

DYNAMICAL SYSTEMS

FINAL EXAM

[HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/](http://www.its.caltech.edu/~asgor/dynsys/)

Consider the map $F_{\alpha,\beta} : \Sigma^2 \times S^1 \rightarrow \Sigma^2 \times S^1$, $\omega \in \Sigma^2$, $\varphi \in S^1$,

$$F_{\alpha,\beta}(\omega, \varphi) = \begin{cases} (\sigma(\omega), R_\alpha(\varphi)), & \text{if } \omega_0 = 0; \\ (\sigma(\omega), R_\beta(\varphi)), & \text{if } \omega_0 = 1. \end{cases}$$

1. For which pairs (α, β) the map $F_{\alpha,\beta}$ is transitive?

Consider the map $g : [-1, 1] \times [-1, 1] \times [-1, 1] \rightarrow \mathbb{R}^3$,

$$g(x, y, z) = \begin{cases} (\frac{x}{100} + \frac{1}{2}, 100y - 10, 100z), & \text{if } y \in [0, 1]; \\ (\frac{x}{100} - \frac{1}{2}, 100y + 10, 100z), & \text{if } y \in [-1, 0]. \end{cases}$$

Let Λ be the set of points such that all iterates of g (positive and negative) are defined.

2. Describe Λ . Find $h_{\text{top}}(g|_\Lambda)$.
3. What is $\dim_H \Lambda$?

Consider the map $f : S^1 \rightarrow S^1$,

$$f(x) = \begin{cases} 4x \pmod{1}, & \text{if } x \in [0, \frac{1}{2}); \\ 6x \pmod{1}, & \text{if } x \in [\frac{1}{2}, 1). \end{cases}$$

Let μ be Lebesgue measure on S^1 .

4. Show that f is an expanding map and preserve μ .
5. Is f topologically conjugate to a linear expanding map?
6. Is $f : (S^1, \mu) \rightarrow (S^1, \mu)$ ergodic? Is it mixing?
7. Calculate $h_{\text{top}}(f)$ and $h_\mu(f)$.