Ma 147, Hamiltonian Dynamics

Spring 2007

HOMEWORK # 7

Due Wednesday May 23, 2:30pm, 2007.

1) Show that for any values of the masses, the only non-coplanar central configuration for the four-body problem is the regular tetrahedron configuration.

2) Introduce a symplectic structure on the space of oriented lines in $\mathbb{R}^n$. Prove that this space is diffeomorphic to the tangent bundle of the unit sphere in $\mathbb{R}^n$.

3) Consider a billiard inside of a unit sphere in $\mathbb{R}^3$. Describe the phase space of a billiard map, dynamics, invariant foliations.

4) Consider a convex billiard table in $\mathbb{R}^2$. Prove that the volume of the phase space of a billiard map is equal to $2L$, where L is the length of the boundary of a billiard table.

5) Consider the "mushroom" billiard table (see the picture). Consider the set A of orbits that never leave the round top of the "mushroom" (never enter its steam). Show that A is an invariant subset of the phase space with positive area and that the billiard map is completely integrable in A.