Ma 4, Introduction to Mathematical Chaos

Spring 2007

HOMEWORK # 1

Due Wednesday April 4, 1:00pm, 2007.

1) Let $f : [a, b] \to [a, b]$ be a homeomorphism (i.e. f is a continuous bijection and f^{-1} is also continuous) such that $f(a) = b$ and $f(b) = a$. Prove that f has exactly one fixed point.

2) Assume that all prime periods of periodic orbits of a continuous map $f : [0, 1] \to [0, 1]$ are uniformly bounded (i.e. there exists $N \in \mathbb{N}$ such that the prime period of every periodic orbit of f is smaller than N). What can you say about periods of periodic orbits of f? For example, can f have a periodic orbit of period 2007? Of period 2048?

3) Prove that $\sup_{n\in\mathbb{Z}}(\sin n) = 1$.

4) Assume that $\alpha \notin \mathbb{Q}$ and $\beta \notin \mathbb{Q}$. Is it true that the map $F : \mathbb{T}^2 \to \mathbb{T}^2$ of the two-dimensional torus $\mathbb{T}^2 = S^1 \times S^1$, $F(x, y) = (R_\alpha(x), R_\beta(y))$, is minimal? Prove or give a counterexample.

5) Find the asymptotic distribution of the first digits of the sequence $a_n = 5^n + 2$.