Ma 147, Hamiltonian Dynamics

Spring 2007

FINAL

Due Wednesday May 30, 2:30 pm, 2007.

You may use your notes for this course, your solutions to course homework, the textbooks, and all other the sources you may need.

This should be your own work. No collaboration or a direct assistance of other people.

No time limits.

1) Consider a hamiltonian system with two degrees of freedom with Hamiltonian

$$H(q_1, q_2, p_1, p_2) = ap_1^2 + bp_2^2 + cq_1^2 + dq_2^2$$

Find all values of a, b, c, and d for which the origin is a Lyapunov stable equilibrium point.

2) Show that for all values of a, b, c, and d the hamiltonian system from the previous problem is completely integrable.

3) Consider a motion of a countable collection of particles of masses $m_1, m_2, m_3, \ldots$, $\sum_{i\in\mathbb{N}} m_i = 1$, in $\mathbb{R}^3$, under the Newtonian law of mutual gravitation. Try to derive a theory for this problem (let us call it $\mathbb{N}$-body problem). How to write the equations of motion? Are there any first integrals? What can you say about central configuration? Try to state as many questions as you can, and try to answer at least some of them. Introduce some additional assumptions if needed, consider some partial cases (e.g a collinear case) in more details. Try to formulate some conjectures. Research!