MIDTERM EXAM

HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/

1. What is the number of periodic points of (not necessarily minimal) period 8 of the topological Markov chain σ_A given by the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}?$$

2. Calculate the topological entropy of the topological Markov chain σ_A given by the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

3. Suppose the continuous map $f : [0,1] \to [0,1]$ has a periodic orbit $\{x_1 < x_2 < x_3 < x_4\}$ such that $f(x_i) = x_{i+1}$ for $i < 4$ and $f(x_4) = x_1$. Show that f has periodic points of all periods.

4. Prove that the map $f_\alpha : \mathbb{T}^2 \to \mathbb{T}^2$,

$$f_\alpha(x, y) = (x + \alpha, y + x) \pmod 1,$$

is transitive if and only if $\alpha \notin \mathbb{Q}$.

5. Consider the following map $\alpha : \Sigma_2^+ \to \Sigma_2^+$,

$$(\alpha(\omega))_i = \begin{cases} 1 - \omega_i & \text{if } \omega_j = 1 \text{ for all } j < i, \\ \omega_i & \text{otherwise.} \end{cases}$$

Show that $\alpha : \Sigma_2^+ \to \Sigma_2^+$ is a homeomorphism and calculate its topological entropy.

6. Find $\sup_{n \in \mathbb{N}} \{ \sin n + \cos \sqrt{2} n \}$.

7. Describe all the ergodic measures of the homeomorphism of the torus $f : \mathbb{T}^2 \to \mathbb{T}^2$,

$$f(x, y) = (x, x + y) \pmod 1.$$