DYNAMICAL SYSTEMS

HOMEWORK #2

HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/

1. Give an example of a homeomorphism $f : M \to M$ of a compact metric space such that

$$d(f^n(x), f^n(y)) \to 0 \ \ as \ \ n \to \infty$$

for every pair $x, y \in M$.

2. Prove that an isometry of a compact metric space that contains more than one point can not be topologically mixing.

Definition 0.0.1. A product of dynamical systems $f : M \to M$ and $g : N \to N$ is a map $(f \times g) : M \times N \to M \times N$ such that $(f \times g)(x, y) = (f(x), g(y))$.

3. Is the product of two topologically transitive (minimal, topologically mixing, expansive) systems topologically transitive (minimal, topologically mixing, expansive)?

4. Which of the following maps are topologically mixing? Which of them are expansive?

- Expending maps of S^1;
- Rotations of a circle;
- Translations on $\mathbb{T}^k$;
- Shift $\sigma : \Sigma_N \to \Sigma_N$;
- Smale horseshoe;
- Hyperbolic automorphism of $\mathbb{T}^2$;
- Solenoid.

5. Consider homeomorphisms $f, g : [0, 1] \to [0, 1]$, such that $f(0) = g(0) = 0$, $f(1) = g(1) = 1$, and there are no other fixed points. Prove that f and g are topologically conjugate.