Ma 147, Hamiltonian Dynamics

Spring 2007

HOMEWORK # 6

Due Wednesday May 16, 2:30pm, 2007.

1) Consider the one-dimensional Kepler problem, $H(p,q) = \frac{p^2}{2} - \frac{1}{|q|}$. Plot the level curves of H in the (p,q)-plane. Explain what motions of a particle on the line corresponds to each type of curve.

2) Consider the one-dimensional Kepler problem for a particle with a negative mass, $H(p,q) = \frac{p^2}{2} + \frac{1}{|q|}$. Plot the level curves of H in the (p,q)-plane. Describe the properties of a motion of a particle in this case.

3) The inverse s force law ($s \neq 1$) defines the potential $U = -\frac{1}{s-1}\sum_{1\leq j<i\leq N} \frac{m_j m_i}{\rho_{ij}^{s-1}}$, $\rho_{ij} = \|q_i - q_j\|$. Notice that for $s = 2$ we get the Newtonian potential. Suggest a definition of a central configuration for this case and show that a central configuration corresponds to the critical point

$$\nabla I U^{\frac{2}{s-1}} = 0,$$

where I is a moment of inertia.

4) Consider the restricted planar circular three body problem. Calculate the positions of the five libration points in the case when primaries have equal masses, $m_1 = m_2 = \frac{1}{2}$.

5) Consider the three body problem on the plane when all three particles have the same mass, $m_1 = m_2 = m_3 = \frac{1}{3}$. Find all the solutions for which all three particles are moving uniformly in circles (including the case of a "circle of zero radius"), and with the same angular velocities.