

Ma 147, Hamiltonian Dynamics

Spring 2007

HOMEWORK # 2

*Due Wednesday April 11, 2:30pm, 2007.*

1) Suppose that a particle in  $\mathbb{R}^3$  in a central force field  $F(\vec{r}) = -f(|\vec{r}|)e_{\vec{r}}$  starts from the rest, i.e. that at some point  $t = 0$  we have  $\dot{\vec{r}} = 0$ . Then the motion is linear. Suppose, moreover, that  $f(r)$  is positive for  $0 < r < \infty$ . Prove that the particle must collide with the center of force in a finite time  $t_0$ .

2) In the preceding problem, can you tell where the particle will be at each moment of time between 0 and  $t_0$ ? Give an explicit answer for  $f(r) = \mu r^{-3}$  and for  $f(r) = \mu r^{-2}$ .

3) Three equal masses start at rest from the vertices of an equilateral triangle (the law of attraction is newtonian). Find out when they collide.

4) Consider the motion of a particle in a central force field  $F(\vec{r}) = -f(|\vec{r}|)e_{\vec{r}}$ . Show that for the direct first power law,  $f(r) = \mu r$ , the orbits are ellipses with center (*not* focus) at the origin.

5) Consider  $n$ -body problem with the linear law of attraction (rather than the inverse square law), i.e. attraction between two bodies is proportional to the mass of each body and to the distance between them. Describe the motion.