Ma 4, Introduction to Mathematical Chaos

Spring 2007

MIDTERM EXAM

Due Wednesday May 2, 1:00 pm, 2007.

You may use your notes for this course, your solutions to course homework, and the textbooks. No collaboration or a direct assistance of other people.

All problems are weighted equally.

Time limit 3 hours. If you need more time, you should mark which part of the work was done after 3 hours; you will be given a partial credit for that work.

1) Let $f : [0,1] \to [0,1]$ and $g : [0,1] \to [0,1]$ be continuous maps. Consider the map

$$F : [0,1] \times [0,1] \to [0,1] \times [0,1], \quad F = f \times g$$

(i.e. $F(x,y) = (f(x), g(y))$). Show that if F has a periodic point of prime period 3 then it has periodic points of all periods. Does a general Sharkovskij Theorem hold for the map F?

2) Find $\limsup_{n\to\infty} n\left(\sin^2\left(\frac{n^2+1}{2n}\right) - \sin^2\left(\frac{n^2-1}{2n}\right)\right)$.

3) What is the rotation number of the map

$$f : S^1 \to S^1, \quad f(x) = x + \frac{1}{4} + \frac{1}{100}\sin 4\pi x \quad ?$$

4) Consider the map of the torus $F_L : \mathbb{T}^2 \to \mathbb{T}^2$ defined by the matrix $L = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ (i.e. $F_L(x,y) = (2x+3y, x+2y) (\text{mod } 1)$). Find the number of periodic points of the map F_L of each period $n \in \mathbb{N}$.

5) Find the number of periodic points of period n for the topological Markov chain determined by the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$