ANTON GORODETSKI

253-37 Mathematics, Caltech, Pasadena CA 91125 *(626) 395-4350, (626) 403-0386; asgor@caltech.edu*
www.its.caltech.edu/∼asgor

Description of research interests and results

My research interests are mostly in the field of smooth dynamical systems, including

- partially hyperbolic and non-uniformly hyperbolic dynamics;
- structure of Newhouse domains;
- nonlocal and homoclinic bifurcations;
- applications to problems in other fields of mathematics (including celestial mechanics).

Every field of mathematics begins with the definition of objects and an equivalence relation between objects. The properties to be studied are those that are preserved by the equivalence relation. In Dynamical Systems (we will consider here only smooth invertible discrete dynamical systems) the objects are diffeomorphisms, and two diffeomorphisms are equivalent (or *topologically conjugate*) if they coincide up to a continuous change of coordinates.

The first internal problem of any field is a problem of classification. In the space $\mathrm{Diff}^r(M)$ this problem is very far from being solved if the manifold M is more complicated than a circle. There is a part of $\mathrm{Diff}^r(M)$ that consists of diffeomorphisms with dynamical behavior that does not change after a small perturbation. Those are *hyperbolic* dynamical systems, and a very complete classification and understanding is achieved there. Almost every question regarding hyperbolic dynamics can be answered in a short and precise way, see [1] as an example. But the same questions become much more delicate in general setting, see [2].

Amazingly enough, if $\dim M \geq 2$, there are domains in $\mathrm{Diff}^r(M)$ in the complement to the set of hyperbolic diffeomorphisms. One of the first examples was constructed by S.Newhouse, and now we use the term "Newhouse domains".

The most surprising feature of Newhouse domains is that diffeomorphisms with infinite number of attractors are topologically generic there. J.Palis conjectured that from the measure theoretical point of view a typical diffeomorphism has only finite number of attractors (that would give a hope to provide some kind of classification). In [7] we give a partial solution to Palis' conjecture, namely, we show that only a finite number of attracting periodic orbits with bounded cyclicity (a natural condition on combinatorial complexity) can coexist for typical diffeomorphism. The machinery of perturbations by Newton Interpolation Polynomials is used for the proof, see [8] for a separate discussion of the method.

Another approach that provides some description of non-hyperbolic part of $\mathrm{Diff}^r(M)$ is to consider systems that are in some sense related or close to hyperbolic. There are three major ways to do that: using partial hyperbolicity, non-uniform hyperbolicity (Pesin theory), or bifurcations.

The easiest way to get a partially hyperbolic system is to cosider a product $f \times \mathrm{id}_{M_2}$, where $f : M_1 \to M_1$ is hyperbolic. In [6] the Hölder structure in the space of central leaves of an unfolding of $f \times \mathrm{id}_{M_2}$ is established. This gives the way to explore small perturbations of $f \times \mathrm{id}_{M_2}$, and that was done in details in [3], [4], [5].

Non-uniformly hyperbolic map is defined as a diffeomorphism with an invariant measure without zero Lyapunov exponents. Is it true that all invariant measures of a typical diffeomorphism have this property? The answer is "No", and in [5] we have made the first step towards this answer. Moreover, in [9] we suggest that the existence of an invariant measure with zero Lyapunov exponent could be a "positive" description of a non-hyperbolic dynamical system, and justify this conjecture in some cases.

The description of non-hyperbolic dynamics is not just an intrinsic problem in Dynamical Systems. For example, using the most recent results about homoclinic bifurcations, renormalization, and non-integrable Hamiltonian dynamics, we show in [10] that in some restricted three body problems (Sitnikov problem, planar circular three body problem) the set of oscillatory motions has full Hausdorff dimension for many values of a parameter. This result is related to the famous Kolmogorov conjecture (that the set of oscillatory motions has zero measure).

References

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