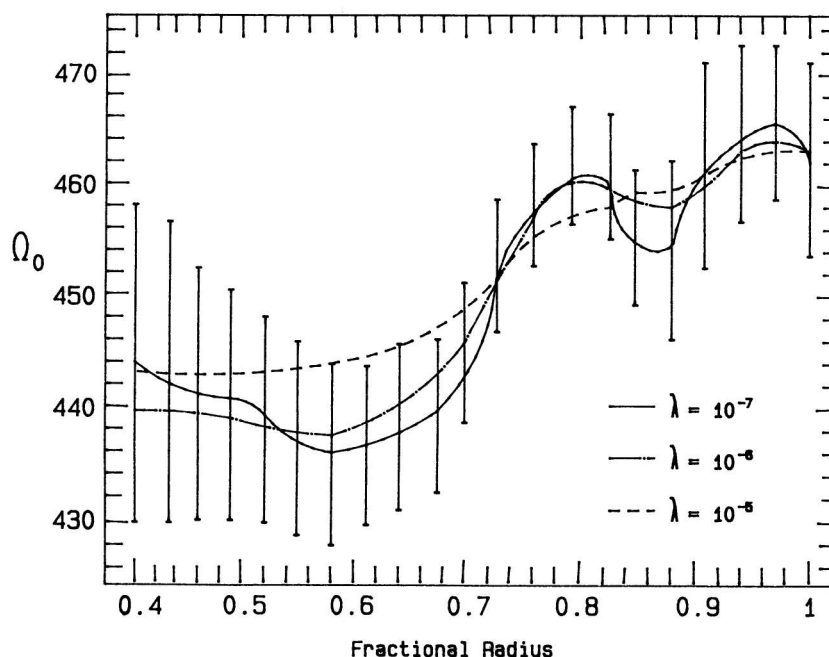
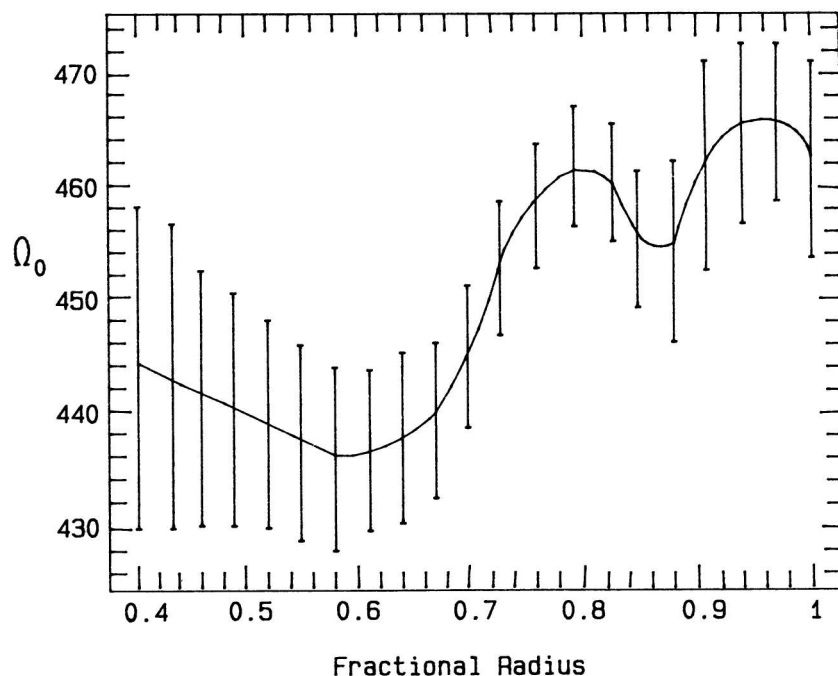FIG. 2.—$\Omega_0(r)$ (nHz) vs. fractional radius for $\lambda = 10^{-5}$, 10^{-6}, and 10^{-7}
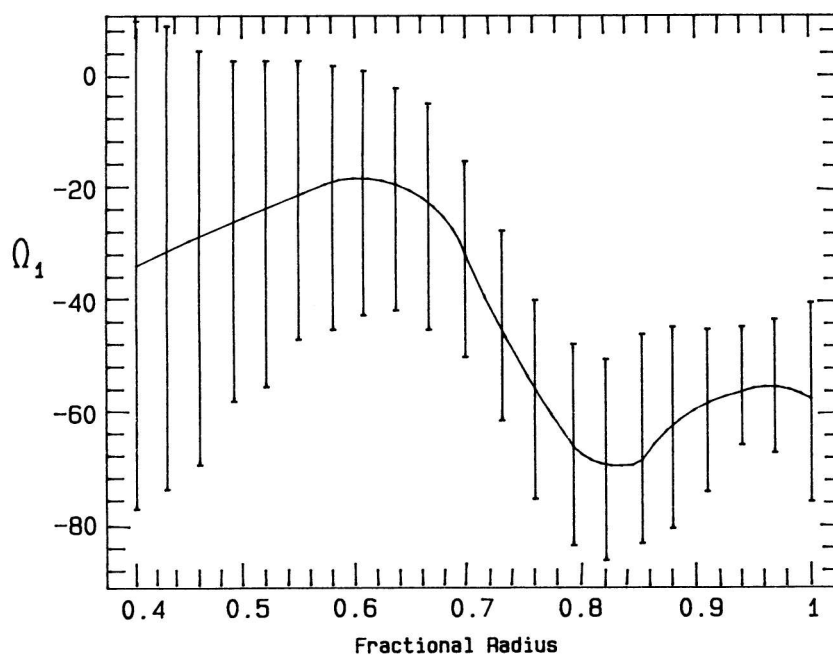
Fractional Radius

