Ma 4, Introduction to Mathematical Chaos

Spring 2007

HOMEWORK # 4

Due Wednesday April 25, 1:00pm, 2007.

1) A map f of a metric space is said to be *eventually contracting* if there are constants $C > 0$, $\lambda \in (0, 1)$ such that

$$d(f^n(x), f^n(y)) \leq C\lambda^n d(x, y)$$

for all $n \in \mathbb{N}$. Prove the analog of the Contraction Mapping Principle for eventually contracting maps in complete metric spaces.

2) Prove that (Σ_m, d_{Σ_m}) is a complete metric space.

3) Consider the following map of a torus $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$:

$$f : \mathbb{T}^2 \to \mathbb{T}^2, \quad f(x, y) = (2x, 3y) \,(\mathrm{mod}\, 1).$$

Prove that f is topologically mixing and periodic points of f are dense in $\mathbb{T}^2$.

4) Find $\#Per_n(f)$ (the number of periodic points of period n) for the map $f : \mathbb{T}^2 \to \mathbb{T}^2$ in Problem 3.

5) Find the number of periodic points of period n for the topological Markov chain determined by the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$