Classical Analysis – Math 108b

Midterm exam
due 5 pm Tuesday, February 8, 2005.

You can use your notes for this course, your solutions to course homework, and the textbooks. No collaboration or a direct assistance of other people!

All problems are weighted equally.

1. Let $(r(t), \theta(t))$ for t in $[a, b]$ describe the polar coordinates of a curve in the plane. Assuming $r(t)$ and $\theta(t)$ are C^1 functions, find a formula for the length of the curve.

Find the length of the curve $(r(t), \theta(t)) = (2 \sin \pi t, \pi t)$, $t \in [0, 1]$.

2. Suppose that ϕ is a strictly increasing continuous function from $[c, d]$ onto $[a, b]$. Given $f \in R_\alpha[a, b]$, show that $g = f \circ \phi \in R_\beta[c, d]$, where $\beta = \alpha \circ \phi$. Moreover, $\int_c^d g d\beta = \int_a^b f d\alpha$.

3. Given two measurable sets $E, F \subset [0, 1]$, define $d(E, F) = m(E \triangle F)$ (here $E \triangle F = (F \cup E) \backslash (F \cap E))$, and $E \sim F$ if $d(E, F) = 0$. Prove that $\sim$ is an equivalence relation on a space of measurable subsets of $[0, 1]$, and d induces a metric on the set of equivalence classes.

4. Let $f : [0, 1] \to \mathbb{R}$ be a measurable function. For t real define $\phi(t) = m\{x \,|\, f(x) < t\}$. Show that ϕ is monotone nondecreasing, continuous on the left, $\lim_{t \to +\infty} \phi(t) = 1$. (The function $\phi : \mathbb{R} \to \mathbb{R}$ is called the *distribution function*)

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Show that f' is (Lebesgue) measurable.

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Ma108b, Classical Analysis
due Wednesday, February 9, 5 p.m.

Please, spend not more than 3 hours.