DYNAMICAL SYSTEMS

HOMEWORK #2

HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/

1. Show that the topological entropy of an isometry is zero.

2. Let X, Y be compact metric spaces, $f : X \to X$ and $g : Y \to Y$ be homeomorphisms. Assume that there exists a continuous map (onto) $h : X \to Y$ such that $h \circ f = g \circ h$. Show that $h_{top}(f) \geq h_{top}(g)$.

3. Denote by $P_n(\sigma)$ the number of periodic points of (not necessarily minimal) period n for a topological Bernoulli shift $\sigma : \Sigma_m \to \Sigma_m$. Show that

$$\lim_{n\to\infty} \frac{1}{n} \log P_n(\sigma) = h_{top}(\sigma).$$

Check if this is true for an expending endomorphism $E_m : S^1 \to S^1$.

4. The same question as in the previous problem, for a hyperbolic automorphism of a torus, $A : \mathbb{T}^2 \to \mathbb{T}^2$, $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$.

5. Construct an example of a topologically transitive diffeomorphism of a two dimensional torus for which the only minimal set is a fixed point.