Math 4, Introduction to Mathematical Chaos

Spring 2006

HOMEWORK #7

Due Thursday May 25, 2:30pm, 2006

(1) Sketch the phase curves of the vector field

$$\begin{cases} \dot{x} = y \\ \dot{y} = x \end{cases}$$

(2) Sketch the phase curves of the vector field

$$\begin{cases} \dot{x} = \ \ y + (x^2+y^2)(1-x^2-y^2) \\ \dot{y} = -x + (x^2+y^2)(1-x^2-y^2) \end{cases}$$

What is $\omega(p)$, if $p = (\frac{1}{2}, \frac{1}{2})$? If $p = (0,0)$? If $p = (10,10)$?

(3) Sketch the set

$$A = \{z \in \mathbb{C} \mid |z| \leq 1 \text{ or } |\sqrt{2}z - 1 - i| \leq 1 \text{ or } |\sqrt{2}z - 1 + i| \leq 1 \text{ or } |\sqrt{2}z + 1 - i| \leq 1 \text{ or } |\sqrt{2}z + 1 + i| \leq 1\}.$$

(4) Find all the fixed points for each of the following complex functions and determine whether they are attracting, repelling, or neutral.
   a) $Q_2(z) = z^2 + 2$
   b) $F(z) = z^2 + z + 1$
   c) $F(z) = iz^2$
   d) $F(z) = -\frac{1}{z}$
   e) $F(z) = 2z(i - z)$
   f) $F(z) = -iz(1 - z)/2$
   g) $F(z) = z^3 + (i + 1)z$

(5) Show that $z_0 = e^{2\pi i/7}$ is a periodic point of period 3 for the map $Q_0(z) = z^2$. Is this periodic orbit attracting, repelling, or neutral?