

DYNAMICAL SYSTEMS

MIDTERM

[HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/](http://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/)

Consider expanding maps

$$E_2 : S^1 \rightarrow S^1, E_2(x) = 2x(\text{mod } 1), \quad \text{and}$$

$$E_3 : S^1 \rightarrow S^1, E_3(x) = 3x(\text{mod } 1).$$

Denote by F the product map, $F : \mathbb{T}^2 \rightarrow \mathbb{T}^2$, $F = E_2 \times E_3$.

1. Is F minimal? Transitive? Topologically mixing?
2. Find $P_n(F)$ - the number of periodic points of (not necessarily minimal) period n .
3. Find $h_{top}(F)$.
4. Prove that there exists a semiconjugacy between a topological Bernoulli shift $\sigma : \Sigma_6^+ \rightarrow \Sigma_6^+$ and $F : \mathbb{T}^2 \rightarrow \mathbb{T}^2$.
5. Is it possible to find a point $x \in \mathbb{T}$ such that $\omega(x)$ is homeomorphic to a circle? To a Cantor set? To a product of a circle and a Cantor set?

Hint to Problem 5: use Proposition 1.7.3 from "Introduction to the Modern Theory of Dynamical Systems" by A.Katok and B.Hasselblatt.