

DYNAMICAL SYSTEMS

HOMEWORK #1

[HTTP://WWW.ITS.CALTECH.EDU/~ASGOR/DYNSYS/](http://www.its.caltech.edu/~asgor/dynsys/)

1. Which of the following maps are topologically transitive (minimal, topologically mixing)?

- identity map on a circle;
- irrational rotation of a circle;
- expanding endomorphisms of a circle;
- hyperbolic automorphism of a torus;
- topological Bernoulli shift.

Definition 0.0.1. A product of a dynamical systems $f_1 : M_1 \rightarrow M_1$ and $f_2 : M_2 \rightarrow M_2$ is a map $F : M_1 \times M_2 \rightarrow M_1 \times M_2$, $F(x, y) = (f_1(x), f_2(y))$, $(x, y) \in M_1 \times M_2$.

2. Is the product of two topologically transitive (minimal, topologically mixing) systems topologically transitive (minimal, topologically mixing)?

3. Let α be irrational and $f : \mathbb{T}^2 \rightarrow \mathbb{T}^2$ be the homeomorphism of the 2-torus given by $f(x, y) = (x + \alpha, x + y)$. Prove that f is topologically transitive.

4. Prove that f from the Problem 3 is minimal.

5. Is it possible to construct a transitive homeomorphism of S^2 that has exactly one dense orbit? Only countable number of points with dense orbits?