DYNAMICAL SYSTEMS

HOMEWORK #8

HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/

1. Assume that a map $f : [0,1] \to [0,1]$ is Hölder continuous with exponent $\alpha > 1$. Prove that $f \equiv const$.

2. Consider the following metrics on Σ_2

$$d_2(\omega,\omega') = \begin{cases} 0, & \text{if } \omega = \omega'; \\ 2^{-l}, & \text{if } l = \min\{i \in \mathbb{Z}_+ \,|\, \omega_i \neq \omega'_i \text{ or } \omega_{-i} \neq \omega'_{-i}\}, \end{cases}$$

$$d_5(\omega,\omega') = \begin{cases} 0, & \text{if } \omega = \omega'; \\ 5^{-l}, & \text{if } l = \min\{i \in \mathbb{Z}_+ \,|\, \omega_i \neq \omega'_i \text{ or } \omega_{-i} \neq \omega'_{-i}\}. \end{cases}$$

Prove that $\mathrm{id}_{\Sigma^2} : (\Sigma_2, d_2) \to (\Sigma_2, d_5)$ is Hölder continuous. What is the Hölder exponent?

3. Construct an example of a locally maximal hyperbolic set Λ such that periodic points are not dense in Λ.

4. Let a diffeomorphism $f : M \to M$ satisfy Axiom A. Prove that if $g : M \to M$ is C^1-close to f then $h_{top}(g) \geq h_{top}(f)$.

5. Consider the map $f : D^2 \to \mathbb{R}^2$, $f(x,y) = (10x, \frac{y}{10})$, where $D^2 = \{(x,y) \,|\, x^2 + y^2 < 1\}$, and the map $F : D^2 \times S^1 \to \mathbb{R}^2 \times S^1$, $F = f \times id_{S^1}$. The map F has an invariant circle $\{0\} \times S^1$. Prove that any map G which is C^1-close to F also has an invariant closed C^1-curve.