

Ma 147, Hamiltonian Dynamics

Spring 2007

HOMEWORK # 1

Due Wednesday April 4, 2:30pm, 2007.

- 1) Draw the phase curves for the "equation of a pendulum on a rotation axis"

$$\ddot{x} = -\sin x + M$$

for different values of M .

- 2) Sketch the phase curves of the system with one degree of freedom with potential energies $U(x) = \pm x \sin x$, $U(x) = \pm \frac{\sin x}{x}$, $U(x) = \pm \sin x^2$.

- 3) Sketch the phase curves of Newton's equation with the force fields $F(x) = \pm x \sin x$, $F(x) = \pm \frac{\sin x}{x}$, $F(x) = \pm \sin x^2$.

- 4) For the system with one degree of freedom find the tangents to the separatrices of a saddle singular point $(\xi, 0)$ ($U''(\xi) < 0$).

- 5) For the system with one degree of freedom let E_0 be the value of the potential at a minimum point ξ . Find the period $T_0 = \lim_{E \rightarrow E_0} T(E)$ of small oscillations in a neighborhood of the point $(\xi, 0)$.