

DYNAMICAL SYSTEMS

HOMEWORK #1

[HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/](http://www.its.caltech.edu/~asgor/dynsys/)

1. Give an example of an invertible contracting map of a compact metric space. Prove that this is the only example.

2. Prove the following. Let $f : M \rightarrow M$ be a C^1 -map of a complete Riemannian manifold to itself. Then f is a contracting map if and only if the norm of the differential is bounded by a constant $\lambda < 1$.

Definition 0.0.1. A topological dynamical system is called minimal if every orbit is dense.

3. Prove that if α is irrational then the rotation R_α is minimal.

4. Prove that the decimal expansion of the number 2^n may begin with any finite number of digits.

5. Show that the set of points with dense orbits under E_2 has Lebesgue measure 1.

6. Is the solenoid pathwise-connected?