Ma 4, Introduction to Mathematical Chaos

Spring 2006

HOMEWORK # 1

Due Tuesday April 4, 2:30pm, 2006.

1) (10) A map f of a metric space is said to be *eventually contracting* if there are constants $C > 0$, $\lambda \in (0, 1)$ such that

$$d(f^n(x), f^n(y)) \leq C\lambda^n d(x, y)$$

for all $n \in \mathbb{N}$. Prove the analog of the Contraction Mapping Principle for eventually contracting maps in complete metric spaces.

2) (10) Let $f : [a, b] \to [a, b]$ be a homeomorphism such that $f(a) = b$ and $f(b) = a$. Prove that f has exactly one fixed point, and all other periodic points have prime period two.

3) (10) Prove that $\sup_{n\in\mathbb{Z}}(\sin n) = 1$.

4) a) (7) Prove that for any closed set $K \subset \mathbb{R}$ there is a continuous strictly increasing map $f : \mathbb{R} \to \mathbb{R}$ such that $\mathrm{Fix}(f) = K$, where $\mathrm{Fix}(f)$ is a set of fixed points of f.

b)* (3) Prove that for any closed set $K \subset \mathbb{R}$ there is a continuously differentiable strictly increasing map $f : \mathbb{R} \to \mathbb{R}$ such that $\mathrm{Fix}(f) = K$.

5)* (10) Prove that if a continuous map $f : [a, b] \to [a, b]$ has a periodic point of prime period greater than 1, then it has a periodic point of prime period 2. (Do not use Sharkovsky Theorem! We did not prove it!)