Ma 4, Introduction to Mathematical Chaos

Spring 2007

HOMEWORK # 5

Due Wednesday May 9, 1:00pm, 2007.

1) Assume that a compact (bounded and closed) set $F \subset \mathbb{R}^n$ is represented as a finite union $F = F_1 \cup F_2 \cup \ldots \cup F_m$. Prove that

$$\dim_B F = \max_{1 \le i \le m} \dim_B F_i.$$

2) Assume that a compact (bounded and closed) set $F \subset \mathbb{R}^n$ is represented as a countable union $F = \cup_{i=1}^{\infty} F_i$. Prove that

$$\dim_H F = \sup_{i \in \mathbb{N}} \dim_H F_i.$$

3) Consider a set $F = \{0, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\} \subset \mathbb{R}^1$. Find $\dim_B F$ and $\dim_H F$.

4) Consider the following similarities $S_1, S_2, S_3 : \mathbb{R}^2 \to \mathbb{R}^2$:

$$S_1(x, y) = \left(\frac{x}{2}, \frac{y}{2}\right), \quad S_2(x, y) = \left(\frac{x}{2} + \frac{1}{2}, \frac{y}{2}\right), \quad S_3(x, y) = \left(\frac{x}{2}, \frac{y}{2} + \frac{1}{2}\right).$$

Sketch the set F such that $F = S_1(F) \cup S_2(F) \cup S_3(F)$. What is $\dim_H F$?

5) Consider $S_1, S_2 : \mathbb{R}^1 \to \mathbb{R}^1$,

$$S_1(x) = \frac{x}{2}, \quad S_2(x) = \frac{x}{4} + \frac{3}{4}.$$

Let F be the unique compact set such that $F = S_1(F) \cup S_2(F)$. What is $\dim_H F$?