Ma 147, Hamiltonian Dynamics

Spring 2007

HOMEWORK # 3

Due Wednesday April 18, 2:30pm, 2007.

1) Show that a definition of a Hamiltonian system in $\mathbb{R}^{2n}$ is a partial case of a general definition (using a symplectic structure). What the symplectic structure in $\mathbb{R}^{2n}$ was implicitly used in the first definition?

2) Let $H(t,z) = \frac{1}{2}z^T S(t) z$ and $\zeta(t)$ be a solution of the linear system with Hamiltonian H. Show that

$$\frac{d}{dt}H = \frac{\partial}{\partial t}H,$$

i.e.,

$$\frac{d}{dt}H(t,\zeta(t)) = \frac{\partial}{\partial t}H(t,\zeta(t)).$$

3) Show that every differential 1-form on the line is a differential of some function; every differential 2-form on a plane is a differential of some 1-form.

4) Find differential 1-forms on the circle and the plane which are not a differential of any function.

5) Suppose that we are given two coordinate systems on $\mathbb{R}^3 : x_1, x_2, x_3$ and y_1, y_2, y_3. Let ω be a 2-form on $\mathbb{R}^3$. Then ω can be written in the system of x-coordinates as

$$\omega = X_1 dx_2 \wedge dx_3 + X_2 dx_3 \wedge dx_1 + X_3 dx_1 \wedge dx_2,$$

where X_1, X_2, X_3 are functions of x_1, x_2, x_3, and in the system of y-coordinates as

$$\omega = Y_1 dy_2 \wedge dy_3 + Y_2 dy_3 \wedge dy_1 + Y_3 dy_1 \wedge dy_2,$$

where Y_1, Y_2, Y_3 are functions of y_1, y_2, y_3.

Given the form written in the x-coordinates (i.e., the X_i) and the change of variables $x_i = x_i(y_1, y_2, y_3), i = 1, 2, 3$, write the form in y-coordinates, i.e., find Y_1, Y_2, Y_3.