DYNAMICAL SYSTEMS

HOMEWORK #4

HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/

1. Let p be a saddle fixed point of a diffeomorphism $f : M \to M$. Let x be a homoclinic point, i.e. $x \in W^s_f(p) \cap W^u_f(p)$. Prove that x is non-wandering point but not recurrent.

Set $D = \{(x, y) | \, x^2 + y^2 < 1\}$ and consider a map $f : D \times S^1 \to D \times S^1$, $f(x, y, \phi) = (\frac{x}{10}, \frac{y}{10}, \phi)$. Let $g : D \times S^1 \to D \times S^1$ be C^1-close to f.

2. Prove that maximal attractor of g is homeomorphic to a circle.

3. Prove that maximal attractor of g is a smooth closed curve.

Set $B = \{(x, y) | \, |x| < 1, |y| < 1\}$ and consider a map $h : B \times S^1 \to B \times S^1$, $h(x, y, \phi) = (10x, \frac{y}{10}, \phi)$. Let $k : B \times S^1 \to B \times S^1$ be C^1-close to h.

4. Prove that $\cap_{n \in \mathbb{Z}} k^n(B \times S^1)$ is homeomorphic to a circle.

5. Prove that $\cap_{n \in \mathbb{Z}} k^n(B \times S^1)$ is a smooth closed curve.