

Ma 147, Hamiltonian Dynamics

Spring 2007

HOMework # 4

Due Wednesday April 25, 2:30pm, 2007.

1) Show that the 1-form dH is an integral invariant of the phase flow with hamiltonian function H .

2) Show that in \mathbb{R}^n (equipped with the standard symplectic structure) every one-parameter group of diffeomorphisms preserving the symplectic form is a hamiltonian flow.

3) Show that the Poisson bracket of two locally hamiltonian fields is a hamiltonian field. *Hint:* Consider $H = \omega^2(v_1, v_2)$.

4) Let F_1, F_2, H be smooth functions on M^{2n} . Prove that $(F_1 F_2, H) = F_1(F_2, H) + F_2(F_1, H)$.

5) Prove that any two commuting vector fields on S^2 must have a common singular point. Is it true for commuting vector fields on \mathbb{T}^2 ?