Complex Dynamics

Final Exam

due Wednesday, June 1, 2005.

1. Produce some computer graphics of the Julia set of the following maps:

a) $z \mapsto z^2 + c$ for several different $c \in \mathbb{C}$;

b) $z \mapsto z^2 \frac{2z^2-1}{2-z^3}$; $z \mapsto e^{i\pi\sqrt{5}} z^2 \frac{z-4}{4z-1}$;

c) $z \mapsto z - z^2 + \frac{z^3}{z_0}$, $z_0 = -0.41 + 0.54i$;

d) $z \mapsto z - z^2$, $z \mapsto z - z^4$, $z \mapsto i(z + z^2)$;

e) $z \mapsto e^{i\pi\sqrt{5}z} + iz^2$;

f) $z \mapsto \frac{-iz^2}{1+z^2}$;

g) $z \mapsto \frac{1}{10} e^z$.

What properties of Fatou and Julia sets did you use to generate these pictures? Can you explain (or guess) what kind of Fatou components one can see there?

2. Show that the two polynomials $z + z^2$ and $z + z^2 + z^3$ are topologically, but not analytically, nor formally, conjugate near the origin.

3. Let $R(z) = \frac{z}{2-z^2}$. Show that $F(R)$ has an attracting component of infinite connectivity.

4. Prove that if rational functions R and S are permutable ($RS \equiv SR$), then any periodic Fatou component of R is also a periodic Fatou component of S, and vice versa.

5. Prove that the Julia set of a Blaschke product is either S^1 or a Cantor set contained in S^1.