

Ma 4, Introduction to Mathematical Chaos

Spring 2006

HOMEWORK # 5

Due Tuesday May 9, 2:30pm, 2006.

Let Γ be a set of points $x \in [0, 1]$ that have a decimal representation $x = 0.x_1x_2 \dots x_n \dots$ that contains only digits 0 and 9.

- 1) (10) Prove that Γ is a Cantor set. What is $\dim_H \Gamma$?
- 2) (10) Prove that $\Gamma - \Gamma$ is a Cantor set. What is $\dim_H(\Gamma - \Gamma)$?
- 3) (10) Consider the following contractions S_1, S_2, S_3 on \mathbb{R}^2 :

$$S_1 \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}, \quad S_2 \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x-1 \\ y \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad S_3 \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x \\ y-1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Describe the unique compact set F such that $F = S_1(F) \cup S_2(F) \cup S_3(F)$ and find its Hausdorff dimension.

- 4) (10) Can a fractal that is totally disconnected have a Hausdorff dimension larger than 1?
- 5) (10) Compute the area of the Koch snowflake. What is the Hausdorff dimension of its boundary?