Ma 147, Hamiltonian Dynamics

Spring 2007

HOMEWORK # 5

Due Wednesday May 9, 2:30pm, 2007.

1) Show that if you scale time by $t \to \mu t$, then you should scale the Hamiltonian by $H \to \mu^{-1} H$.

2) Scale the Hamiltonian on the N-body problem in rotating coordinates so that $\omega = 1$.

3) What is the Hamiltonian of Kepler problem in polar coordinates?

4) Consider a transformation of positions given by $Q = f(q) = A^T q$, where A is a nonsingular matrix. Show that a Mathieu transformation in this case is a linear symplectic transformation. What is its matrix?

5) Consider the three body problem on the plane in fixed Jacobi coordinates with center mass at the origin and linear momentum zero. Introduce polar coordinates for x_1 and x_2 (so called Jacobi-polar coordinates). That is, let

$$x_1 = (r_1 \cos\theta_1, r_1 \sin\theta_1), \quad x_2 = (r_2 \cos\theta_2, r_2 \sin\theta_2),$$
$$y_1 = (R_1 \cos\theta_1 - (\Theta_1/r_1)\sin\theta_1, R_1 \sin\theta_1 + (\Theta_1/r_1)\cos\theta_1),$$
$$y_2 = (R_2 \cos\theta_2 - (\Theta_2/r_2)\sin\theta_2, R_2 \sin\theta_2 + (\Theta_2/r_2)\cos\theta_2).$$

Write down the Hamiltonian of the three body problem in these Jacobi-polar coordinates.