Ma 4, Introduction to Mathematical Chaos

Spring 2006

HOMEWORK # 2

Due Tuesday April 11, 2:30pm, 2006.

1) (10) Give an example of a continuous map f of a compact metric space X such that

$$\frac{1}{n}\sum_{k=0}^{n-1}\phi(f^k(x))$$

converges uniformly (in x) for every continuous function ϕ, but f is not uniquely ergodic.

2) (10) Prove that the homeomorphism $f : S^1 \to S^1$, $f(x) = x + \frac{1}{100}\sin \pi x$ is uniquely ergodic.

3) (10) Consider a map $f : \mathbb{T}^3 \to \mathbb{T}^3$, $f(x, y, z) = (x + \frac{\sqrt{2}}{10}, y + \frac{\sqrt{3}}{10}, z + \frac{\sqrt{2}}{5}) \pmod 1$. Is it minimal? What is the closure of the orbit of zero?

4) Let $f_\omega : S^1 \to S^1$ be a rotation $f_\omega(x) = x + \omega \pmod 1$. Find the rotation number $\rho(f_\omega)$.

5) (10) Prove that every orientation reversing homeomorphism of a circle has exactly two fixed points.