Fig. 3a

Fractional Radius

Fig. 3b

Fig. 3.—$\Omega_0(r)$, $\Omega_1(r)$, and $\Omega_2(r)$ vs. fractional radius

feature is the radial gradient in $\Omega_0(r)$. We note that the point of inflection for this gradient coincides with 0.73 of the radius—the base of the convection zone as located by Christensen-Dalsgaard et al. (1985). Secondary robust features are the dip in rotation at 0.85 of the radius and the gradual inward increase in $\Omega_0(r)$ below the convection zone. We regard the inversion with $\lambda_0 = 10^{-7}$ as our result because it shows the robust features while being properly stiffened by the constraint.

Figure 3 shows our result for $\Omega_0(r)$, $\Omega_1(r)$, and $\Omega_2(r)$ with

$\lambda_s = 10^{-7}$. The regularized inversion has translated the significant trends in large variance a versus l data into significant trends in large error Ω_s versus r results. The radial gradient in $\Omega_0(r)$, at the base of the convection zone, is quite sharp even though it has been smoothed. The frequency $\Omega_1(r)$, with less certainty, shows a sharp gradient inflected at 0.73 of the radius. The most uncertain frequency is $\Omega_2(r)$, and it shows no structure. These gradients are much sharper and better specified in radius than those reported by Brown et al. (1988), reflecting
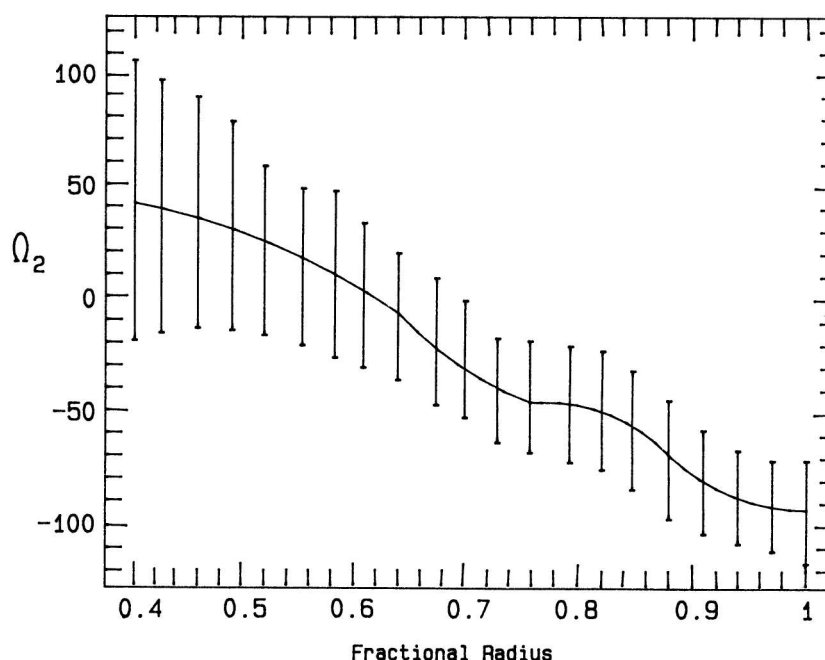
FIG. 3c

our improved resolution. Nonetheless, it would be no surprise if the true gradient were even sharper than the smoothed one we report. Our overall rotation law is consistent, at 1 σ, with surface differential rotation through the convection zone and a sharp transition to rigid rotation below. We note that ρ_s^2 is better minimized if a discontinuity is allowed in rotation at the base of the convection zone. The result then is an abrupt change in rotation at the discontinuity from surface-like rotation to rigid rotation beneath the convection zone. In particular, $\Omega_0(r)$ drops, going inward, from 462 to 442 nHz, and $\Omega_1(r)$ and $\Omega_2(r)$ exhibit much larger drops in magnitude with less relative certainty. This last inversion also preserves the other robust features of rotation illustrated in Figure 3.

If the sharp radial gradient is used as the criterion for locating the solar dynamo, then that dynamo is seated near the base of the convection zone. We speculate that the dip in rotation could be due to either a propagating torsional wave induced by dynamo action below it or the back torque due to a local propagating toroidal field. If either speculation were true, this dip should move on the time scale of the solar cycle and could be associated with the surface torsional oscillations discovered by Howard and LaBonte (1980).

V. SECOND-ORDER EFFECT OF ROTATION AND MAGNETISM

The a_2 and a_4 data of Libbrecht (1989) are consistent with zero; however, they are also systematically negative and positive, respectively. The observed a_2 coefficients become increasingly negative with decreasing l value. If we use the mean rotation law following from $\lambda_s = 1$ in the calculation of the effect of the centrifugal stretching, we find that this dominant second-order effect of rotation accounts for the values of and the trend in a_2. The same mean rotation law was used in the calculation of the solid lines in Figure 1. Centrifugal stretching makes virtually no contribution to any a_4 coefficient because surface-like rotation does not cause an appreciable P_4 distortion. The a_2 and a_4 coefficients could also be due, in part, to a magnetic field near the base of the convection zone. A sizable toroidal field could be generated there by the shear of differential rotation acting on a minuscule poloidal field. Dziembowski and Goode (1989) showed that such a toroidal field would have an upper limit of about 1 MG. Such a limit is probably too large to place an interesting limit on the dynamo field.

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