

Ma 4, Introduction to Mathematical Chaos

Spring 2006

HOMEWORK # 6

Due Tuesday May 16, 2:30pm, 2006.

Consider the logistic family of functions

$$F_\lambda(x) = \lambda x(1 - x).$$

1) (10) For which values of λ does F_λ have an attracting fixed point at $x = 0$? For which values of λ does F_λ have a nonzero attracting fixed point?

2) (10) Describe the bifurcation that occurs when $\lambda = 1$. Sketch the bifurcation diagram near $\lambda = 1$.

3) (10) Describe the bifurcation that occurs when $\lambda = 3$. Sketch the bifurcation diagram near $\lambda = 3$.

4) (10) Describe the bifurcation that occurs when $\lambda = -1$. Sketch the bifurcation diagram near $\lambda = -1$.

5) (10) Compute the Schwarzian Derivative of the function $M(x) = \frac{ax+b}{cx+d}$. Prove that $S(M \circ f) = Sf$ for every smooth function f .