

Ma 4, Introduction to Mathematical Chaos

Spring 2007

HOMEWORK # 3

Due Wednesday April 18, 1:00pm, 2007.

1) Let R_ω be a rotation of S^1 , $R_\omega(x) = x + \omega \pmod{1}$. Which of the following rotations of S^1

$$R_{\frac{\sqrt{2}}{10}}, R_{\frac{2\sqrt{2}}{10}}, R_{\frac{\sqrt{3}}{10}}, R_{\frac{1}{10}}, R_{\frac{9}{10}}, R_{\frac{\sqrt{2}+\sqrt{3}}{10}}$$

are topologically conjugate?

2) For which values of a the function $F_a(x) := x + a \sin 2\pi x$ defines a lift of a circle homeomorphism? Determine the rotation number for those homeomorphisms.

3) Prove that $E_2 : S^1 \rightarrow S^1$ and $E_3 : S^1 \rightarrow S^1$ are not topologically conjugated.

4) Show that $E_2 : S^1 \rightarrow S^1$ has an invariant Cantor set.

5) Prove that for the expanding map $E_m : S^1 \rightarrow S^1, m \geq 2, E_2(x) = 2x \pmod{1}$, rational points are preimages of periodic points.