DYNAMICAL SYSTEMS

HOMEWORK # 7

HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/

1. Consider the following map:

$$f : [-1, 1] \times [-1, 1] \to \mathbb{R}^2, \; f(x, y) = (\frac{1}{2}x, 2y).$$

Prove that any C^1-map g, which is C^1-close to f, has a unique fixed point.

2. How many (up to a topological conjugacy) contracting C^1-diffeomorphisms of $\mathbb{R}^1$ do exist? Does the answer change if $\mathbb{R}^1$ is replaced by $\mathbb{R}^n$?

3. Show that every minimal hyperbolic set consists of exactly one periodic point.

4. Does the tent map have the shadowing property?

5. Consider the map $f : D^2 \to D^2$, $f(x, y) = (\frac{x}{10}, \frac{y}{10})$, where $D^2 = \{(x, y) \,|\, x^2 + y^2 < 1\}$, and the map $F : D^2 \times S^1 \to D^2 \times S^1$, $F = f \times id_{S^1}$. The maximal attractor of of the map F is an invariant circle $\{0\} \times S^1$. Prove that for any map G which is C^1-close to F the maximal attractor of G is C^1-smooth closed curve.