DYNAMICAL SYSTEMS

HOMEWORK #3

HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/

1. Give an example of a topologically transitive homeomorphism $f : M \to M$ of a compact manifold for which the only minimal set is a fixed point.

2. Let X be a C^1-vector field on a projective plane. Prove that if $\omega(x)$ does not contain a singular point then $\omega(x)$ is a periodic orbit of X.

3. Calculate the topological entropy for the following map $A_\alpha : \mathbb{T}^2 \to \mathbb{T}^2$

$$A_\alpha(x, y) = (x + \alpha, y + x) \pmod{1}.$$

4. Is it true that for any $a \in [0, +\infty)$ there exists a topological Markov chain $\sigma_A : \Sigma_A \to \Sigma_A$ such that $h(\sigma_A) = a$? Is it possible to construct a topological Markov chain σ_A such that $h(\sigma_A) = 1$?

5. Is it true that for any $a \in [0, +\infty)$ there exists a homeomorphism $f : S^2 \to S^2$ such that $h(f) = a$?