

DYNAMICAL SYSTEMS

HOMEWORK # 6

[HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/](http://www.its.caltech.edu/~asgor/dynsys/)

1. Suppose a measure-preserving transformation $f : (M, \mu) \rightarrow (M, \mu)$ has a generator with k elements. Prove that $h_\mu(f) \leq \log k$.
2. Calculate the metric entropy of baker's transformation (with respect to Lebesgue measure).
3. Construct an example of a homeomorphism f of a compact metric space with finite topological entropy that does not have a measure of maximal entropy (an invariant measure μ is called "a measure of maximal entropy" if $h_\mu(f) = h_{top}(f)$).
4. Let Λ_i be a hyperbolic set of $f_i : U_i \rightarrow M_i$, $i = 1, 2$. Prove that $\Lambda_1 \times \Lambda_2$ is a hyperbolic set of $f_1 \times f_2 : U_1 \times U_2 \rightarrow M_1 \times M_2$.
5. Consider the following map:

$$f : [-1, 1] \times [-1, 1] \rightarrow \mathbb{R}^2, f(x, y) = \left(\frac{1}{2}x, 2y\right).$$

Prove that any C^1 -map g , which is C^1 -close to f , has a unique fixed point.