Ma 4, Introduction to Mathematical Chaos

Spring 2007

HOMEWORK # 2

Due Wednesday April 11, 1:00pm, 2007.

1) Prove that the homeomorphism $f : S^1 \to S^1$, $f(x) = x + \frac{1}{100}\sin \pi x$ is uniquely ergodic.

2) Prove that every orientation reversing homeomorphism of a circle has exactly two fixed points.

3) Sketch the trajectories of the solutions $(x_1(t), x_2(t))$ of the system

$$\begin{cases} \ddot{x}_1 = -\mu^2 x_1 \\ \ddot{x}_2 = -\omega^2 x_2 \end{cases}$$

a) for $\mu = \omega = 1$;

b) for $\mu = 1$ and $\omega = 2$;

c) for $\mu = 1$ and $\omega = 3$;

d) for $\mu = 2$ and $\omega = 3$.

4) Consider the map $f : \mathbb{T}^3 \to \mathbb{T}^3$, $f(x, y, z) = (x + \frac{\sqrt{2}}{10}, y + \frac{\sqrt{3}}{10}, z + \frac{\sqrt{2}}{5}) (\text{mod } 1)$. Is it minimal? What is the closure of the orbit of zero?

5) Consider the following homeomorphism of the circle:

$$f(x) = \begin{cases} \frac{1}{4} + 2x \,(\text{mod}\, 1), & \text{if } x \in [0, \frac{1}{4}]; \\ \frac{5}{8} + \frac{x}{2} \,(\text{mod}\, 1), & \text{if } x \in [\frac{1}{4}, \frac{3}{4}]; \\ x + \frac{1}{4} \,(\text{mod}\, 1), & \text{if } x \in [\frac{3}{4}, 1]. \end{cases}$$

What is the rotation number of f?